Structure from motion

Digital Visual Effects

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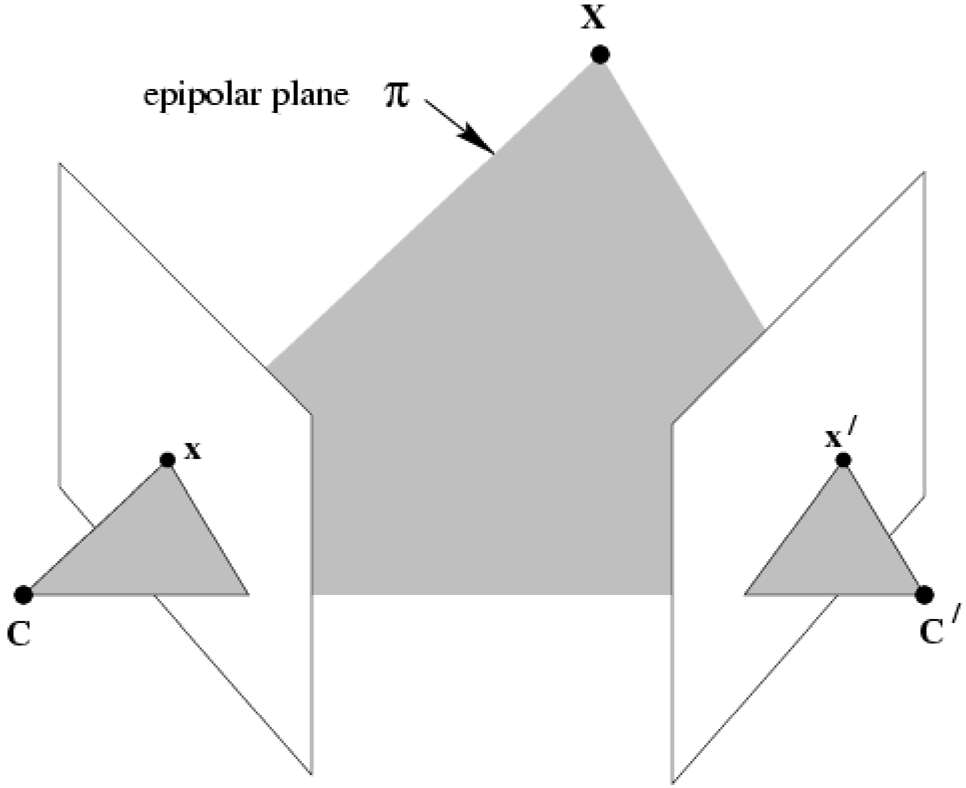
Outline

- Epipolar geometry and fundamental matrix
- Structure from motion
- Factorization method
- Bundle adjustment
- Applications

Epipolar geometry & fundamental matrix

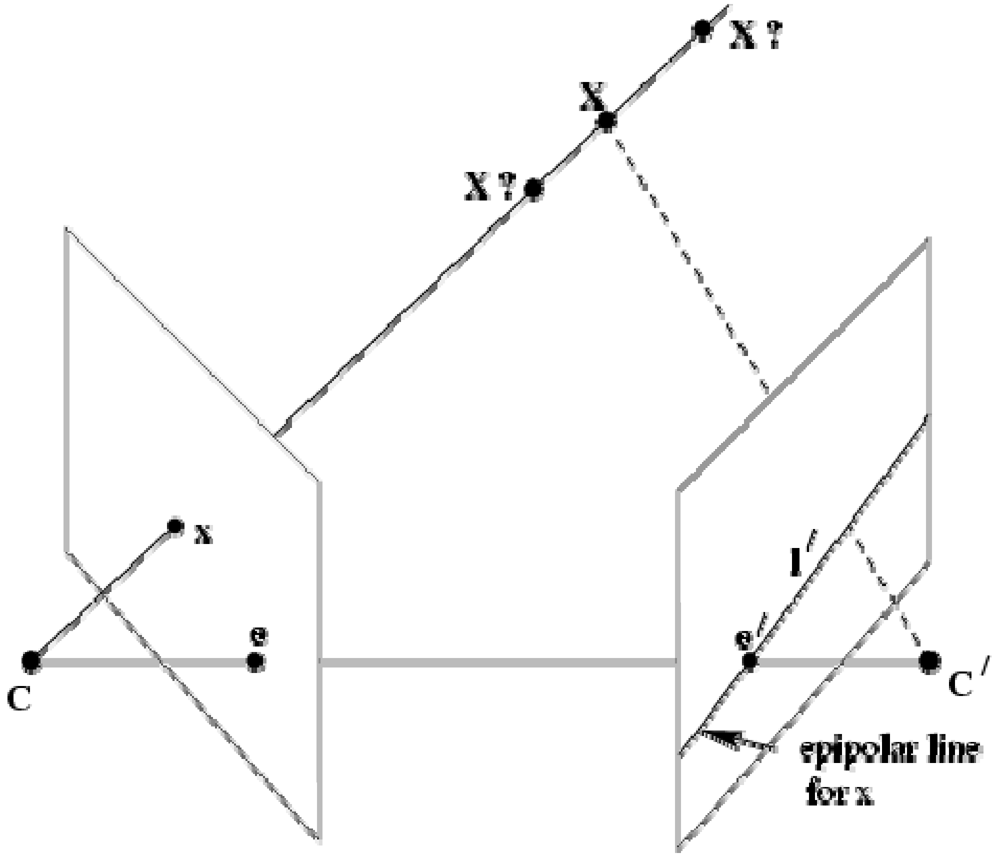
The epipolar geometry

epipolar geometry demo



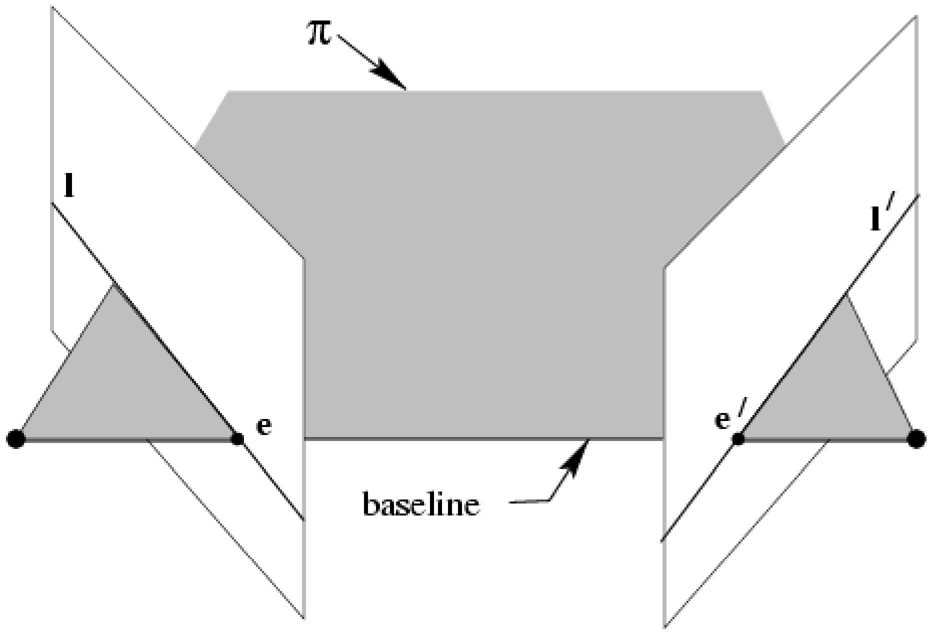
C,C',x,x' and X are coplanar

The epipolar geometry



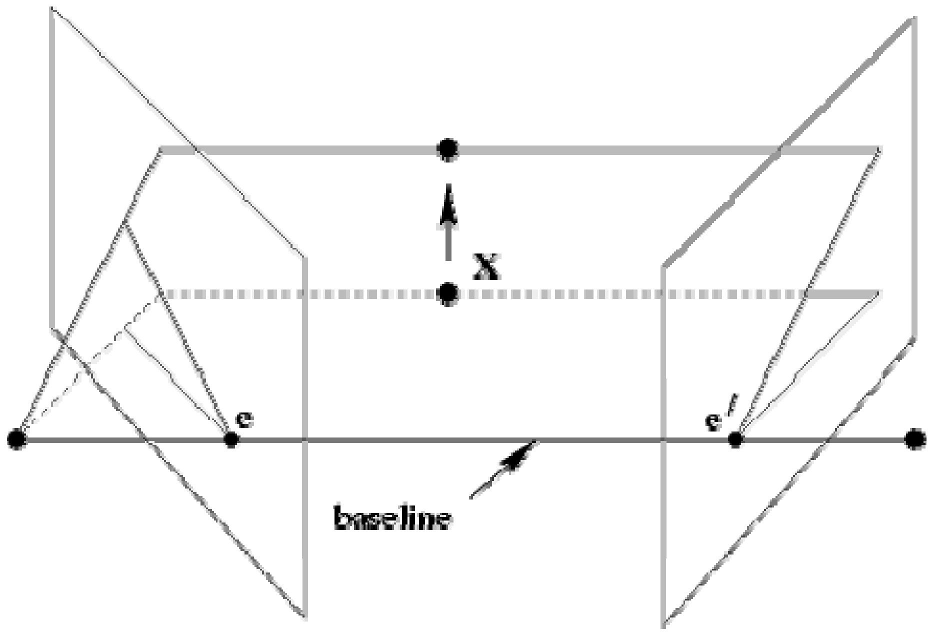
What if only *C*,*C'*,*x* are known?

The epipolar geometry



All points on π project on *l* and *l'*

The epipolar geometry



Family of planes π and lines l and l' intersect at e and e'

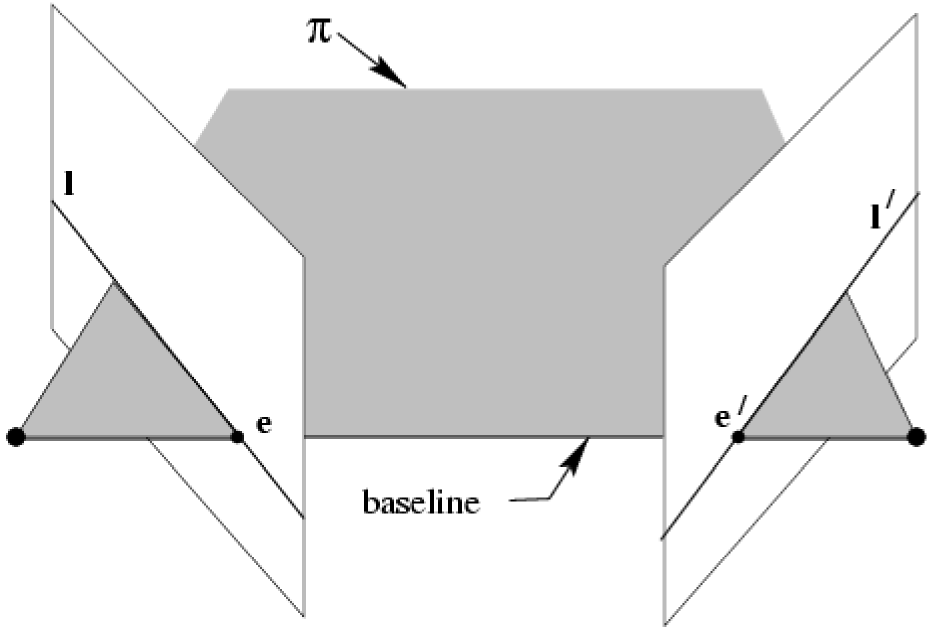
The epipolar geometry

epipolar pole

epipolar geometry demo

= intersection of baseline with image plane

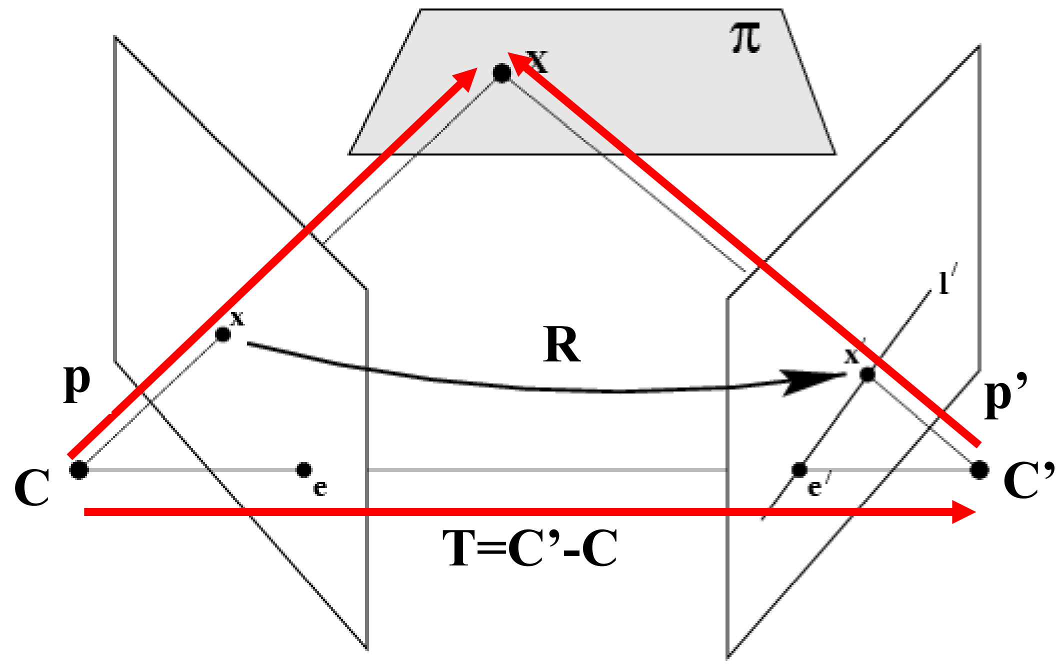
= projection of projection center in other image



epipolar plane = plane containing baseline

epipolar line = intersection of epipolar plane with image

The fundamental matrix F



Two reference frames are related via the extrinsic parameters

$$\mathbf{p} = \mathbf{R}\mathbf{p}' + \mathbf{T}$$

The fundamental matrix F

$$\mathbf{p} = \mathbf{R}\mathbf{p}' + \mathbf{T}$$

Multiply both sides by $\mathbf{p}^{\mathrm{T}}[\mathbf{T}]_{\times}$

$$[\mathbf{T}]_{\times} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

$$\mathbf{p}^{\mathrm{T}}[\mathbf{T}]_{\times}\mathbf{p} = \mathbf{p}^{\mathrm{T}}[\mathbf{T}]_{\times}(\mathbf{R}\mathbf{p}' + \mathbf{T})$$

$$\Rightarrow \quad 0 = \mathbf{p}^{\mathrm{T}}\boxed{[\mathbf{T}]_{\times}\mathbf{R}}\mathbf{p}'$$

$$\Rightarrow \quad \mathbf{p}^{\mathrm{T}}\boxed{\mathbf{E}}\mathbf{p}' = 0$$

essential matrix

The fundamental matrix F

$$\mathbf{p}^{\mathrm{T}}\mathbf{E}\mathbf{p}' = 0$$

Let $\mathbf{M}$ and $\mathbf{M}'$ be the intrinsic matrices, then

$$\mathbf{p} = \mathbf{M}^{-1}\mathbf{x} \qquad \mathbf{p}' = \mathbf{M}'^{-1}\mathbf{x}'$$

$$\Rightarrow (\mathbf{M}^{-1}\mathbf{x})^{\mathrm{T}}\mathbf{E}(\mathbf{M}'^{-1}\mathbf{x}') = 0$$

$$\Rightarrow \mathbf{x}^{\mathrm{T}}\boxed{\mathbf{M}^{-\mathrm{T}}\mathbf{E}\mathbf{M}'^{-1}}\mathbf{x}' = 0$$

$$\Rightarrow \mathbf{x}^{\mathrm{T}}\boxed{\mathbf{F}}\mathbf{x}' = 0$$

fundamental matrix

The fundamental matrix F

- The fundamental matrix is the algebraic representation of epipolar geometry

- The fundamental matrix satisfies the condition that for any pair of corresponding points x↔x’ in the two images

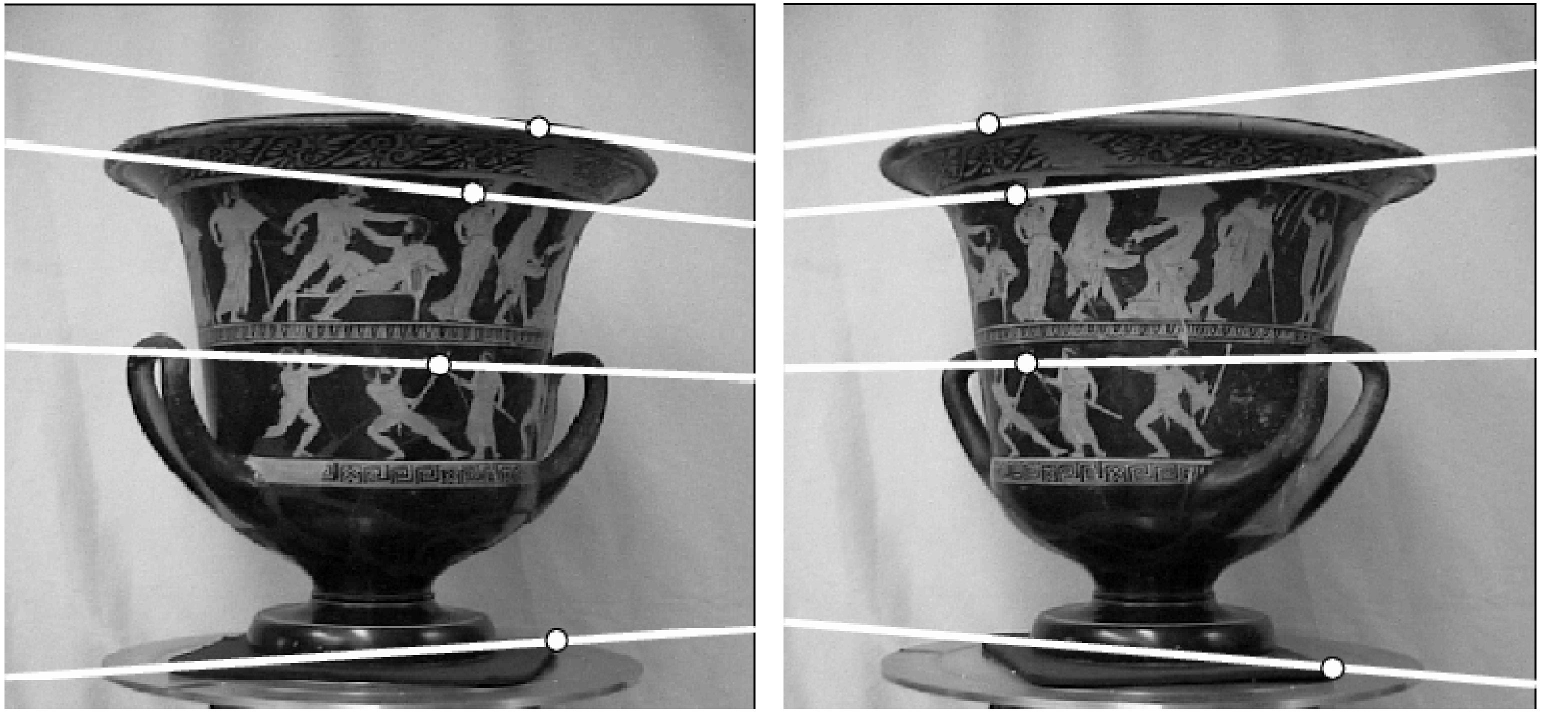
$$x^T F x' = 0 \qquad \left(x^T l = 0\right)$$

The fundamental matrix F

F is the unique 3x3 rank 2 matrix that satisfies $x^T F x' = 0$ for all $x \leftrightarrow x'$

1. **Transpose:** if F is fundamental matrix for (P,P'), then F^T is fundamental matrix for (P',P)
2. **Epipolar lines:** $l = Fx'$ & $l' = F^T x$
3. **Epipoles:** on all epipolar lines, thus $e^T F x' = 0,\ \forall x'$ $\Rightarrow e^T F = 0$, similarly $Fe' = 0$
4. **F** has 7 d.o.f. , i.e. 3x3-1(homogeneous)-1(rank2)
5. **F** is a correlation, projective mapping from a point x to a line $l = Fx'$ (not a proper correlation, i.e. not invertible)

The fundamental matrix F



- It can be used for
  - Simplifies matching
  - Allows to detect wrong matches

Estimation of F — 8-point algorithm

- The fundamental matrix F is defined by

$$\mathbf{x}^{\mathrm{T}}\mathbf{F}\mathbf{x}' = 0$$

for any pair of matches **x** and **x**' in two images.

- Let **x**=(*u*,*v*,*1*)$^{\mathrm{T}}$ and **x**'=(*u*',*v*',*1*)$^{\mathrm{T}}$, $\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$

each match gives a linear equation

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

8-point algorithm

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$$\begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In reality, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\|\mathbf{Af}\|$ subj. $\|\mathbf{f}\| = 1$. Find the vector corresponding to the least singular value.

8-point algorithm

- To enforce that F is of rank 2, F is replaced by F' that minimizes $\|\mathbf{F}-\mathbf{F}'\|$ subject to $\det \mathbf{F}'=0$.
- It is achieved by SVD. Let $\mathbf{F}=\mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}}$, where

$$\Sigma=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix}, \text{ let } \Sigma'=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & 0\end{bmatrix}$$

then $\mathbf{F}'=\mathbf{U}\Sigma'\mathbf{V}^{\mathrm{T}}$ is the solution.

8-point algorithm

```
% Build the constraint matrix
    A = [x2(1,:)‘.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
         x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
         x1(1,:)'             x1(2,:)'            ones(npts,1) ];

    [U,D,V] = svd(A);

% Extract fundamental matrix from the column of V
% corresponding to the smallest singular value.
    F = reshape(V(:,9),3,3)';

% Enforce rank2 constraint
    [U,D,V] = svd(F);
    F = U*diag([D(1,1) D(2,2) 0])*V';
```

8-point algorithm

- Pros: it is linear, easy to implement and fast
- Cons: susceptible to noise

Problem with 8-point algorithm

$$\begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

~10000 ~10000 ~100 ~10000 ~10000 ~100 ~100 ~100 1

! Orders of magnitude difference between column of data matrix
$\rightarrow$ least-squares yields poor results

Normalized 8-point algorithm

1. Transform input by $\hat{\mathbf{x}}_{\mathrm{i}} = \mathbf{T}\mathbf{x}_{\mathrm{i}}$, $\hat{\mathbf{x}}_{\mathrm{i}}' = \mathbf{T}\mathbf{x}_{\mathrm{i}}'$
2. Call 8-point on $\hat{\mathbf{x}}_{\mathrm{i}}, \hat{\mathbf{x}}_{\mathrm{i}}'$ to obtain $\hat{\mathbf{F}}$
3. $\mathbf{F} = \mathbf{T}'^{\mathrm{T}}\hat{\mathbf{F}}\mathbf{T}$

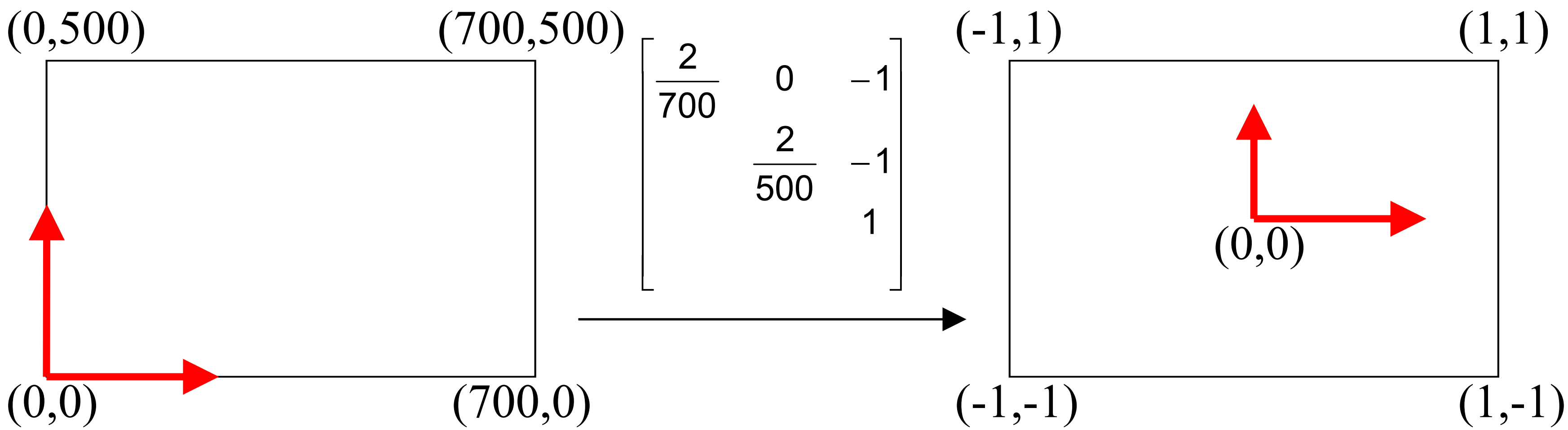
$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0$$

$$\hat{\mathbf{x}}'^{\mathrm{T}}\underbrace{\mathbf{T}'^{-\mathrm{T}}\mathbf{F}\mathbf{T}^{-1}}_{\hat{\mathbf{F}}}\hat{\mathbf{x}} = 0$$

Normalized 8-point algorithm

normalized least squares yields good results

Transform image to ~[-1,1]x[-1,1]



Normalized 8-point algorithm

```
[x1, T1] = normalise2dpts(x1);
[x2, T2] = normalise2dpts(x2);
 A = [x2(1,:)‘.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
        x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
        x1(1,:)'             x1(2,:)'            ones(npts,1) ];

 [U,D,V] = svd(A);

 F = reshape(V(:,9),3,3)';

 [U,D,V] = svd(F);
 F = U*diag([D(1,1) D(2,2) 0])*V';

% Denormalise
 F = T2'*F*T1;
```

Normalization

```
function [newpts, T] = normalise2dpts(pts)

    c = mean(pts(1:2,:)')';   % Centroid
    newp(1,:) = pts(1,:)-c(1); % Shift origin to centroid.
    newp(2,:) = pts(2,:)-c(2);

    meandist = mean(sqrt(newp(1,:).^2 + newp(2,:).^2));
    scale = sqrt(2)/meandist;

    T = [scale      0    -scale*c(1)
            0     scale  -scale*c(2)
            0       0          1      ];
    newpts = T*pts;
```

RANSAC

repeat

select minimal sample (8 matches)

compute solution(s) for F

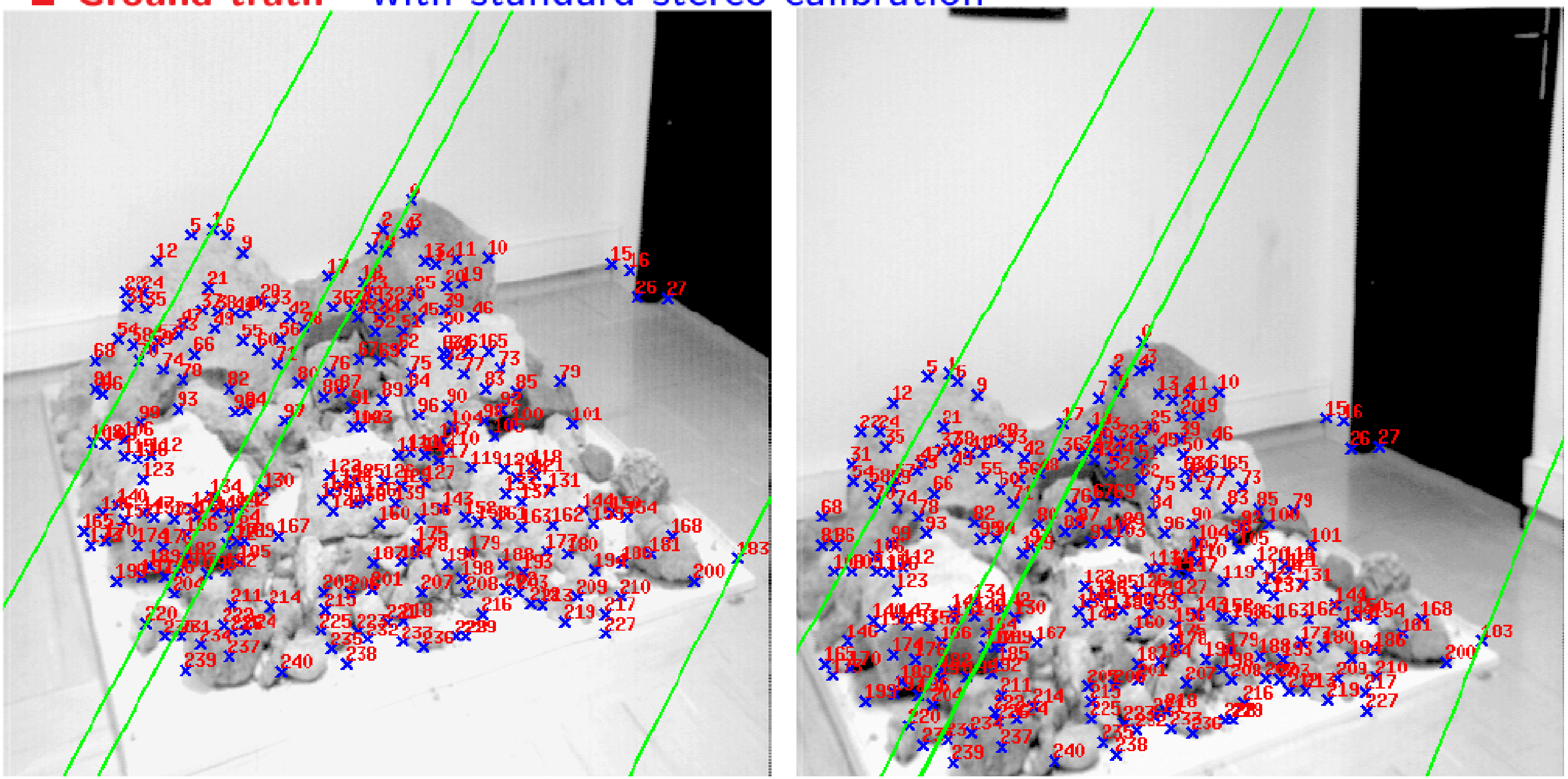
determine inliers

until Γ(*#inliers*,*#samples*)>95% or too many times

compute F based on all inliers

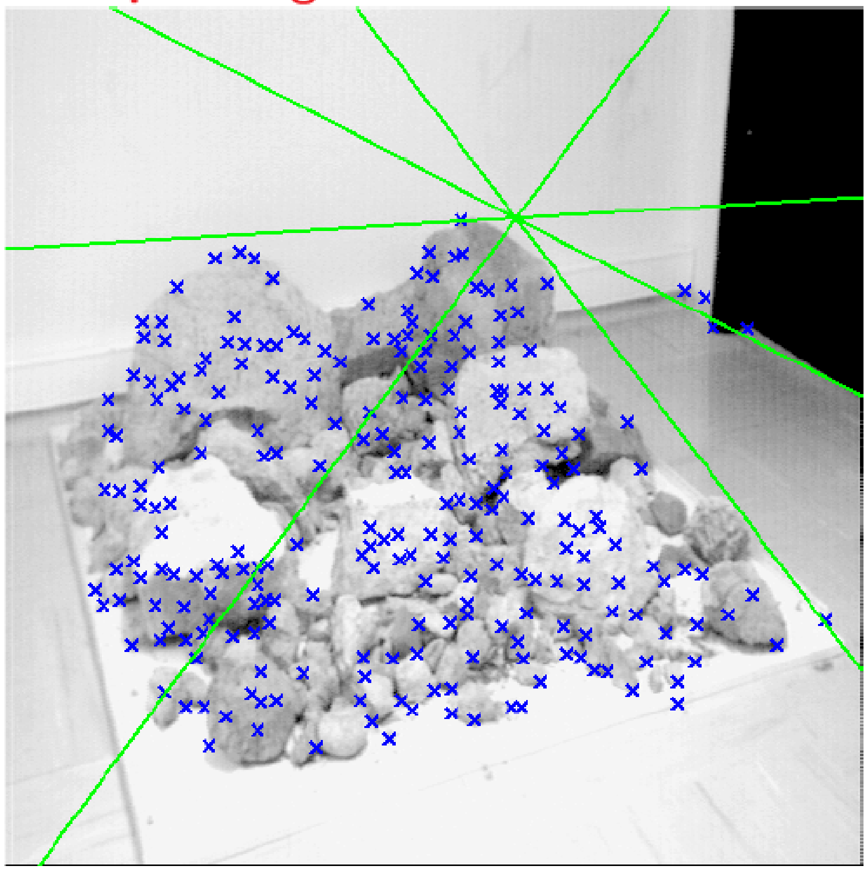
Results (ground truth)

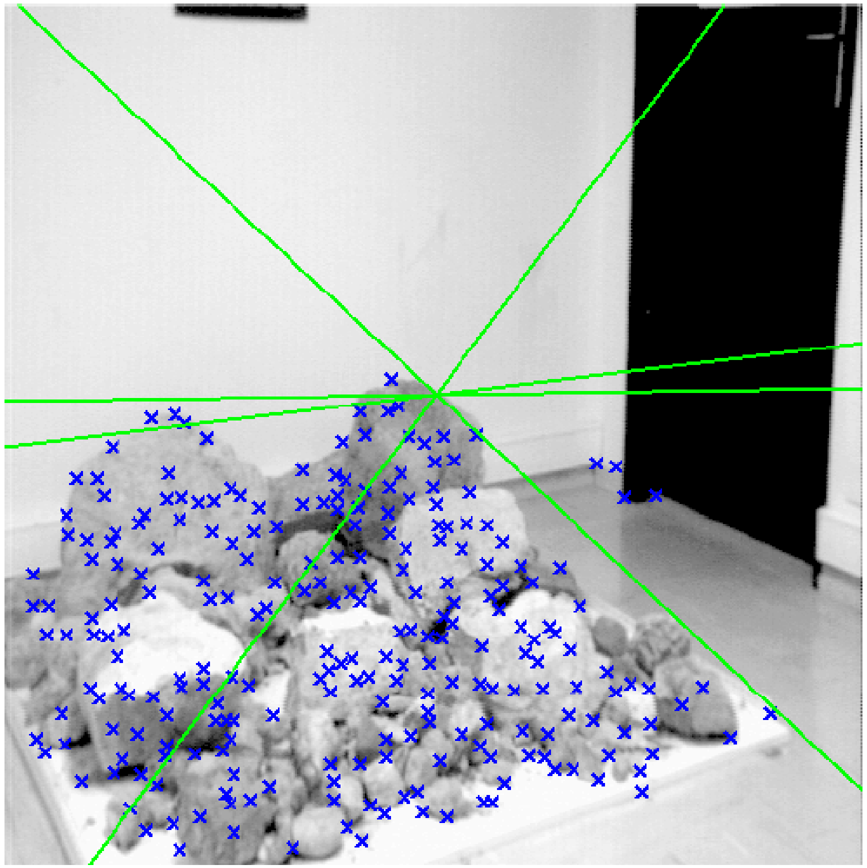
- Ground truth with standard stereo calibration



Results (8-point algorithm)

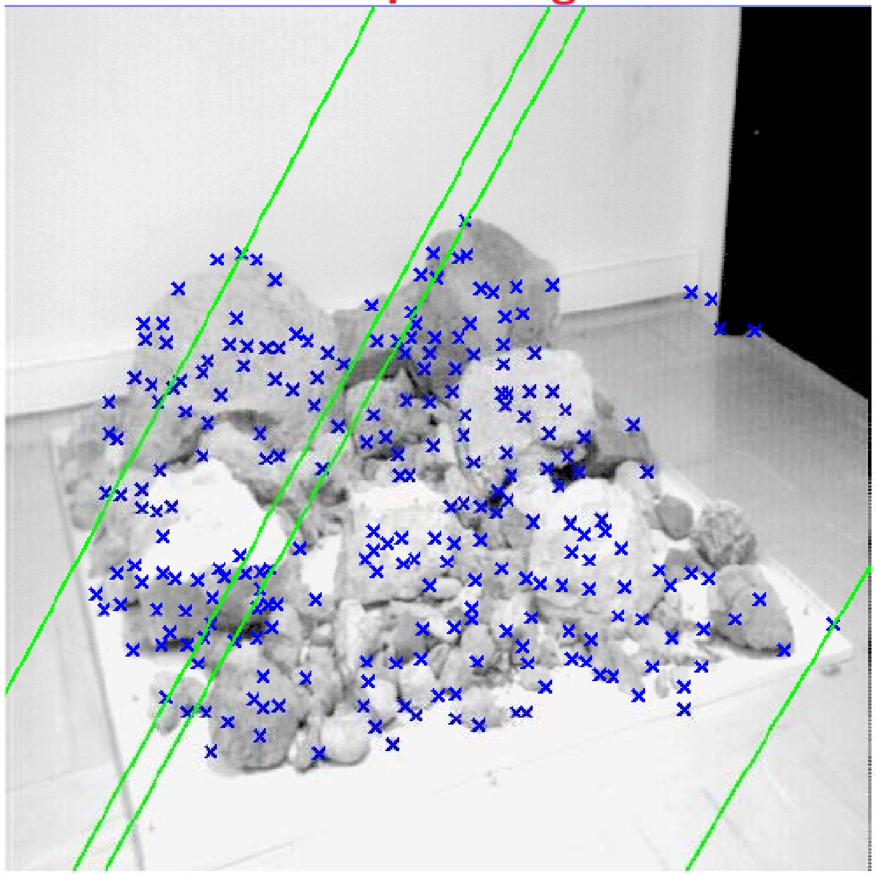
- 8-point algorithm

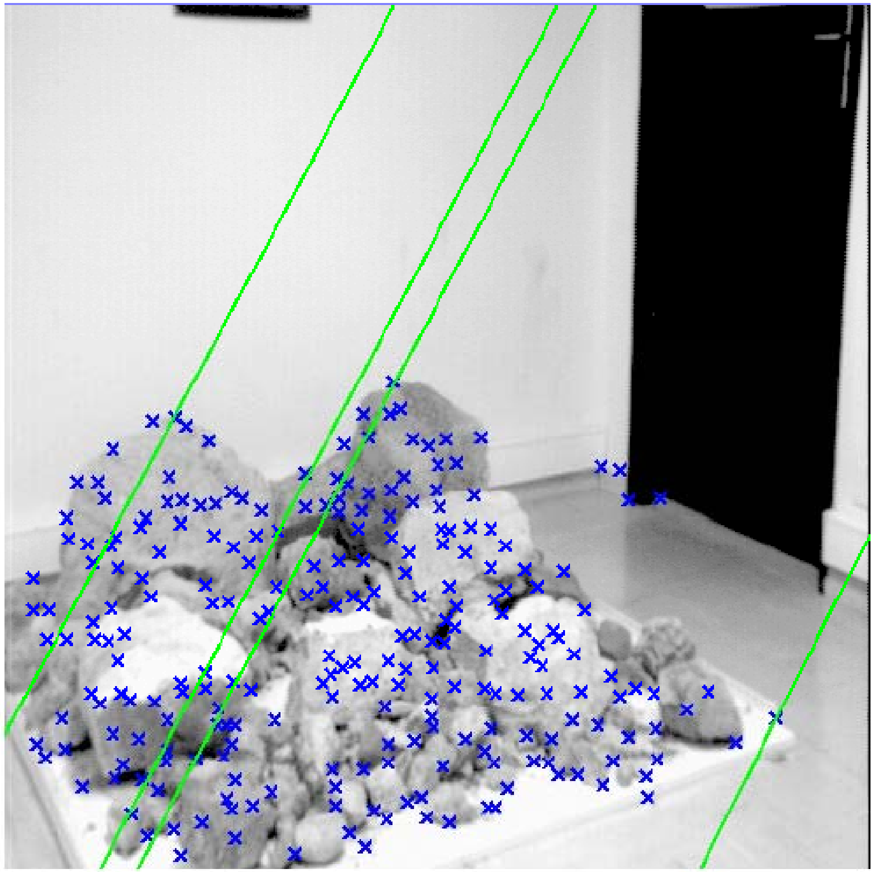




Results (normalized 8-point algorithm)

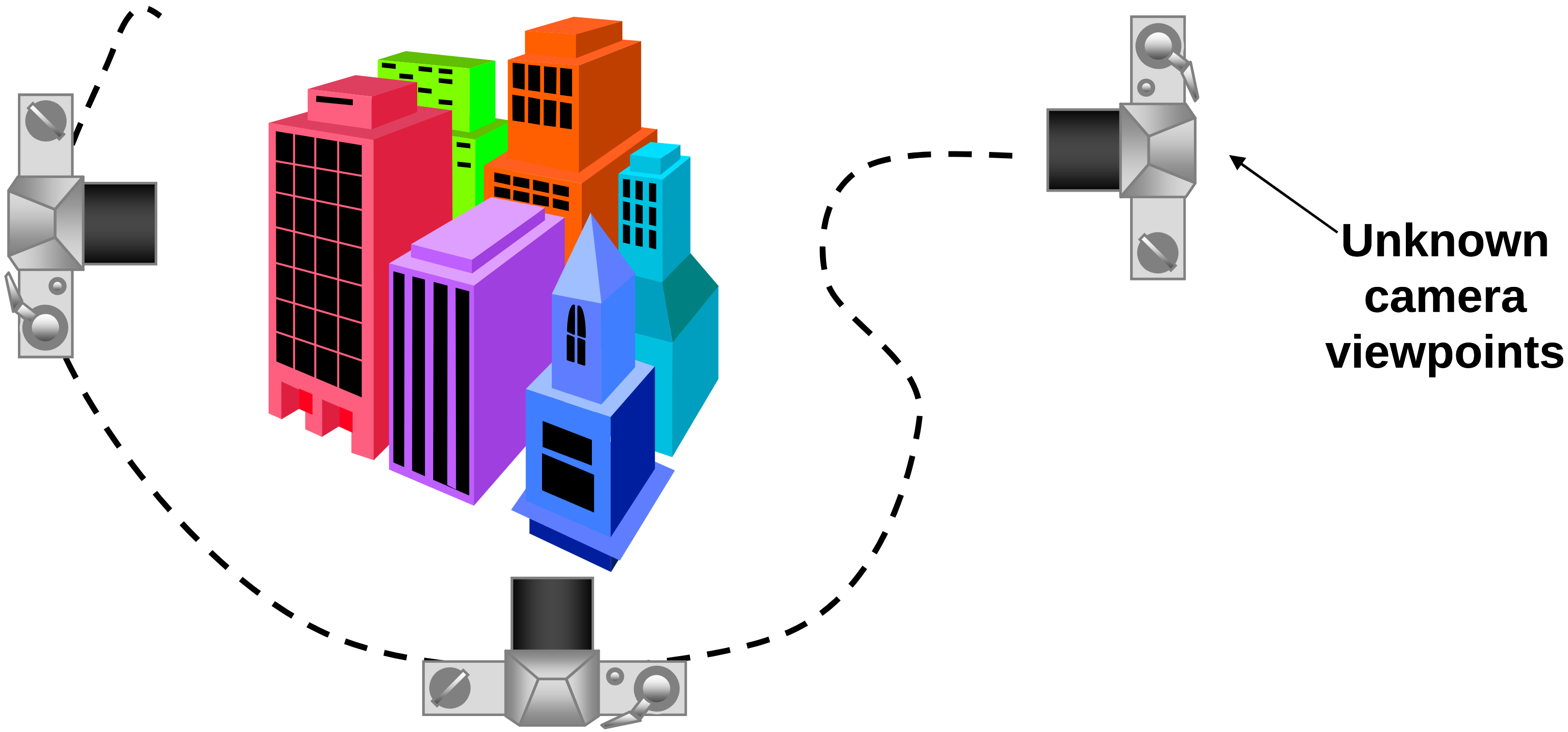
- Normalized 8-point algorithm





Structure from motion

Structure from motion



structure for motion: automatic recovery of **camera motion** and **scene structure** from two or more images. It is a self calibration technique and called *automatic camera tracking* or *matchmoving*.

Applications

- For computer vision, multiple-view shape reconstruction, novel view synthesis and autonomous vehicle navigation.
- For film production, seamless insertion of CGI into live-action backgrounds

Matchmove

[example #1](#) [example #2](#) [example #3](#) [example #4](#)

Structure from motion

2D feature tracking → 3D estimation → optimization (bundle adjust) → geometry fitting

SFM pipeline

Structure from motion

- Step 1: Track Features
  - Detect good features, Shi & Tomasi, SIFT
  - Find correspondences between frames
    - Lucas & Kanade-style motion estimation
    - window-based correlation
    - SIFT matching

KLT tracking

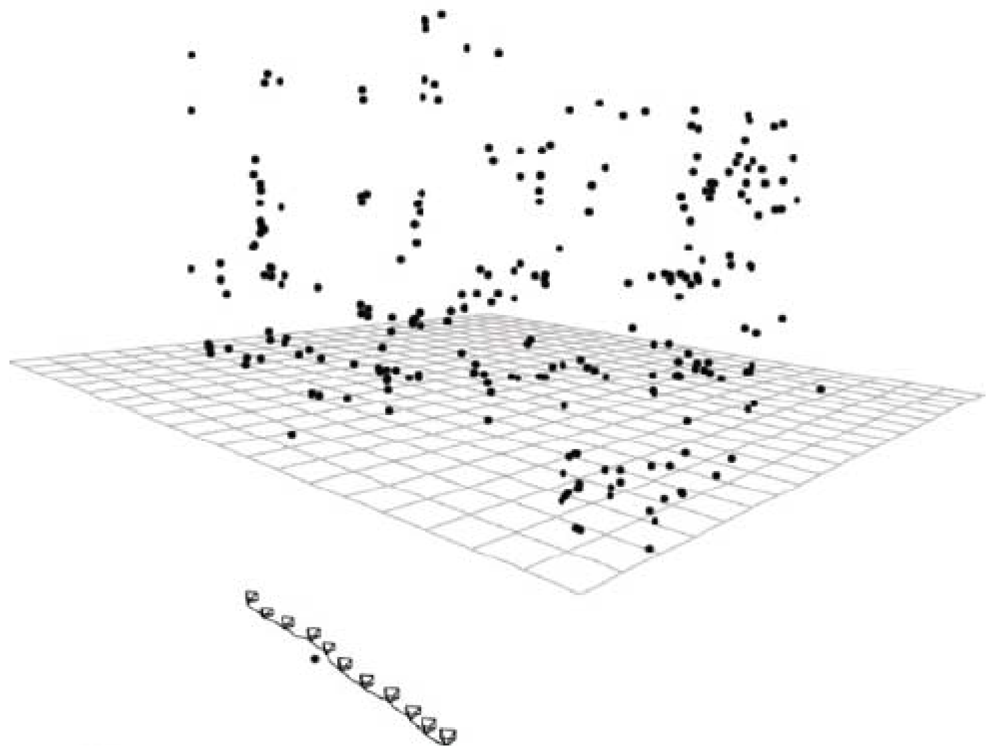
http://www.ces.clemson.edu/~stb/klt/

Structure from Motion

- Step 2: Estimate Motion and Structure
  - Simplified projection model, e.g., **[Tomasi 92]**
  - 2 or 3 views at a time **[Hartley 00]**

Structure from Motion

- Step 3: Refine estimates
  – "Bundle adjustment" in photogrammetry
  – Other iterative methods

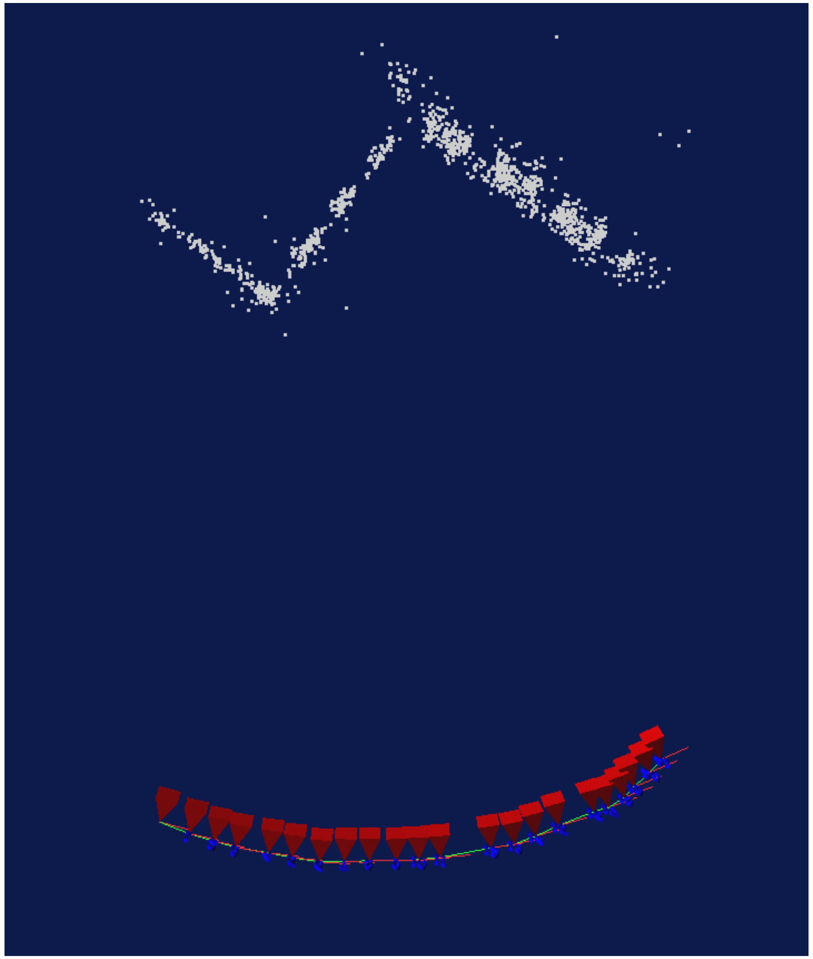


Structure from Motion

- Step 4: Recover surfaces (image-based triangulation, silhouettes, stereo…)

Factorization methods

Problem statement



Notations

- n 3D points are seen in m views
- $\mathbf{q}=(u,v,1)$: *2D image point*
- $\mathbf{p}=(x,y,z,1)$: *3D scene point*
- Π: projection matrix
- π: projection function
- q_{ij} is the projection of the i-th point on image j
- λ_{ij} projective depth of q_{ij}

$$\mathbf{q}_{ij} = \pi(\Pi_j \mathbf{p}_i) \qquad \pi(x,y,z) = (x/z, y/z)$$

$$\lambda_{ij} = z$$

Structure from motion

- Estimate $\prod_j$ and $\mathbf{p}_i$ to minimize

$$\varepsilon(\mathbf{\Pi}_1,\cdots,\mathbf{\Pi}_m,\mathbf{p}_1,\cdots,\mathbf{p}_n)=\sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\log P(\pi(\mathbf{\Pi}_j\mathbf{p}_i);\mathbf{q}_{ij})$$

$$w_{ij}=\begin{cases}1 & \text{if } p_i \text{ is visible in view j}\\ 0 & \text{otherwise}\end{cases}$$

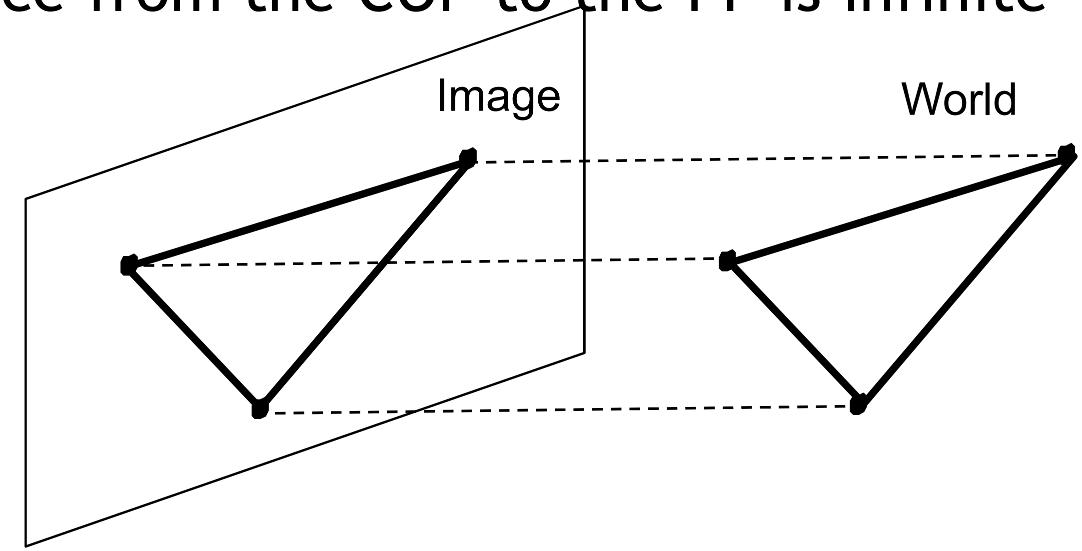
- Assume isotropic Gaussian noise, it is reduced to

$$\varepsilon(\mathbf{\Pi}_1,\cdots,\mathbf{\Pi}_m,\mathbf{p}_1,\cdots,\mathbf{p}_n)=\sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij}\left\|\pi(\mathbf{\Pi}_j\mathbf{p}_i)-\mathbf{q}_{ij}\right\|^2$$

- Start from a simpler projection model

Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

  - Also called “parallel projection”: (x, y, z) $\rightarrow$ (x, y)

SFM under orthographic projection

2D image point

Orthographic projection incorporating 3D rotation

3D scene point

image offset

$$\underset{2\times 1}{\mathbf{q}} = \underset{2\times 3}{\mathbf{\Pi}}\,\underset{3\times 1}{\mathbf{p}} + \underset{2\times 1}{\mathbf{t}}$$

- Trick
  - Choose scene origin to be centroid of 3D points
  - Choose image origins to be centroid of 2D points
  - Allows us to drop the camera translation:

$$\mathbf{q} = \mathbf{\Pi}\mathbf{p}$$

factorization (Tomasi & Kanade)

projection of *n* features in one image:

$$\underset{2\times n}{\begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_n \end{bmatrix}} = \underset{2\times 3}{\prod} \underset{3\times n}{\begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n \end{bmatrix}}$$

projection of *n* features in *m* images

$$\underset{2m\times n}{\begin{bmatrix} \mathbf{q}_{11} & \mathbf{q}_{12} & \cdots & \mathbf{q}_{1n} \\ \mathbf{q}_{21} & \mathbf{q}_{22} & \cdots & \mathbf{q}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{q}_{m1} & \mathbf{q}_{m2} & \cdots & \mathbf{q}_{mn} \end{bmatrix}} = \underset{2m\times 3}{\begin{bmatrix} \mathbf{\Pi}_1 \\ \mathbf{\Pi}_2 \\ \vdots \\ \mathbf{\Pi}_m \end{bmatrix}} \underset{3\times n}{\begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n \end{bmatrix}}$$

W measurement **M** motion **S** shape

Key Observation: *rank*(**W**) <= 3

Factorization

known $\underset{2m\times n}{\mathbf{W}} = \boxed{\underset{2m\times 3}{\mathbf{M}}\ \underset{3\times n}{\mathbf{S}}}$ solve for

- Factorization Technique
  - **W** is at most rank 3 (assuming no noise)
  - We can use *singular value decomposition* to factor **W**:

$$\underset{2m\times n}{\mathbf{W}} = \underset{2m\times 3}{\mathbf{M}'}\ \underset{3\times n}{\mathbf{S}'}$$

  - **S'** differs from **S** by a linear transformation *A*:

$$\mathbf{W} = \mathbf{M}'\mathbf{S}' = (\mathbf{M}\mathbf{A}^{-1})(\mathbf{A}\mathbf{S})$$

  - Solve for **A** by enforcing *metric* constraints on **M**

Metric constraints

- Orthographic Camera
  - Rows of Π are orthonormal: $$\Pi \Pi^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
- Enforcing "Metric" Constraints
  - Compute **A** such that rows of **M** have these properties

$$\mathbf{M}'\mathbf{A} = \mathbf{M}$$

Trick (not in original Tomasi/Kanade paper, but in followup work)

- Constraints are linear in $\mathbf{A}\mathbf{A}^T$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \Pi \Pi^T = \Pi'\mathbf{A}(\mathbf{A}\Pi')^T = \Pi'\mathbf{G}\Pi'^T \qquad \textit{where} \quad \mathbf{G} = \mathbf{A}\mathbf{A}^T$$

- Solve for **G** first by writing equations for every Π_i in **M**
- Then $\mathbf{G} = \mathbf{A}\mathbf{A}^T$ by SVD (since $\mathbf{U} = \mathbf{V}$)

Factorization with noisy data

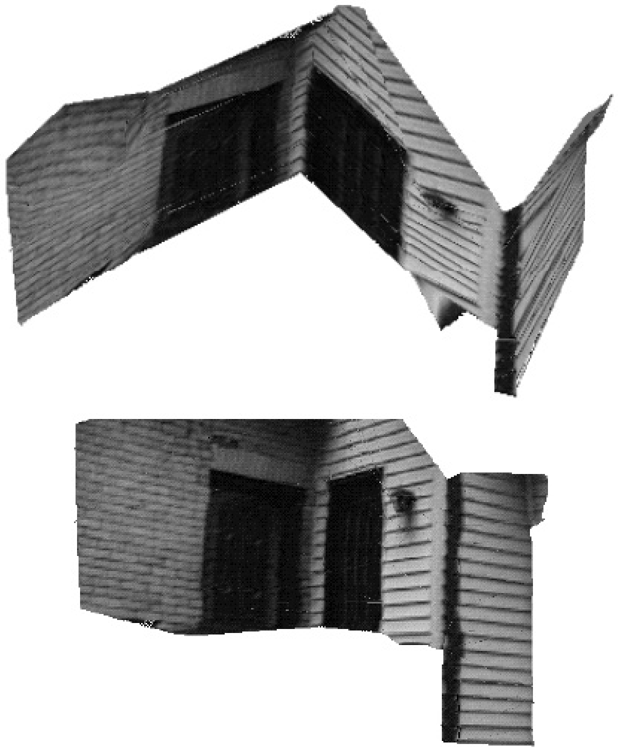
$$\underset{2m\times n}{\mathbf{W}} = \underset{2m\times 3}{\mathbf{M}}\ \underset{3\times n}{\mathbf{S}} + \underset{2m\times n}{\mathbf{E}}$$

- SVD gives this solution
  - Provides optimal rank 3 approximation **W'** of **W**

$$\underset{2m\times n}{\mathbf{W}} = \underset{2m\times n}{\mathbf{W}'} + \underset{2m\times n}{\mathbf{E}}$$

- Approach
  - Estimate **W'**, then use noise-free factorization of **W'** as before
  - Result minimizes the SSD between positions of image features and projection of the reconstruction

Results



Extensions to factorization methods

- Projective projection
- With missing data
- Projective projection with missing data

Bundle adjustment

Levenberg-Marquardt method

- LM can be thought of as a combination of steepest descent and the Newton method. When the current solution is far from the correct one, the algorithm behaves like a steepest descent method: slow, but guaranteed to converge. When the current solution is close to the correct solution, it becomes a Newton's method.

Nonlinear least square

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Given a set of measurements $\mathbf{x}$, try to find the best parameter vector $\mathbf{p}$ so that the squared distance $\varepsilon^T\varepsilon$ is minimal. Here, $\varepsilon = \mathbf{x} - \hat{\mathbf{x}}$, with $\hat{\mathbf{x}} = f(\mathbf{p})$.

Levenberg-Marquardt method

For a small $||\delta_\mathbf{p}||$, $f(\mathbf{p}+\delta_\mathbf{p}) \approx f(\mathbf{p}) + \mathbf{J}\delta_\mathbf{p}$

$\mathbf{J}$ is the Jacobian matrix $\frac{\partial f(\mathbf{p})}{\partial \mathbf{p}}$

it is required to find the $\delta_\mathbf{p}$ that minimizes the quantity

$$||\mathbf{x} - f(\mathbf{p}+\delta_\mathbf{p})|| \approx ||\mathbf{x} - f(\mathbf{p}) - \mathbf{J}\delta_\mathbf{p}|| = ||\epsilon - \mathbf{J}\delta_\mathbf{p}||$$

$$\mathbf{J}^T\mathbf{J}\delta_\mathbf{p} = \mathbf{J}^T\epsilon$$

$$\mathbf{N}\delta_\mathbf{p} = \mathbf{J}^T\epsilon$$

$$\mathbf{N}_{ii} = \mu + \left[\mathbf{J}^T\mathbf{J}\right]_{ii}$$

damping term

Levenberg-Marquardt method

- $\mu=0 \rightarrow$ Newton's method
- $\mu \rightarrow \infty \rightarrow$ steepest descent method

- Strategy for choosing μ
  - Start with some small μ
  - If error is not reduced, keep trying larger μ until it does
  - If error is reduced, accept it and reduce μ for the next iteration

Bundle adjustment

- Bundle adjustment (BA) is a technique for simultaneously refining the 3D structure and camera parameters
- It is capable of obtaining an optimal reconstruction under certain assumptions on image error models. For zero-mean Gaussian image errors, BA is the maximum likelihood estimator.

Bundle adjustment

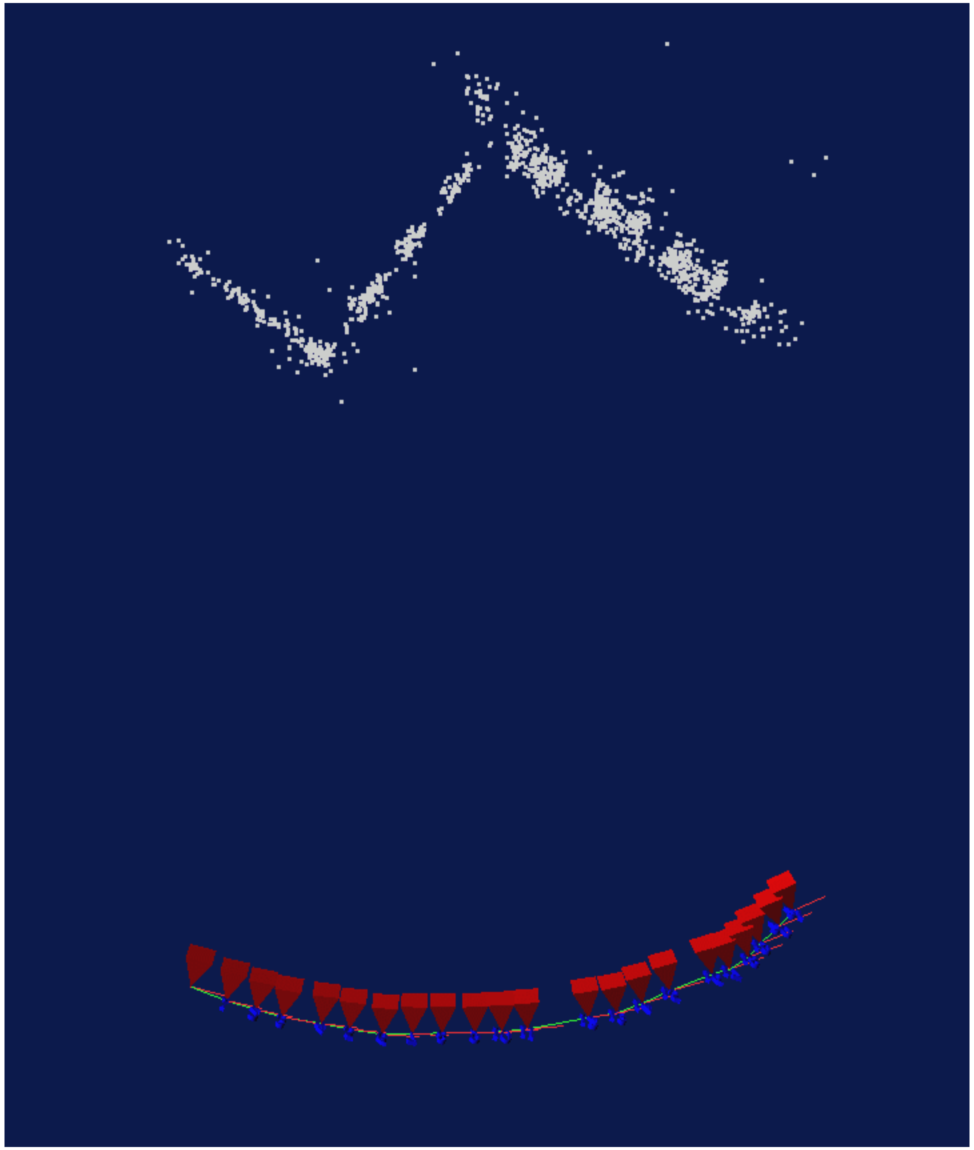
- n 3D points are seen in m views
- x_{ij} is the projection of the i-th point on image j
- a_j is the parameters for the j-th camera
- b_i is the parameters for the i-th point
- BA attempts to minimize the projection error

$$\min_{\mathbf{a}_j, \mathbf{b}_i} \sum_{i=1}^{n} \sum_{j=1}^{m} d(\mathbf{Q}(\mathbf{a}_j, \mathbf{b}_i), \mathbf{x}_{ij})^2$$

predicted projection

Euclidean distance

Bundle adjustment



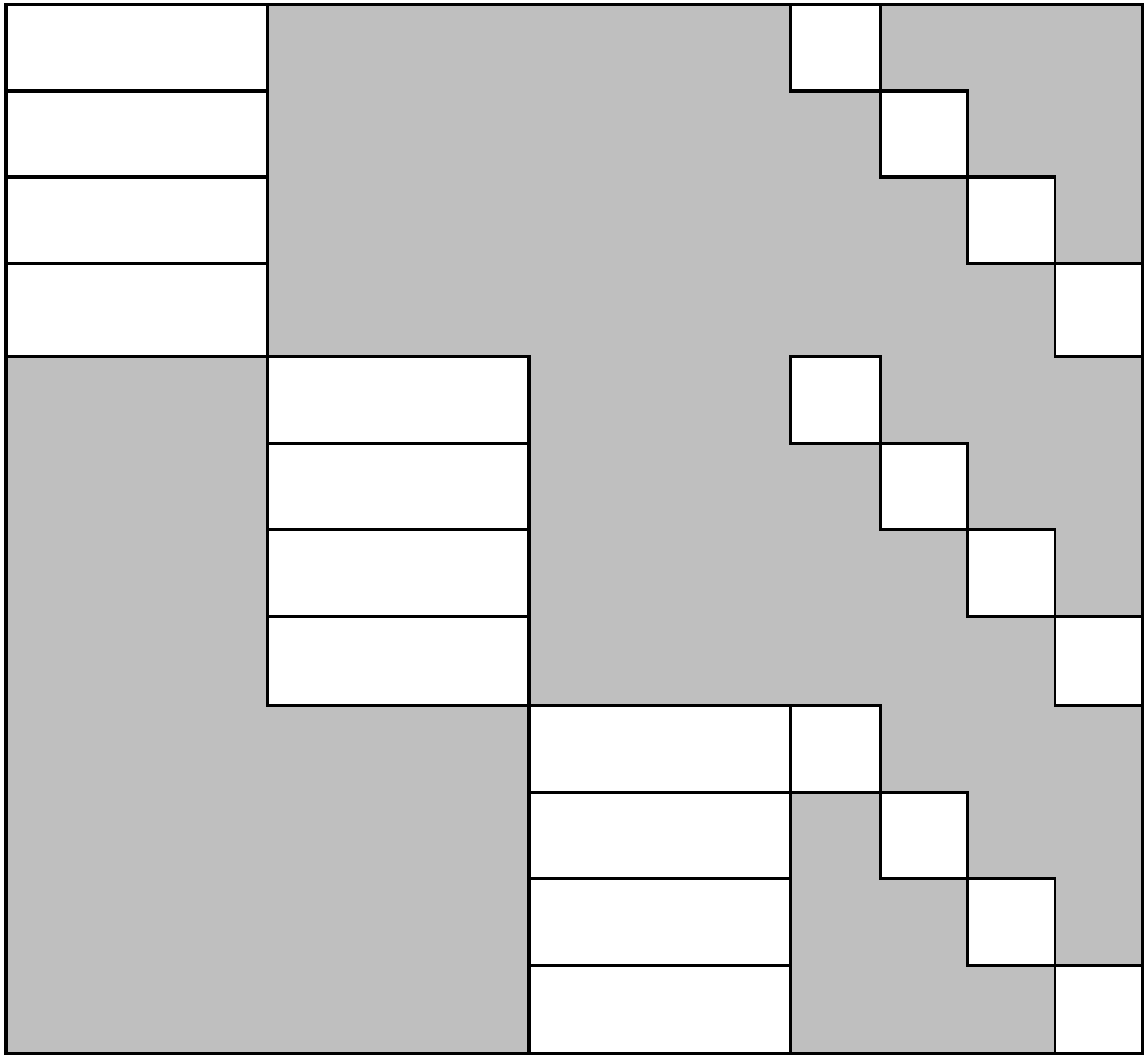
Bundle adjustment

3 views and 4 points $\mathbf{P} = (\mathbf{a}_1{}^T,\ \mathbf{a}_2{}^T,\ \mathbf{a}_3{}^T,\ \mathbf{b}_1{}^T,\ \mathbf{b}_2{}^T,\ \mathbf{b}_3{}^T,\ \mathbf{b}_4{}^T)^T$

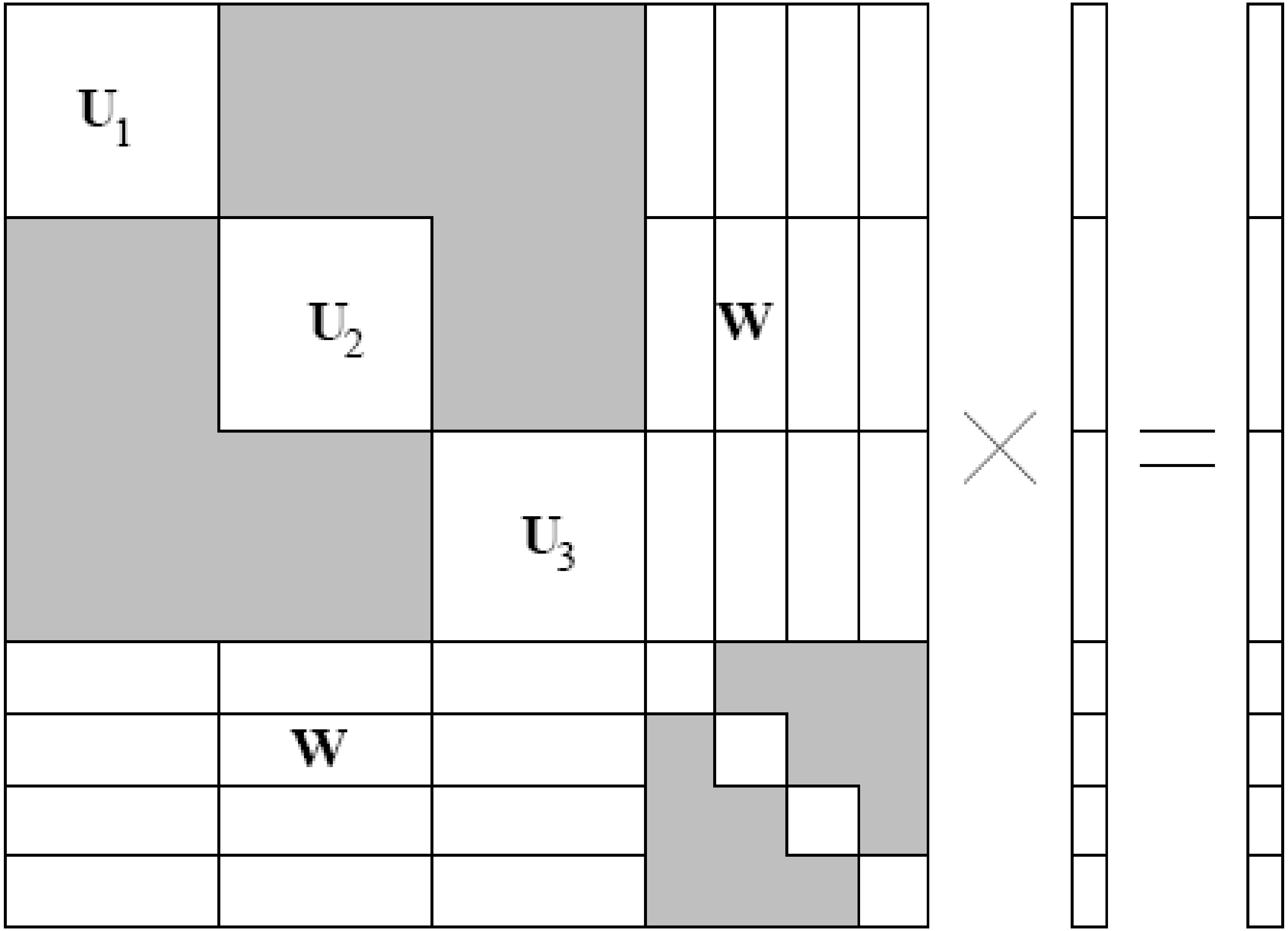
$$\mathbf{X} = (\mathbf{x}_{11}{}^T,\ \mathbf{x}_{12}{}^T,\ \mathbf{x}_{13}{}^T,\ \mathbf{x}_{21}{}^T,\ \mathbf{x}_{22}{}^T,\ \mathbf{x}_{23}{}^T,\ \mathbf{x}_{31}{}^T,\ \mathbf{x}_{32}{}^T,\ \mathbf{x}_{33}{}^T,\ \mathbf{x}_{41}{}^T,\ \mathbf{x}_{42}{}^T,\ \mathbf{x}_{43}{}^T)^T$$

$$\frac{\partial \mathbf{X}}{\partial \mathbf{P}} = \begin{pmatrix}
\mathbf{A}_{11} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{11} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{A}_{12} & \mathbf{0} & \mathbf{B}_{12} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_{13} & \mathbf{B}_{13} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_{21} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{21} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{A}_{22} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{22} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_{23} & \mathbf{0} & \mathbf{B}_{23} & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_{31} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{31} & \mathbf{0} \\
\mathbf{0} & \mathbf{A}_{32} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{32} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_{33} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{33} & \mathbf{0} \\
\mathbf{A}_{41} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{41} \\
\mathbf{0} & \mathbf{A}_{42} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{42} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_{43} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{43}
\end{pmatrix}$$

Typical Jacobian



Block structure of normal equation



Bundle adjustment

$$
\begin{pmatrix}
\mathbf{U}_1 & \mathbf{0} & \mathbf{0} & \mathbf{W}_{11} & \mathbf{W}_{21} & \mathbf{W}_{31} & \mathbf{W}_{41} \\
\mathbf{0} & \mathbf{U}_2 & \mathbf{0} & \mathbf{W}_{12} & \mathbf{W}_{22} & \mathbf{W}_{32} & \mathbf{W}_{42} \\
\mathbf{0} & \mathbf{0} & \mathbf{U}_3 & \mathbf{W}_{13} & \mathbf{W}_{23} & \mathbf{W}_{33} & \mathbf{W}_{43} \\
{\mathbf{W}_{11}}^T & {\mathbf{W}_{12}}^T & {\mathbf{W}_{13}}^T & \mathbf{V}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
{\mathbf{W}_{21}}^T & {\mathbf{W}_{22}}^T & {\mathbf{W}_{23}}^T & \mathbf{0} & \mathbf{V}_2 & \mathbf{0} & \mathbf{0} \\
{\mathbf{W}_{31}}^T & {\mathbf{W}_{32}}^T & {\mathbf{W}_{33}}^T & \mathbf{0} & \mathbf{0} & \mathbf{V}_3 & \mathbf{0} \\
{\mathbf{W}_{41}}^T & {\mathbf{W}_{42}}^T & {\mathbf{W}_{43}}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{V}_4
\end{pmatrix}
\begin{pmatrix}
\delta_{\mathbf{a}_1} \\ \delta_{\mathbf{a}_2} \\ \delta_{\mathbf{a}_3} \\ \delta_{\mathbf{b}_1} \\ \delta_{\mathbf{b}_2} \\ \delta_{\mathbf{b}_3} \\ \delta_{\mathbf{b}_4}
\end{pmatrix}
=
\begin{pmatrix}
\epsilon_{\mathbf{a}_1} \\ \epsilon_{\mathbf{a}_2} \\ \epsilon_{\mathbf{a}_3} \\ \epsilon_{\mathbf{b}_1} \\ \epsilon_{\mathbf{b}_2} \\ \epsilon_{\mathbf{b}_3} \\ \epsilon_{\mathbf{b}_4}
\end{pmatrix}
$$

$$
\mathbf{U}^* = \begin{pmatrix}
\mathbf{U}_1^* & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{U}_2^* & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{U}_3^*
\end{pmatrix},
\mathbf{V}^* = \begin{pmatrix}
\mathbf{V}_1^* & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{V}_2^* & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{V}_3^* & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{V}_4^*
\end{pmatrix},
\mathbf{W} = \begin{pmatrix}
\mathbf{W}_{11} & \mathbf{W}_{21} & \mathbf{W}_{31} & \mathbf{W}_{41} \\
\mathbf{W}_{12} & \mathbf{W}_{22} & \mathbf{W}_{32} & \mathbf{W}_{42} \\
\mathbf{W}_{13} & \mathbf{W}_{23} & \mathbf{W}_{33} & \mathbf{W}_{43}
\end{pmatrix}
$$

$$
\begin{pmatrix}
\mathbf{U}^* & \mathbf{W} \\
\mathbf{W}^T & \mathbf{V}^*
\end{pmatrix}
\begin{pmatrix}
\delta_{\mathbf{a}} \\ \delta_{\mathbf{b}}
\end{pmatrix}
=
\begin{pmatrix}
\epsilon_{\mathbf{a}} \\ \epsilon_{\mathbf{b}}
\end{pmatrix}
$$

Bundle adjustment

Multiplied by $$\begin{pmatrix} \mathbf{I} & -\mathbf{W}\,\mathbf{V}^{*-1} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{U}^* - \mathbf{W}\,\mathbf{V}^{*-1}\,\mathbf{W}^T & \mathbf{0} \\ \mathbf{W}^T & \mathbf{V}^* \end{pmatrix} \begin{pmatrix} \delta_{\mathbf{a}} \\ \delta_{\mathbf{b}} \end{pmatrix} = \begin{pmatrix} \epsilon_{\mathbf{a}} - \mathbf{W}\,\mathbf{V}^{*-1}\,\epsilon_{\mathbf{b}} \\ \epsilon_{\mathbf{b}} \end{pmatrix}$$

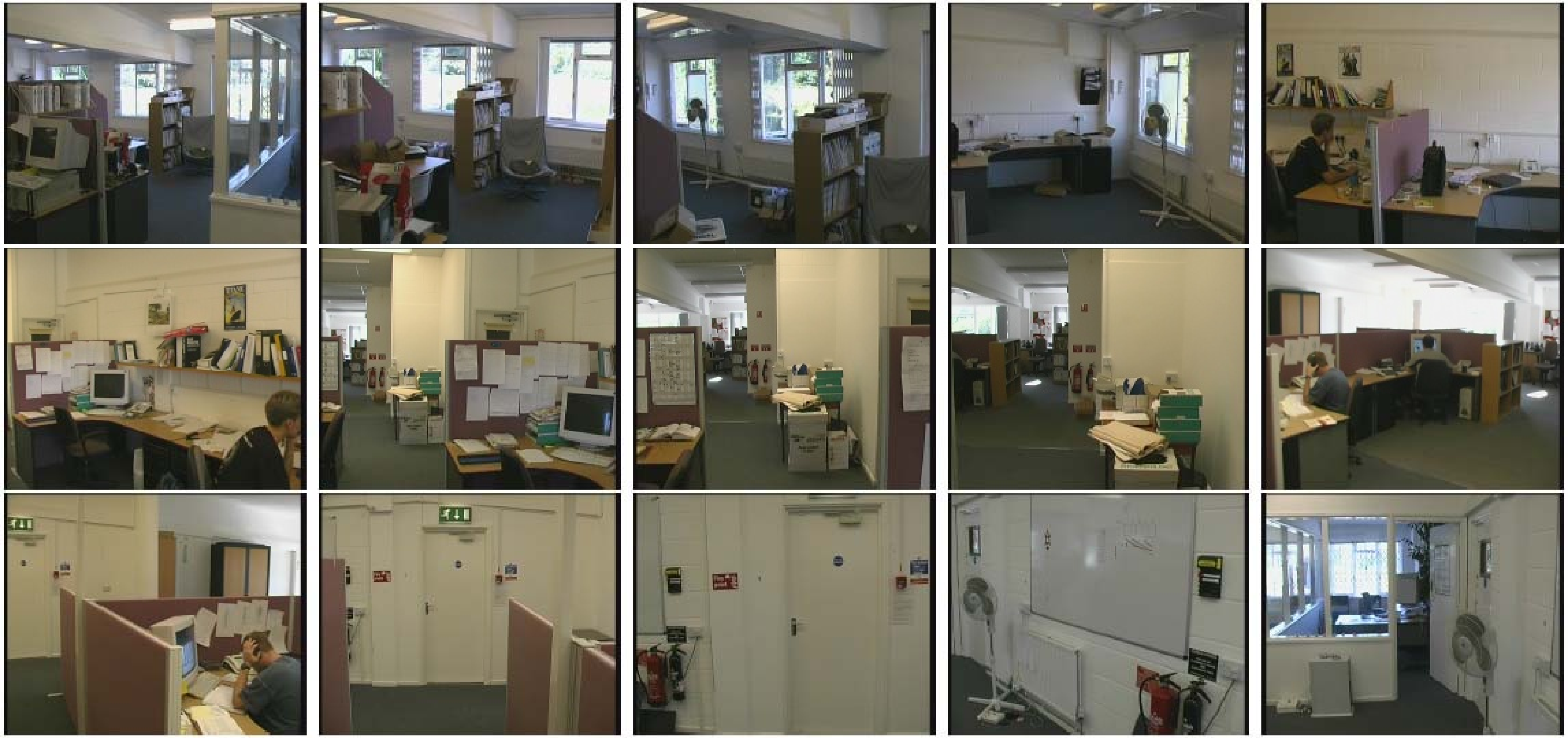
$$(\mathbf{U}^* - \mathbf{W}\,\mathbf{V}^{*-1}\,\mathbf{W}^T)\,\delta_{\mathbf{a}} = \epsilon_{\mathbf{a}} - \mathbf{W}\,\mathbf{V}^{*-1}\,\epsilon_{\mathbf{b}}$$

$$\mathbf{V}^*\,\delta_{\mathbf{b}} = \epsilon_{\mathbf{b}} - \mathbf{W}^T\,\delta_{\mathbf{a}}$$

Issues in SFM

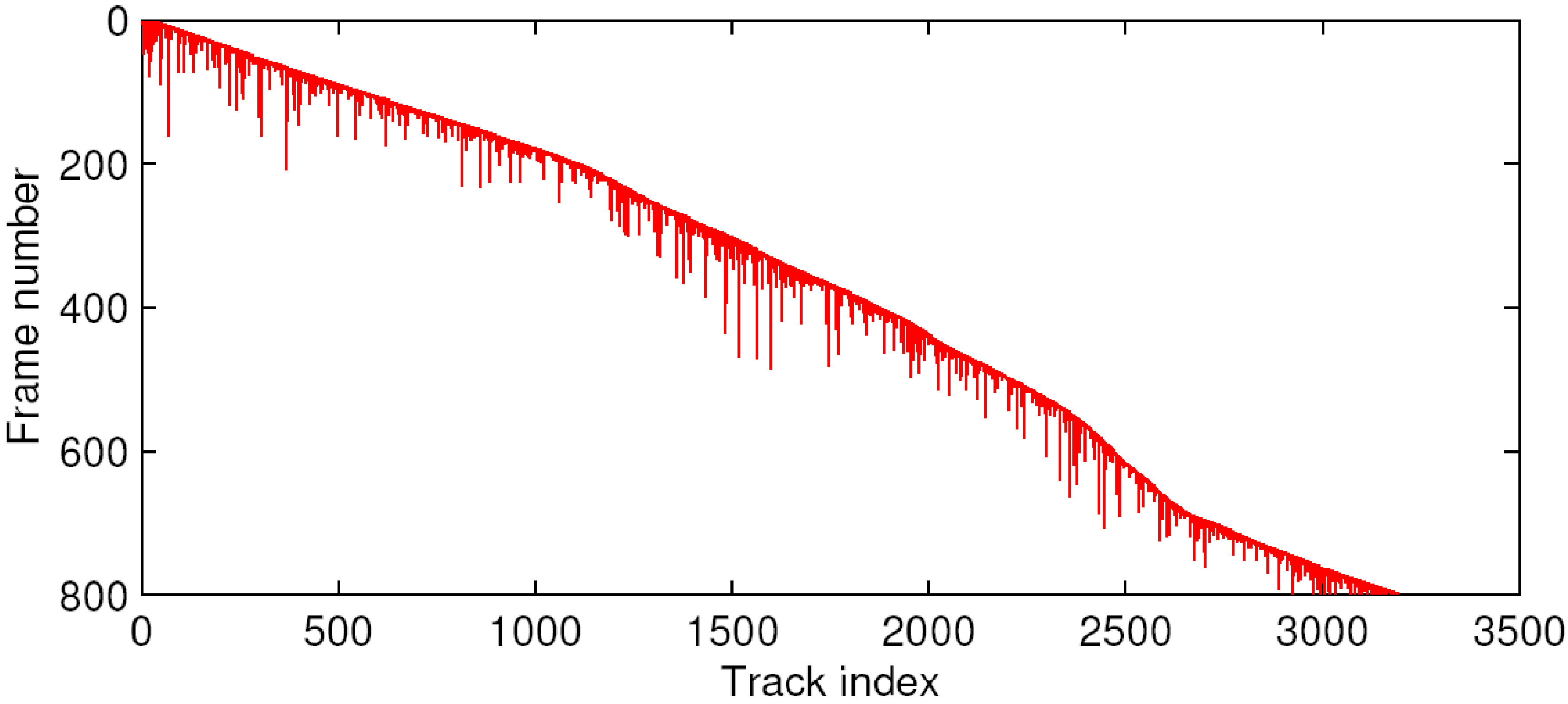
- Track lifetime
- Nonlinear lens distortion
- Degeneracy and critical surfaces
- Prior knowledge and scene constraints
- Multiple motions

Track lifetime



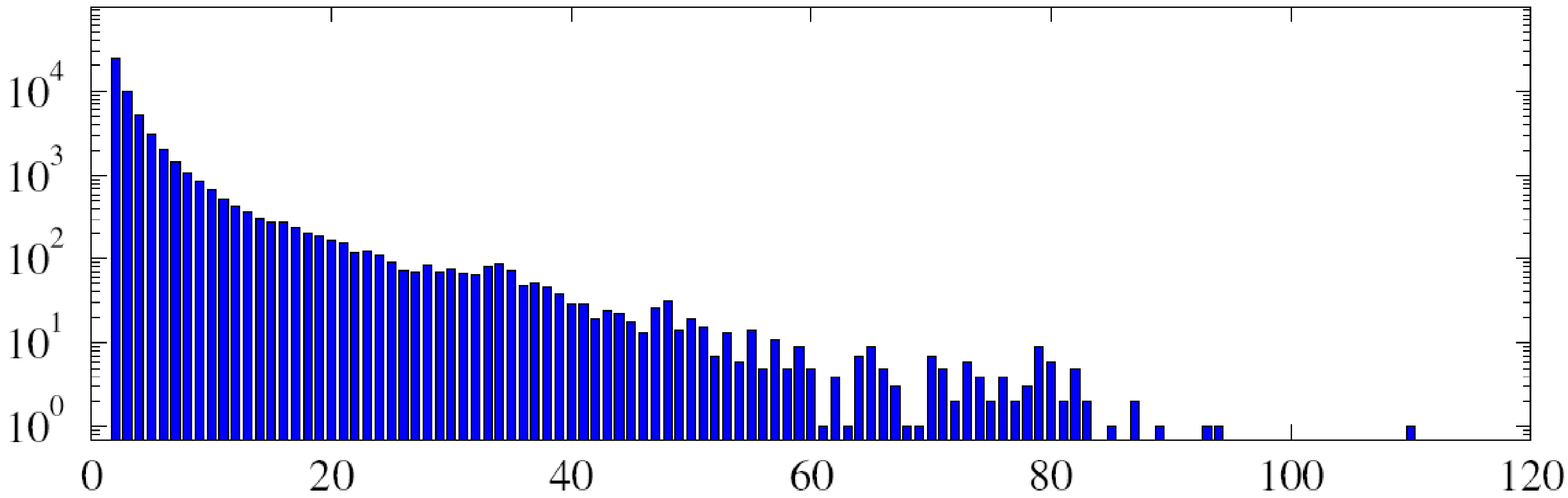
every 50th frame of a 800-frame sequence

Track lifetime



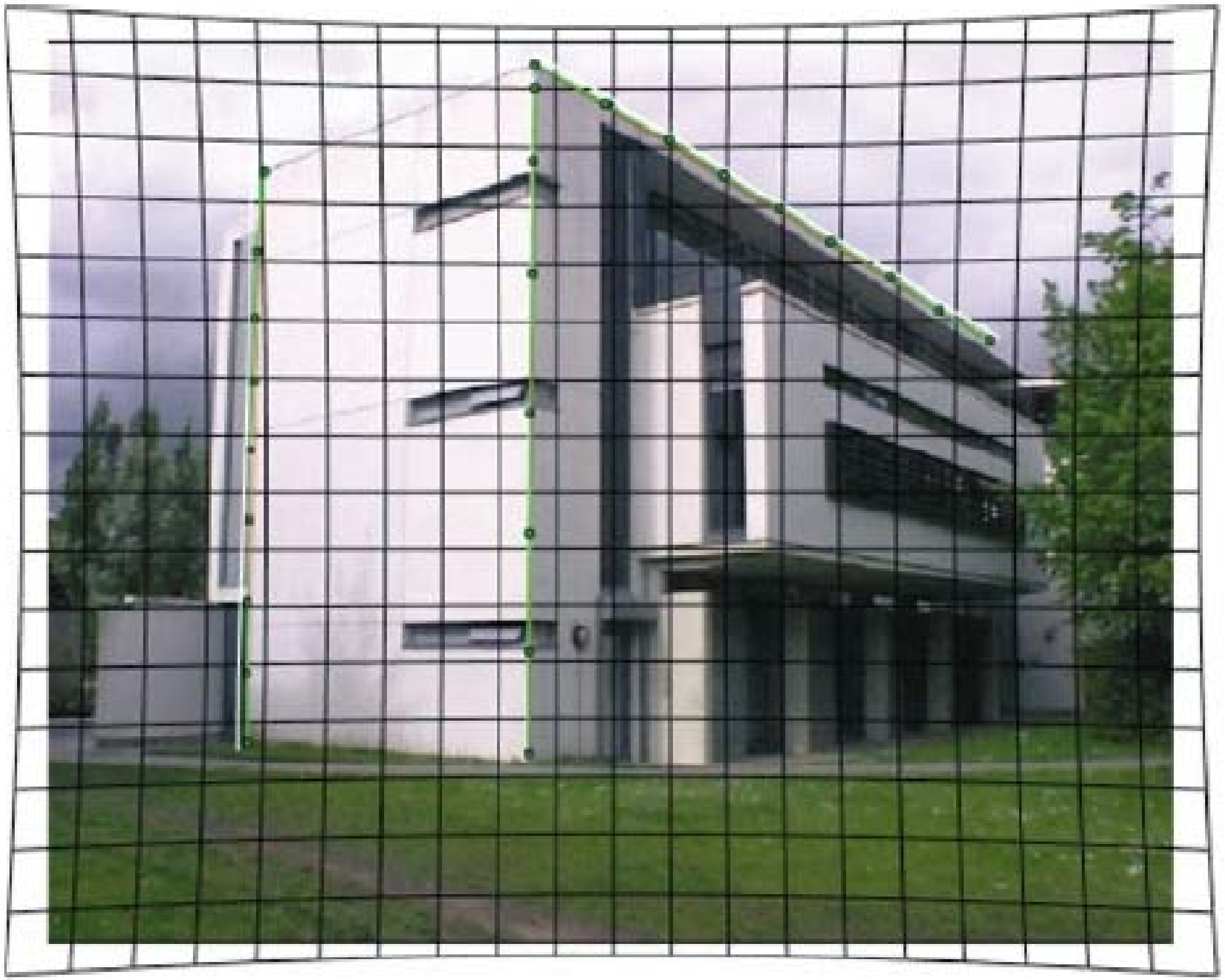
lifetime of 3192 tracks from the previous sequence

Track lifetime

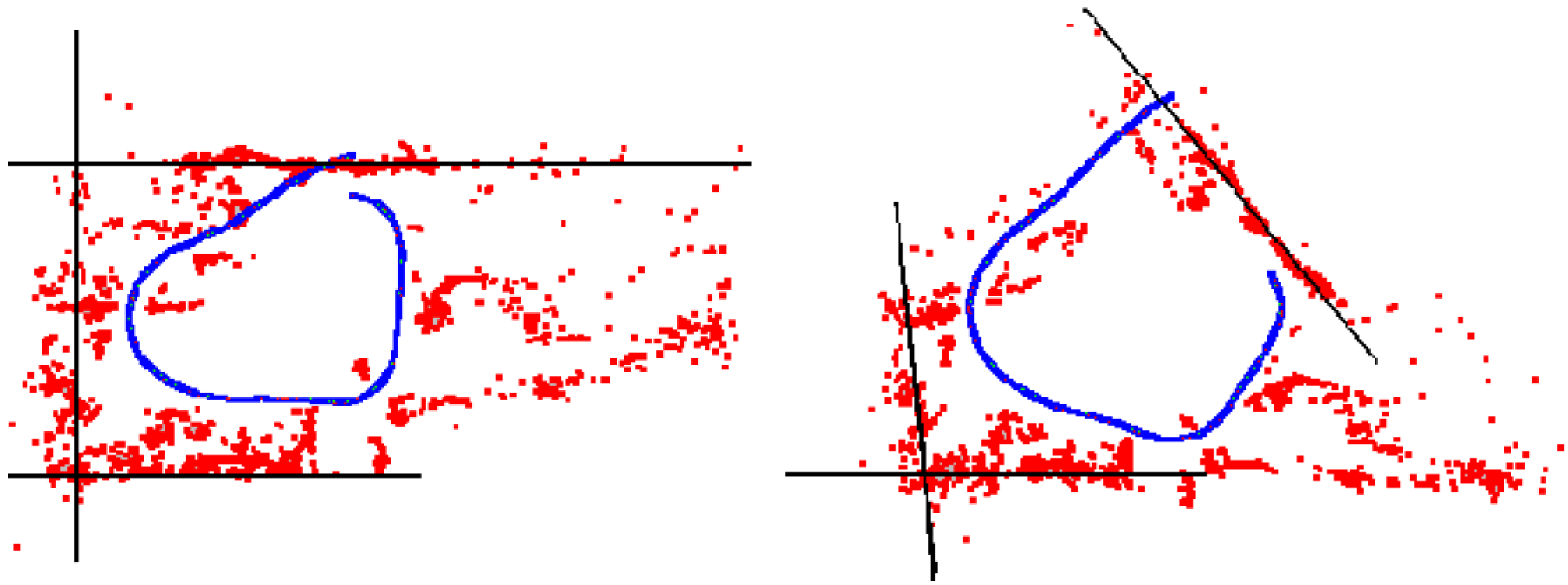


track length histogram

Nonlinear lens distortion



Nonlinear lens distortion

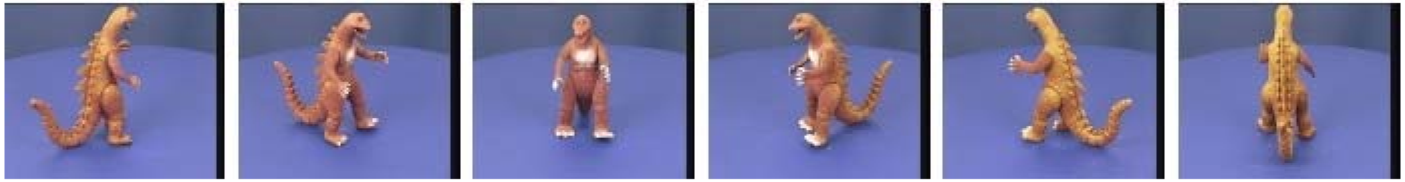


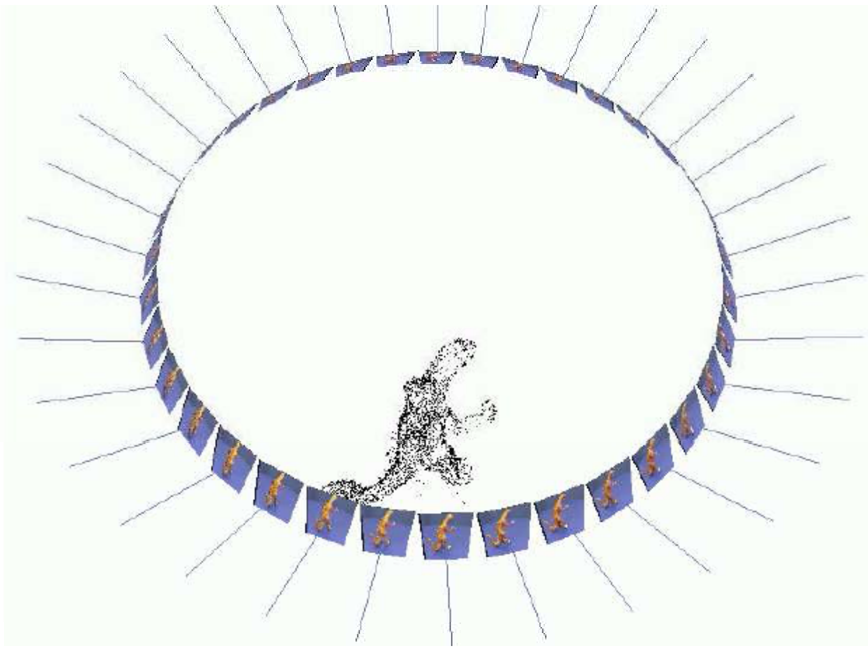
effect of lens distortion

Prior knowledge and scene constraints

add a constraint that several lines are parallel

Prior knowledge and scene constraints





add a constraint that it is a turntable sequence

Applications of matchmove

Jurassic park

2d3 boujou

Enemy at the Gate, Double Negative

2d3 boujou

Enemy at the Gate, Double Negative

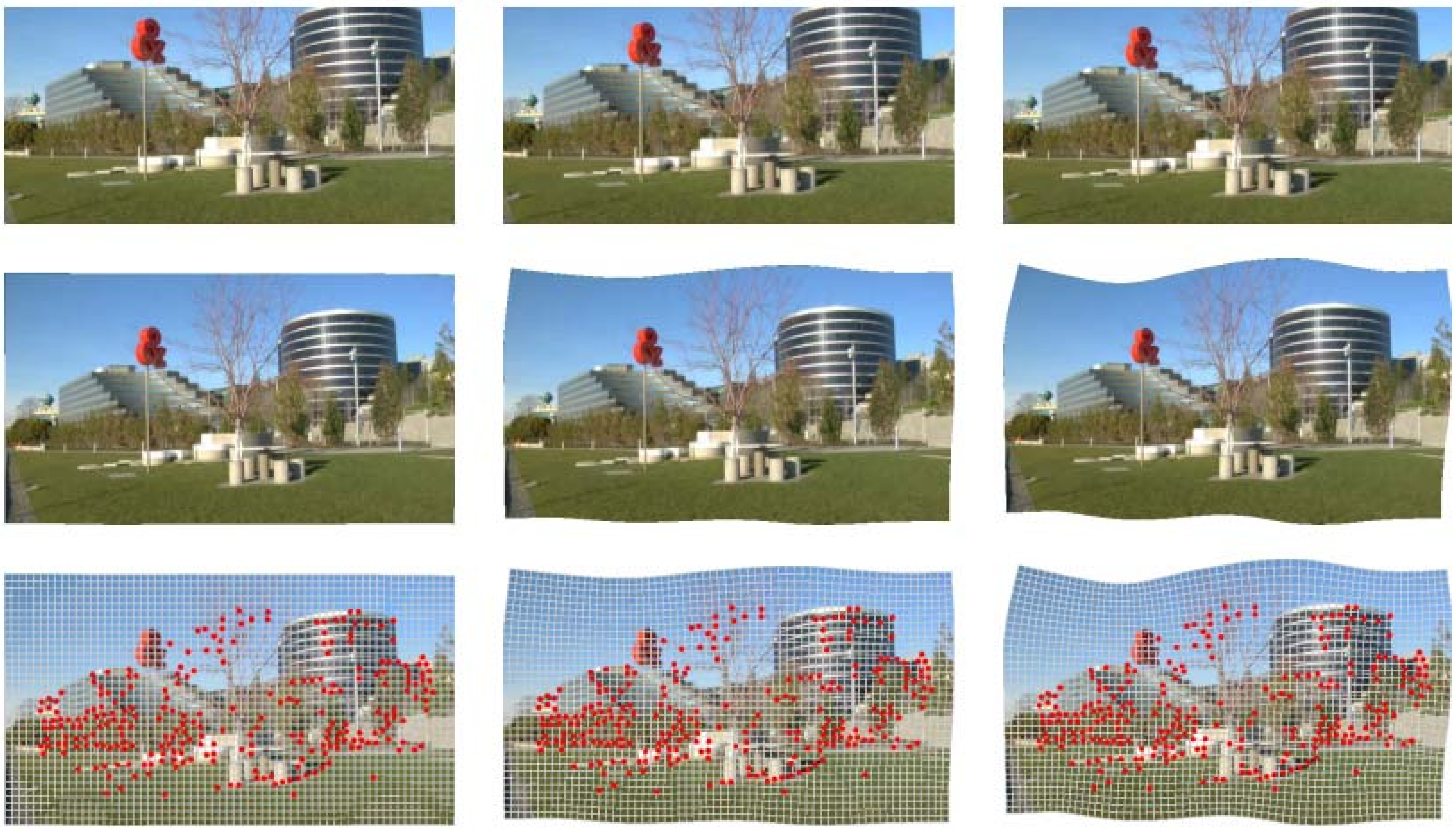
Photo Tourism

(a) (b) (c)

VideoTrace

http://www.acvt.com.au/research/videotrace/

Video stabilization



Project #3 MatchMove

- It is more about using tools in this project
- You can choose either calibration or structure from motion to achieve the goal
- Calibration
- Voodoo/Icarus

- Examples from previous classes, #1, #2

References

- Richard Hartley, In Defense of the 8-point Algorithm, ICCV, 1995.
- Carlo Tomasi and Takeo Kanade, Shape and Motion from Image Streams: A Factorization Method, Proceedings of Natl. Acad. Sci., 1993.
- Manolis Lourakis and Antonis Argyros, The Design and Implementation of a Generic Sparse Bundle Adjustment Software Package Based on the Levenberg-Marquardt Algorithm, FORTH-ICS/TR-320 2004.
- N. Snavely, S. Seitz, R. Szeliski, Photo Tourism: Exploring Photo Collections in 3D, SIGGRAPH 2006.
- A. Hengel et. al., VideoTrace: Rapid Interactive Scene Modelling from Video, SIGGRAPH 2007.