

Image stitching

Digital Visual Effects

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with slides by Richard Szeliski, Steve Seitz, Matthew Brown and Vaclav Hlavac

Image stitching

- Stitching = alignment + blending

↑
geometrical
registration

↑
photometric
registration



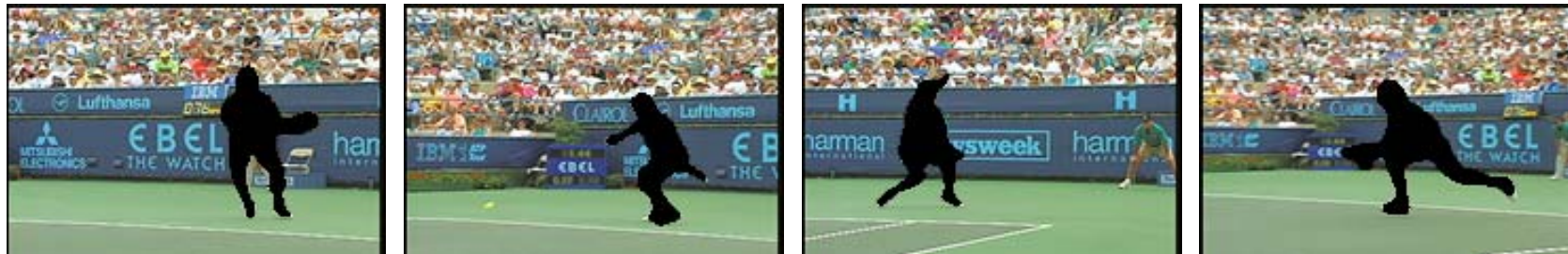
Applications of image stitching

- Video stabilization
- Video summarization
- Video compression
- Video matting
- Panorama creation

Video summarization



Video compression



Object removal



input video

Object removal



remove foreground

Object removal



estimate background

Object removal



background estimation

Panorama creation



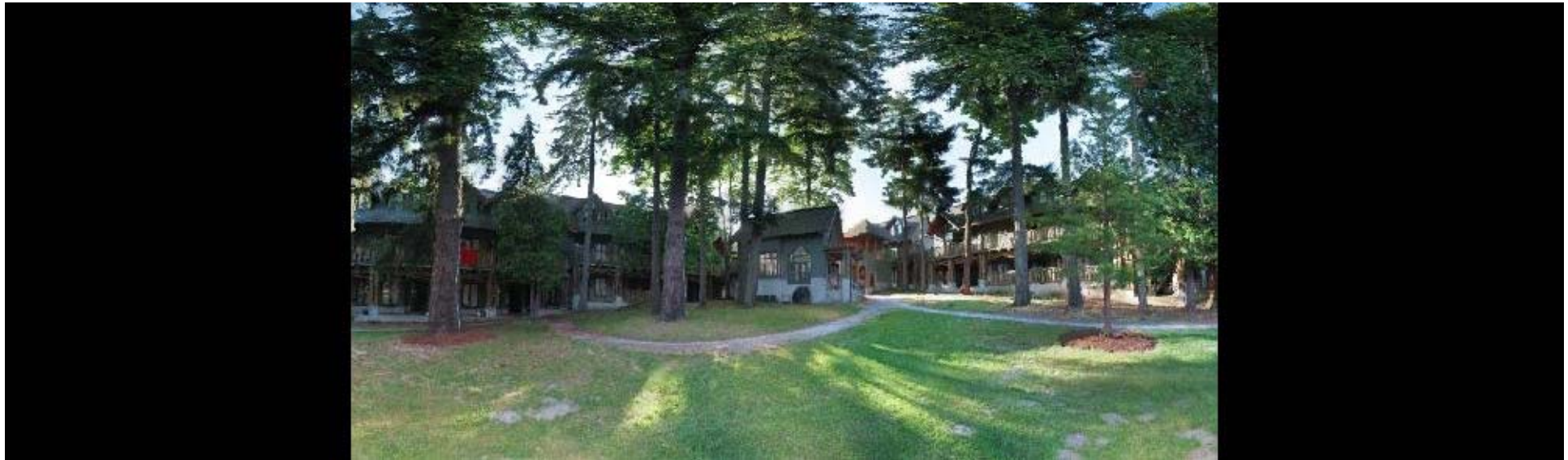
Why panorama?

- Are you getting the whole picture?
 - Compact Camera FOV = 50 x 35°



Why panorama?

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 - Compact Camera FOV = 50 x 35°
 - Human FOV = 200 x 135°



Why panorama?

- Are you getting the whole picture?
 - Compact Camera FOV = 50 x 35°
 - Human FOV = 200 x 135°
 - Panoramic Mosaic = 360 x 180°



Panorama examples

- Like HDR, it is a topic of computational photography, seeking ways to build a better camera mostly in software.
- Most consumer cameras have a panorama mode
- Mars:

http://www.panoramas.dk/fullscreen3/f2_mars97.html

- Earth:

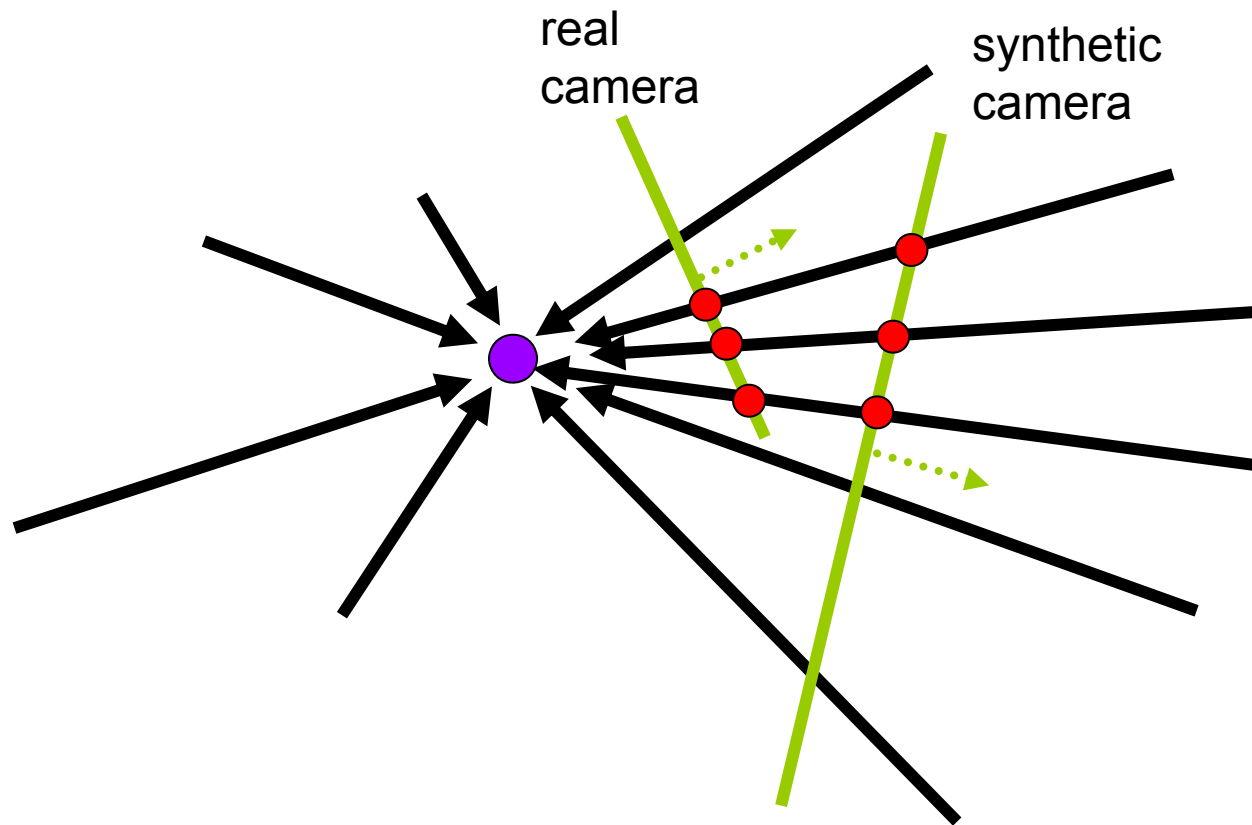
<http://www.panoramas.dk/new-year-2006/taipei.html>

<http://www.360cities.net/>

What can be globally aligned?

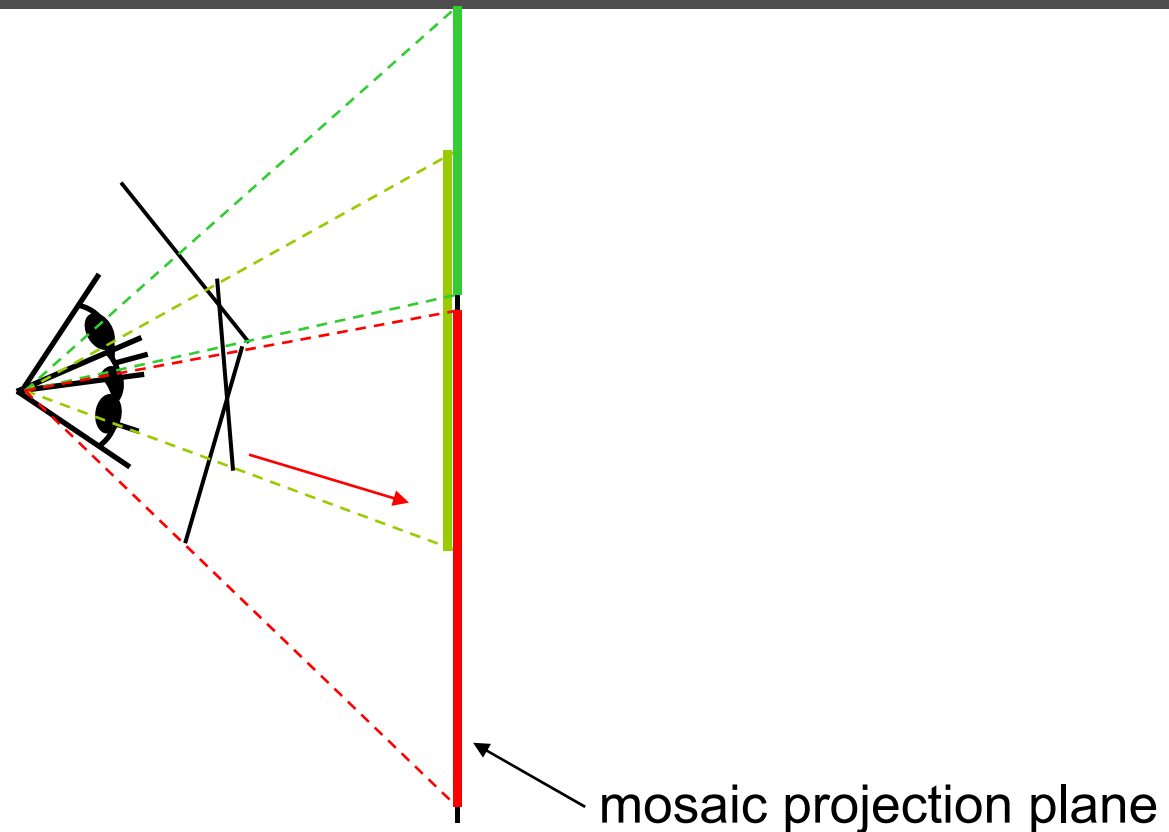
- In image stitching, we seek for a matrix to globally warp one image into another. Are any two images of the same scene can be aligned this way?
 - Images captured with the same center of projection
 - A planar scene or far-away scene

A pencil of rays contains all views



Can generate any synthetic camera view
as long as it has **the same center of projection!**

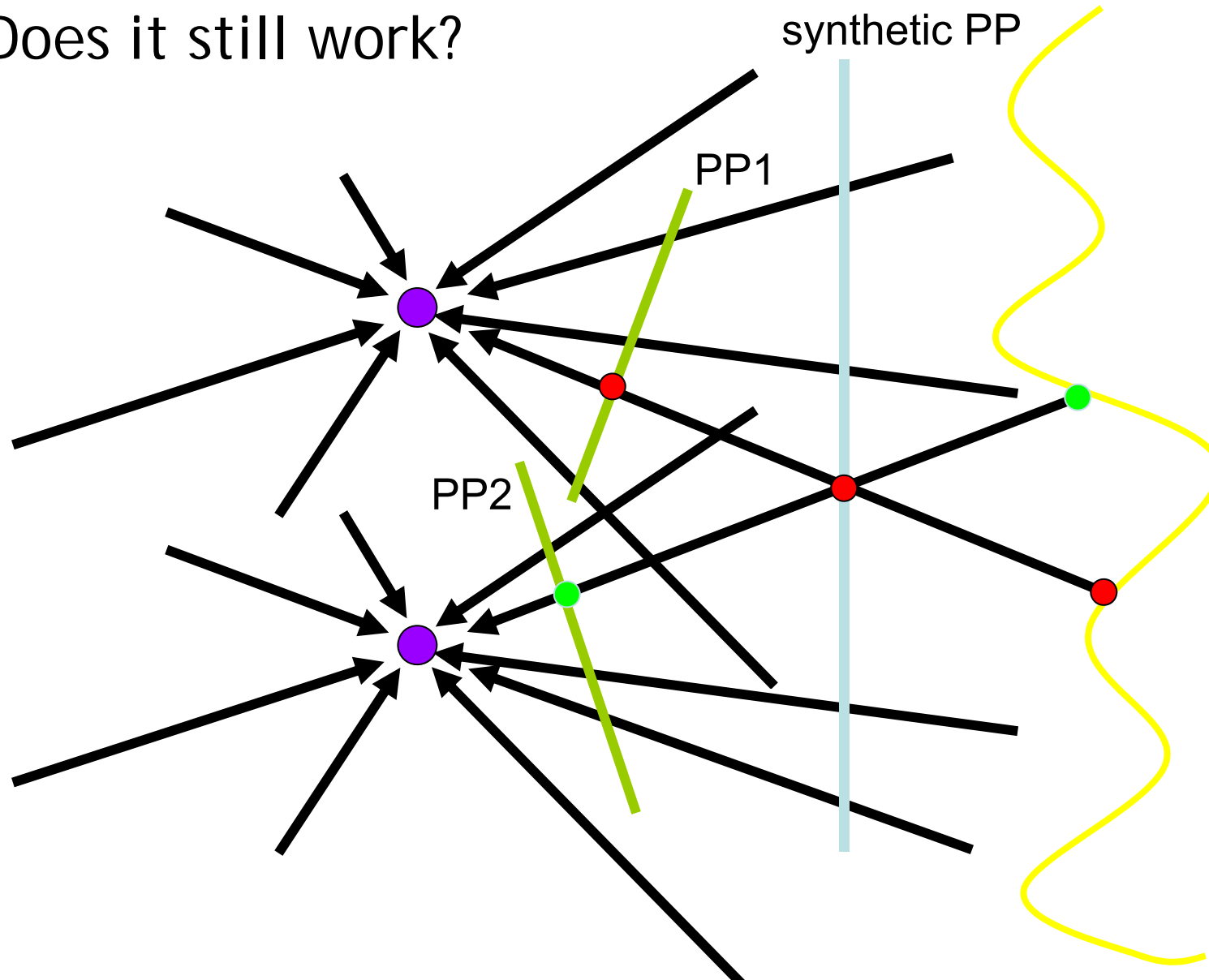
Mosaic as an image reprojection



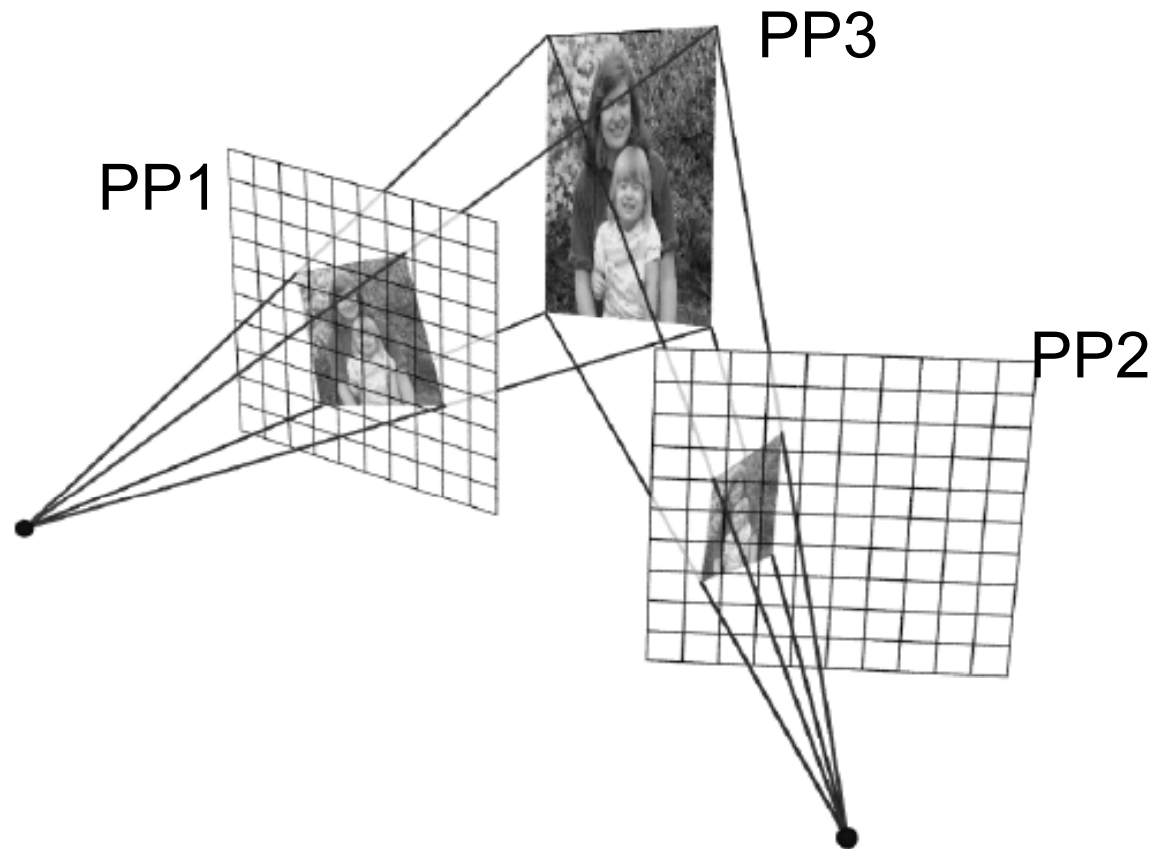
- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

Changing camera center

- Does it still work?



Planar scene (or a faraway one)

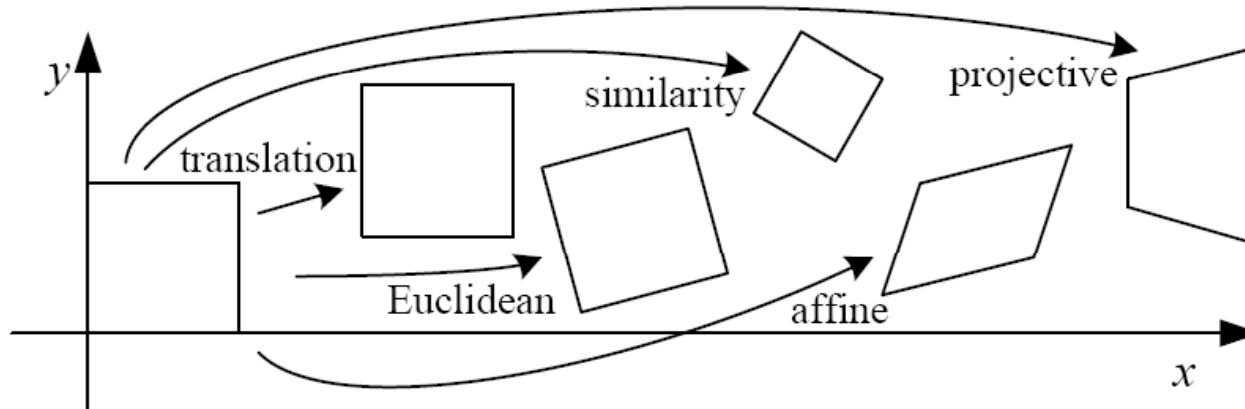


- PP3 is a projection plane of both centers of projection, so we are OK!
- This is how big aerial photographs are made

Motion models

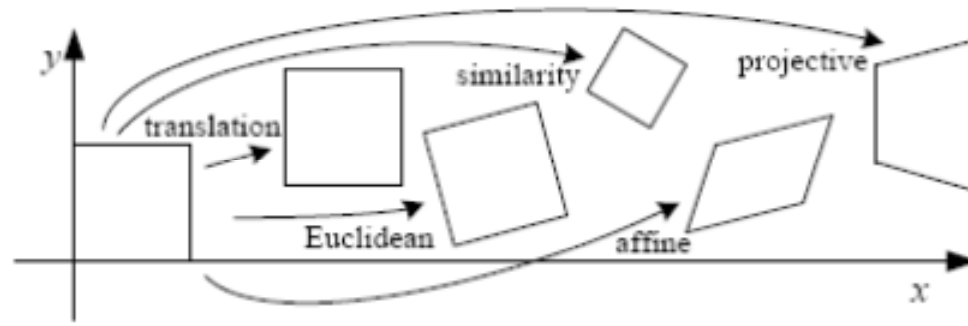
- Parametric models as the assumptions on the relation between two images.

2D Motion models

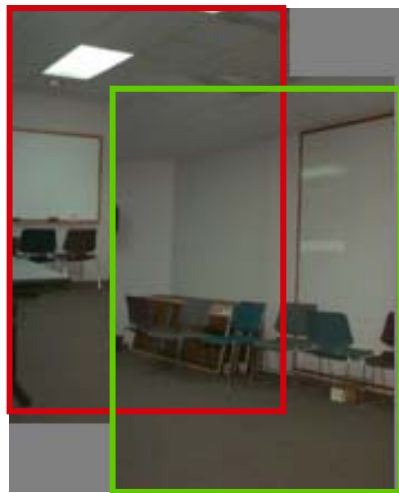


| Name | Matrix | # D.O.F. | Preserves: | Icon |
|-------------------|---|----------|-------------------|------|
| translation | $\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$ | 2 | orientation + ... | |
| rigid (Euclidean) | $\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$ | 3 | lengths + ... | |
| similarity | $\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$ | 4 | angles + ... | |
| affine | $\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$ | 6 | parallelism + ... | |
| projective | $\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$ | 8 | straight lines | |

Motion models

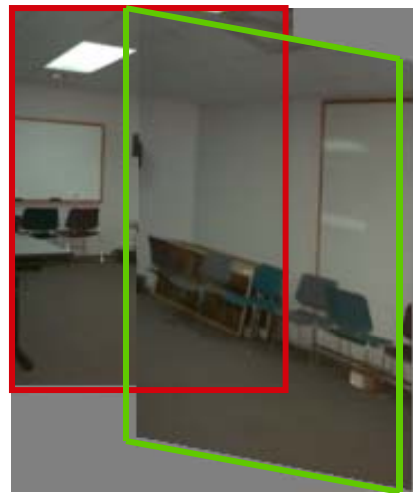


Translation



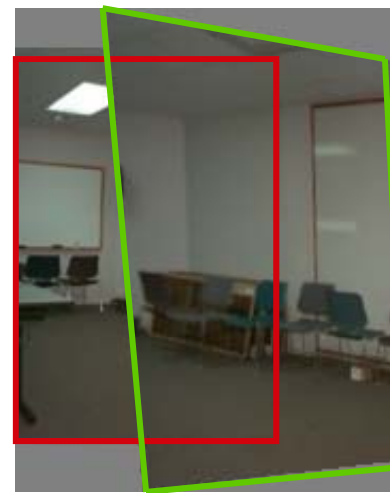
2 unknowns

Affine



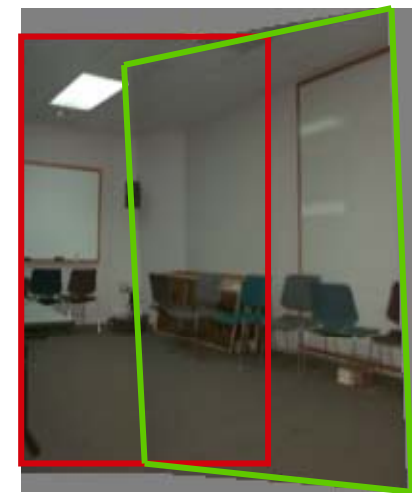
6 unknowns

Perspective



8 unknowns

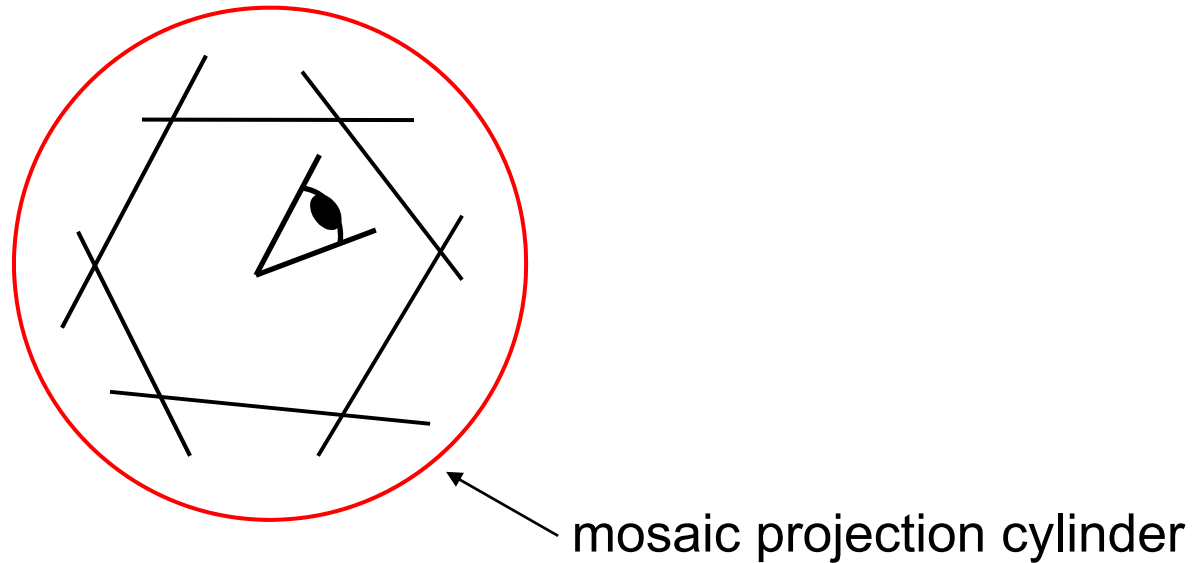
3D rotation



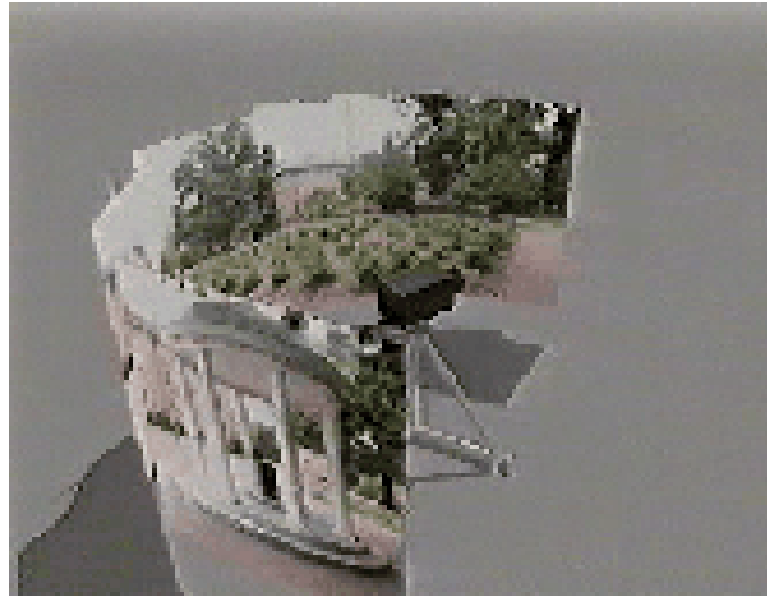
3 unknowns

A case study: cylindrical panorama

- What if you want a 360° field of view?



Cylindrical panoramas



- Steps
 - Reproject each image onto a cylinder
 - Blend
 - Output the resulting mosaic

Cylindrical panorama

1. Take pictures on a tripod (or handheld)
2. Warp to cylindrical coordinate
3. Compute pairwise alignments
4. Fix up the end-to-end alignment
5. Blending
6. Crop the result and import into a viewer

It is required to do radial distortion correction for better stitching results!

Taking pictures



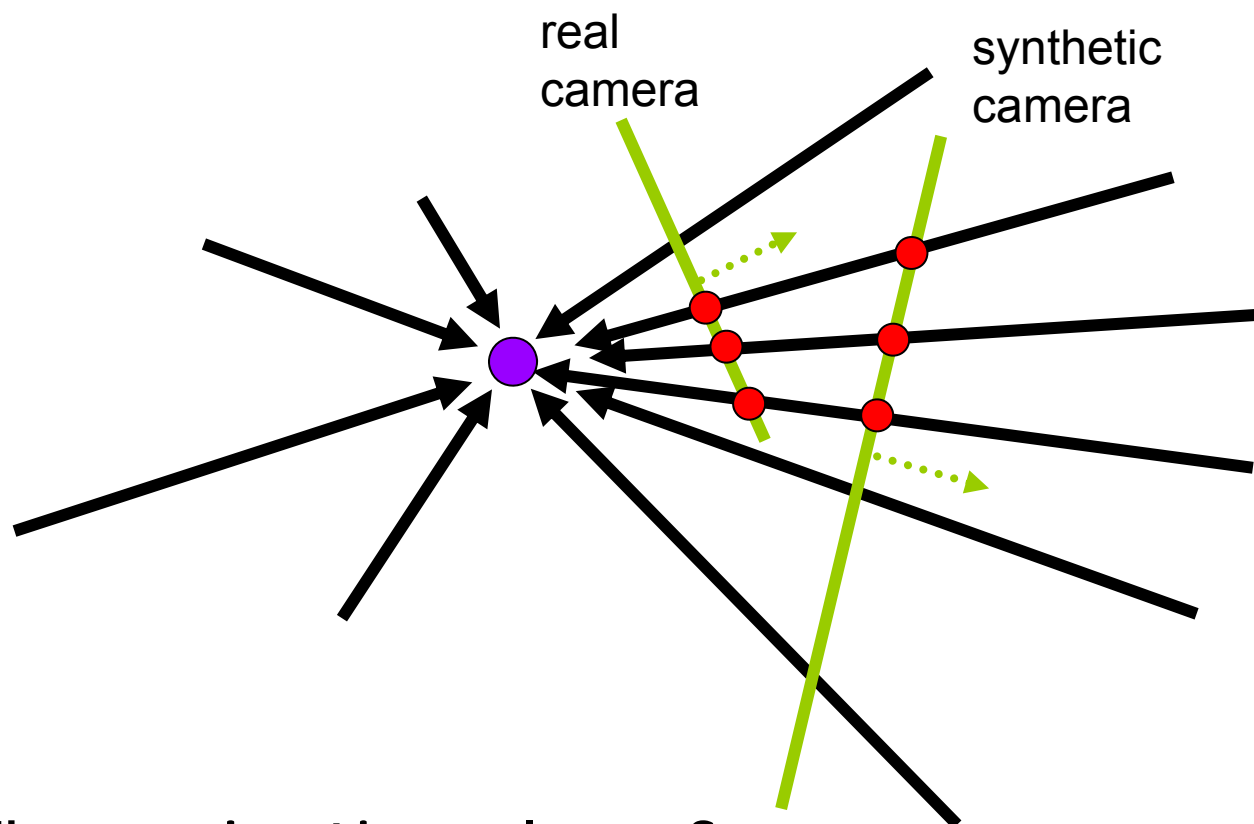
Kaidan panoramic tripod head

Translation model



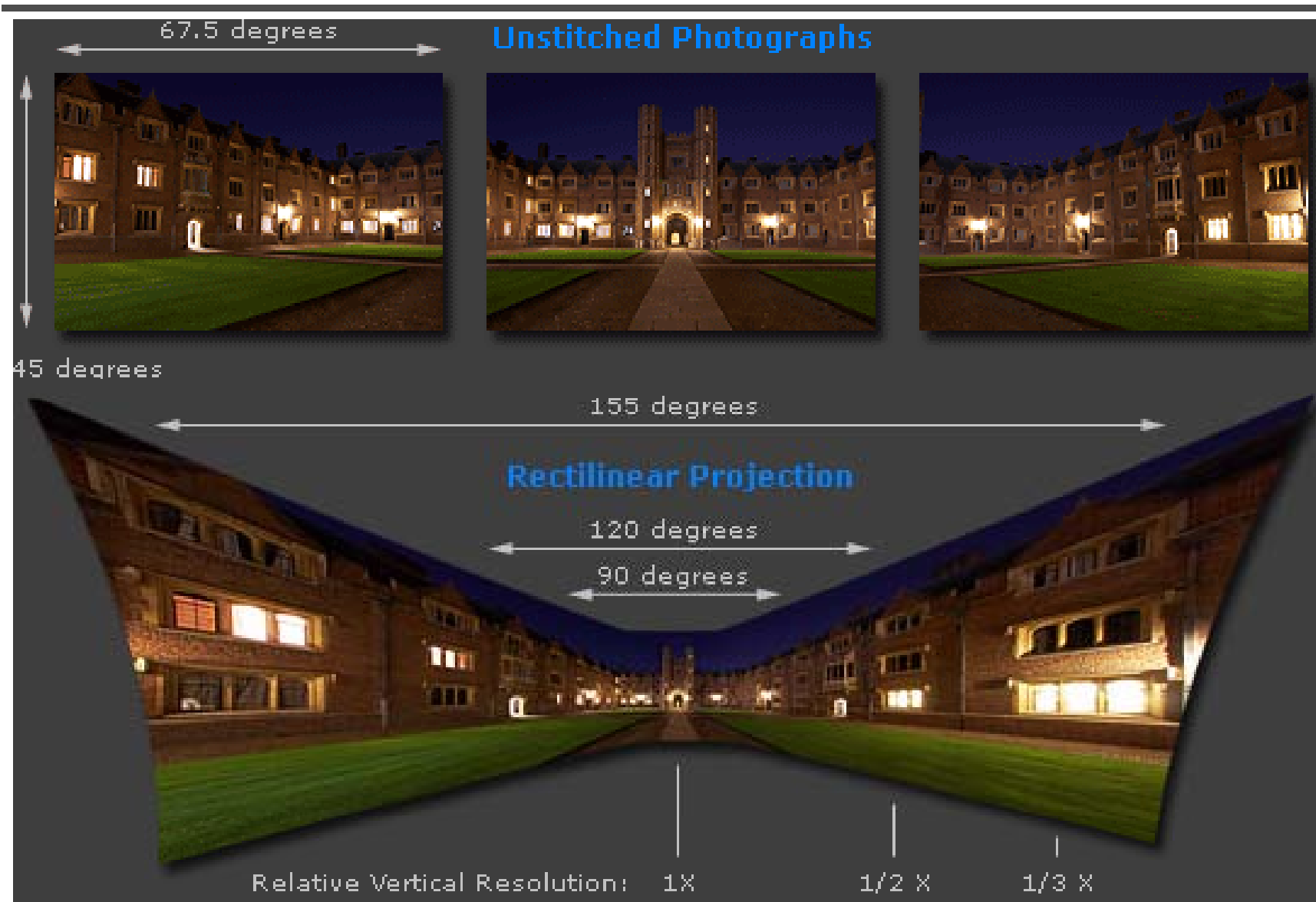
Try to align this in PaintShop Pro

Where should the synthetic camera be

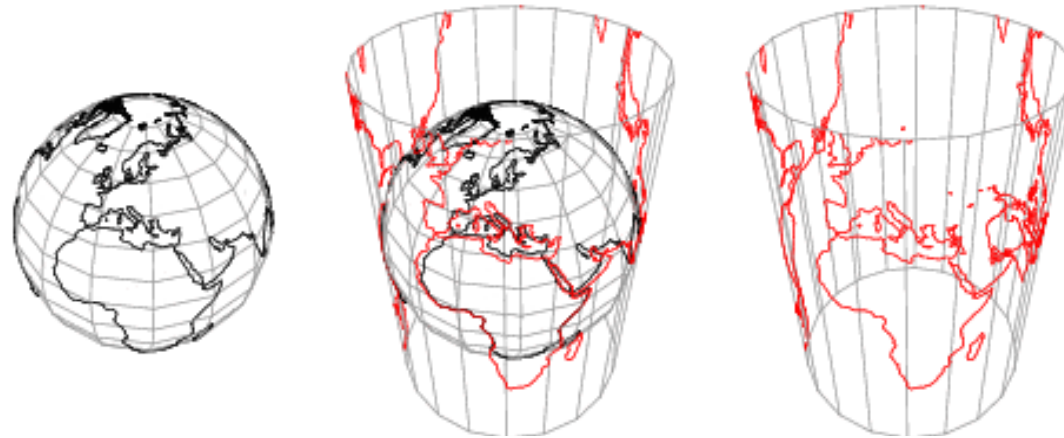


- The projection plan of some camera
- Onto a cylinder

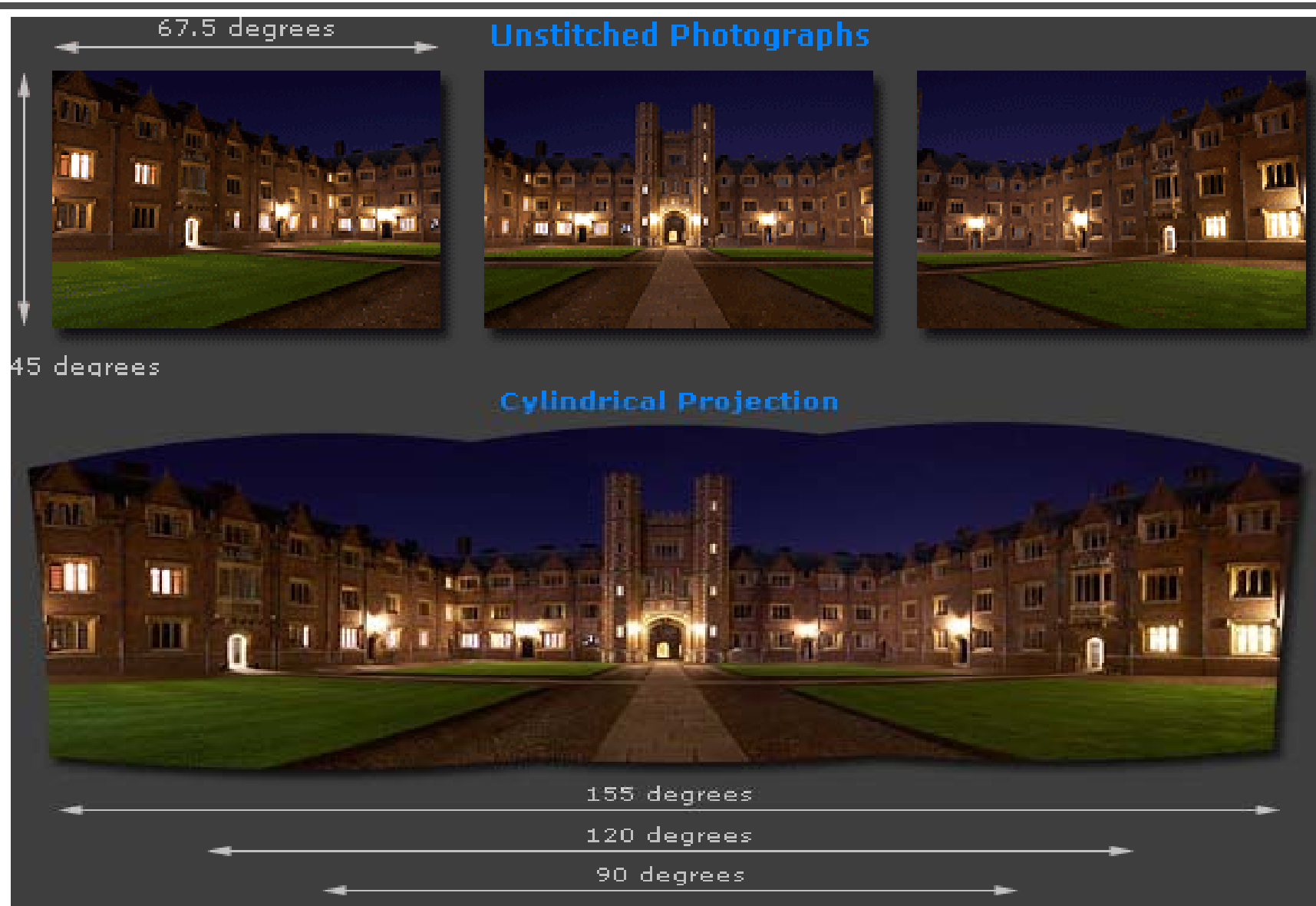
Cylindrical projection



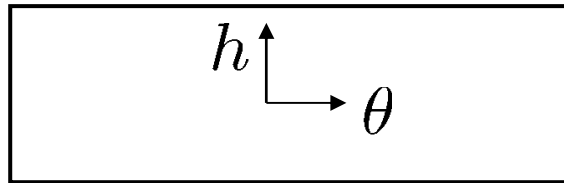
Cylindrical projection



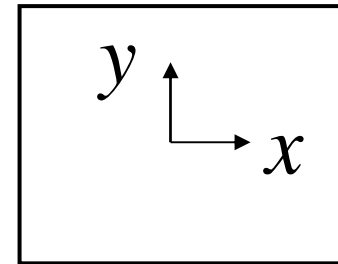
Cylindrical projection



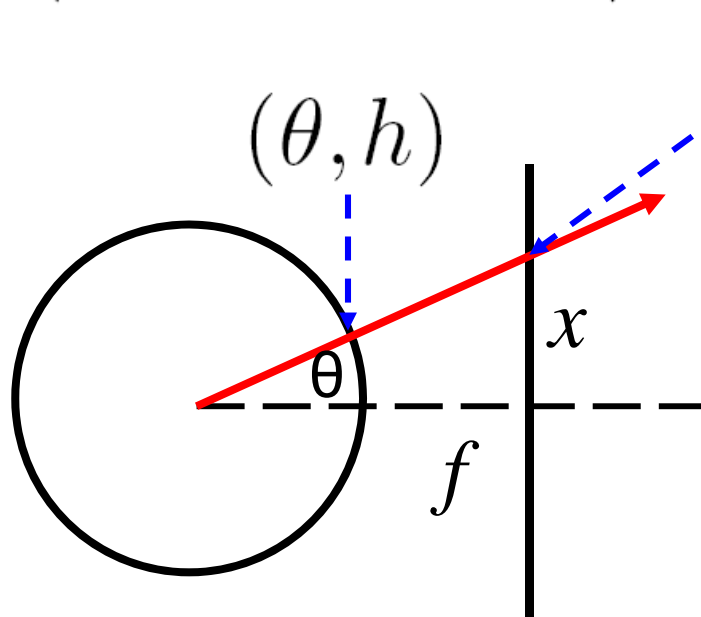
Cylindrical projection



unwrapped cylinder

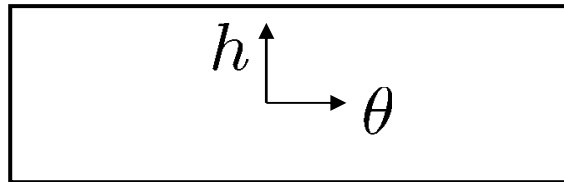


$$(\sin \theta, h, \cos \theta) \propto (x, y, f)$$

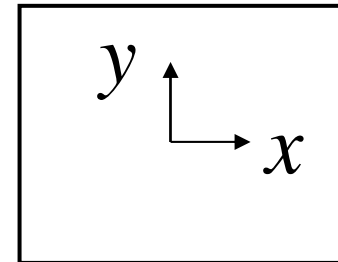


$$\theta = \tan^{-1} \frac{x}{f}$$

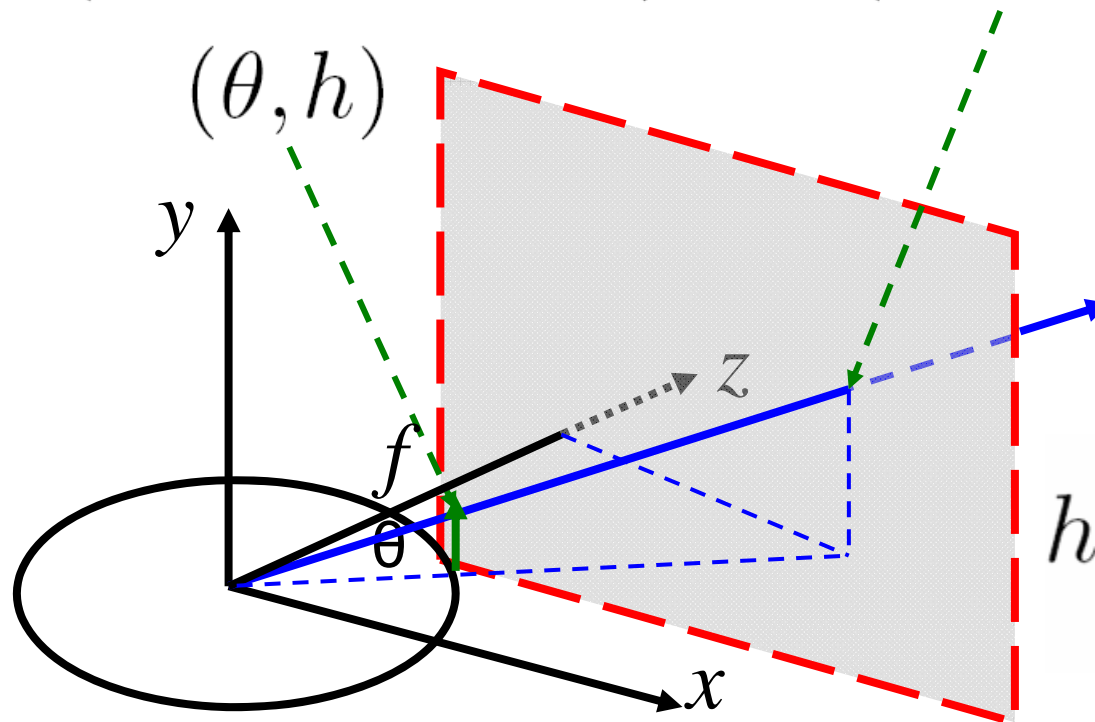
Cylindrical projection



unwrapped cylinder

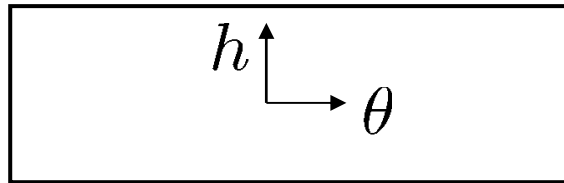


$$(\sin \theta, h, \cos \theta) \propto (x, y, f)$$

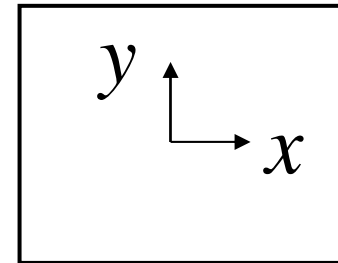


$$h = \frac{y}{\sqrt{x^2 + f^2}}$$

Cylindrical projection



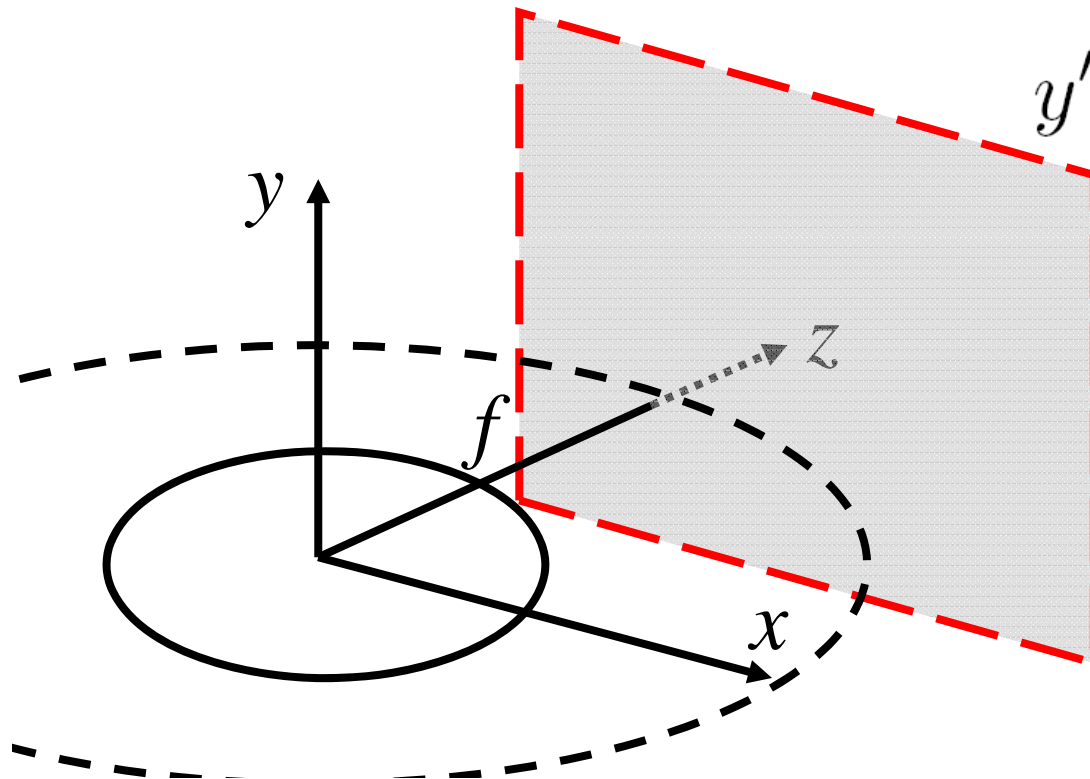
unwrapped cylinder



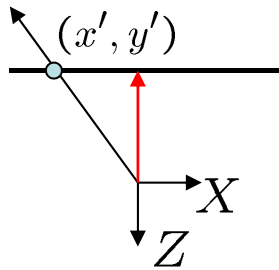
$$x' = s\theta = s \tan^{-1} \frac{x}{f}$$

$$y' = sh = s \frac{y}{\sqrt{x^2 + f^2}}$$

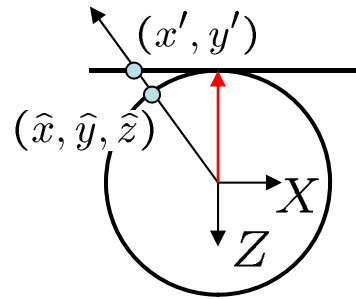
$s=f$ gives less distortion



Cylindrical reprojection



top-down view



Focal length – the dirty secret...

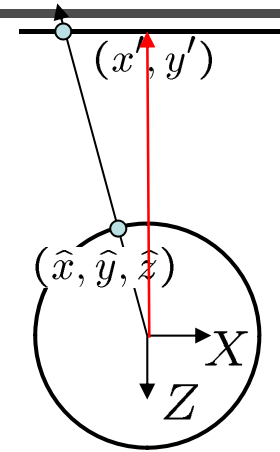
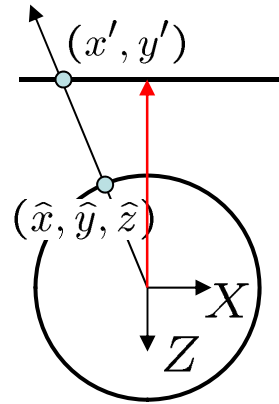


Image 384x300



f = 180 (pixels)

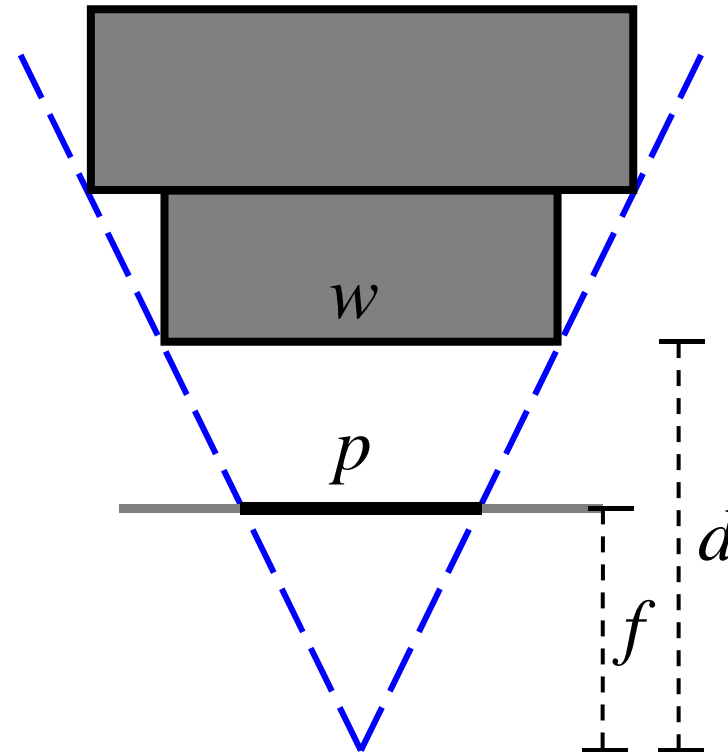
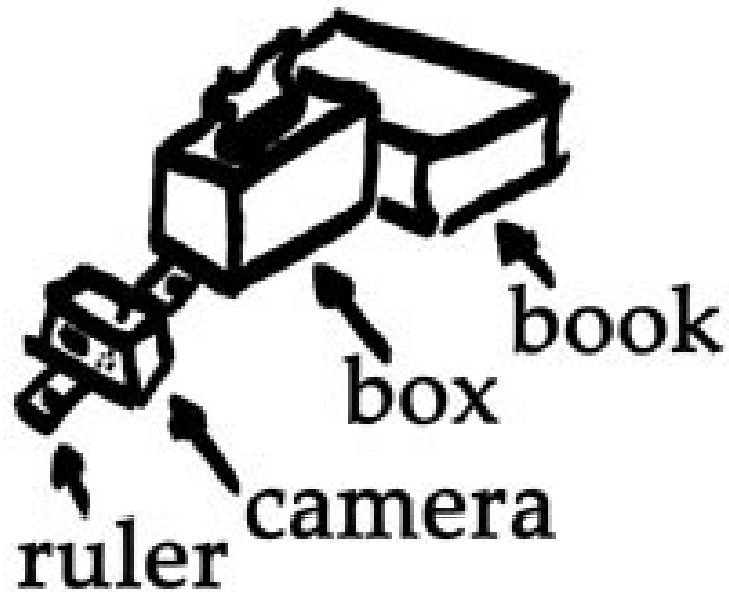


f = 280



f = 380

A simple method for estimating f



Or, you can use other software, such as AutoStitch, to help.

Input images



Cylindrical warping



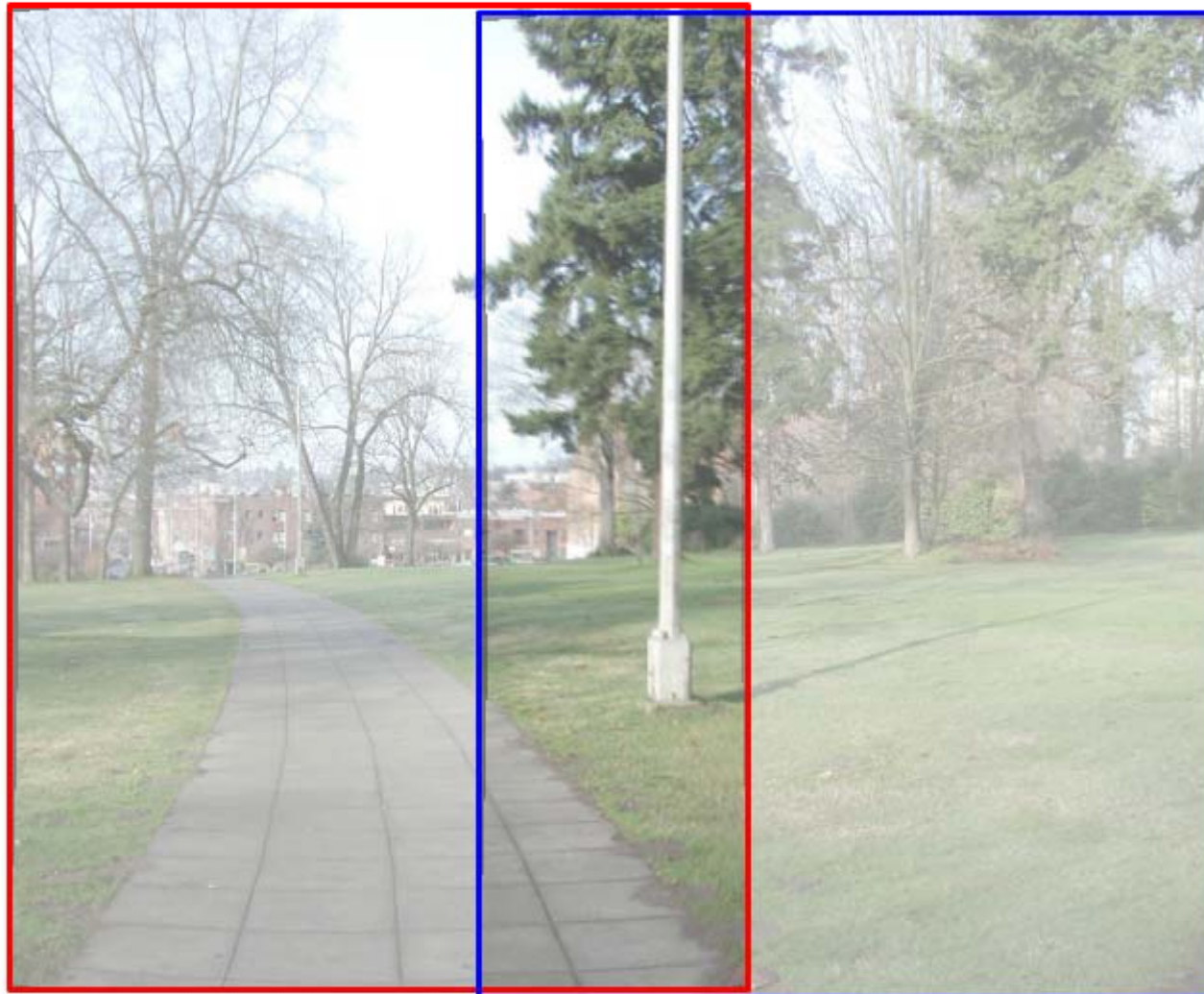
Blending

- Why blending: parallax, lens distortion, scene motion, exposure difference

Blending



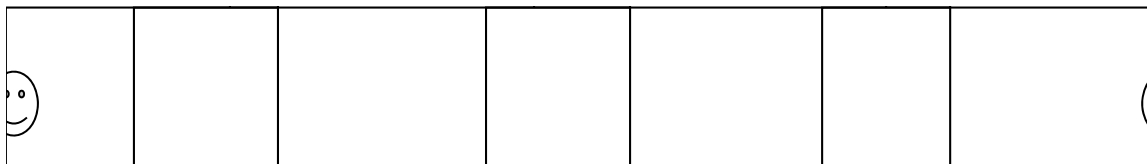
Blending



Blending

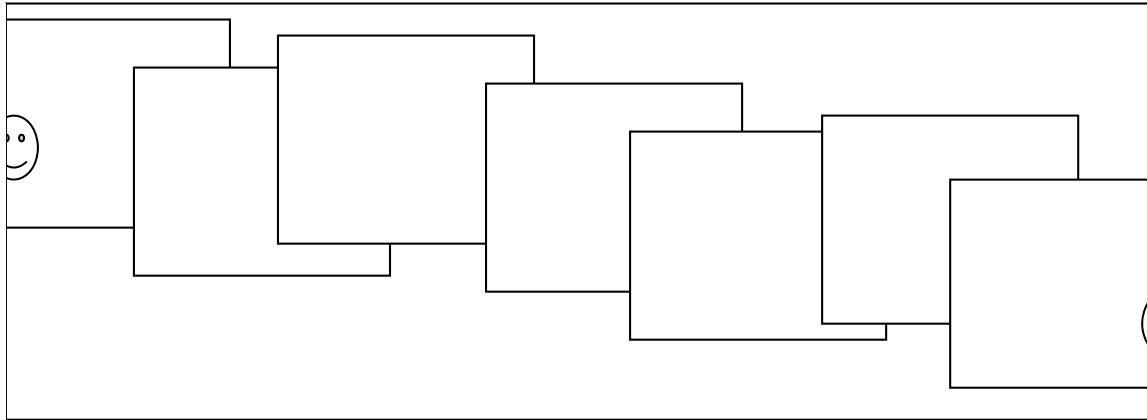


Assembling the panorama



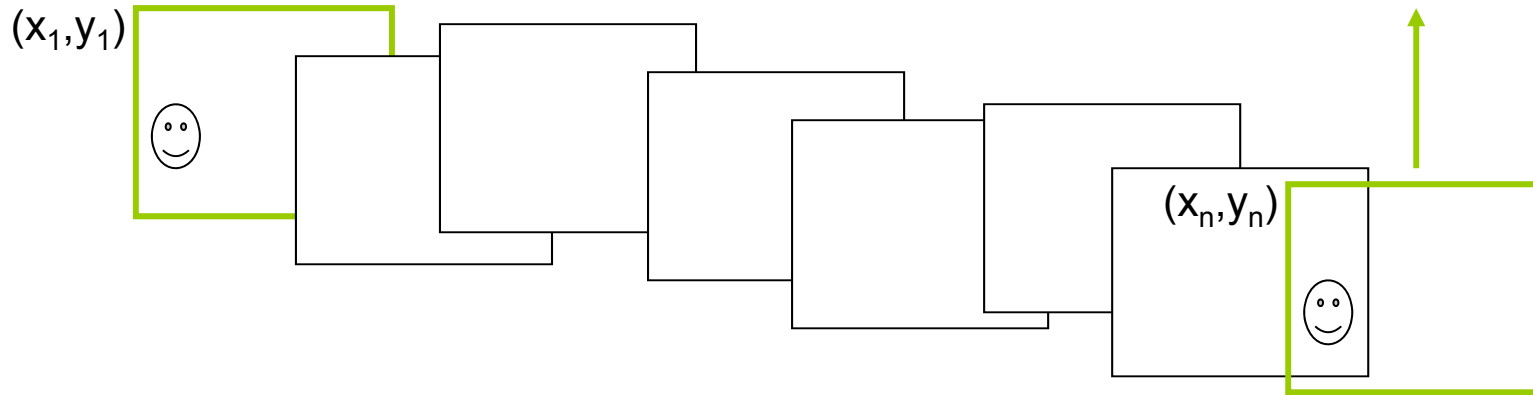
- Stitch pairs together, blend, then crop

Problem: Drift



- Error accumulation
 - small errors accumulate over time

Problem: Drift



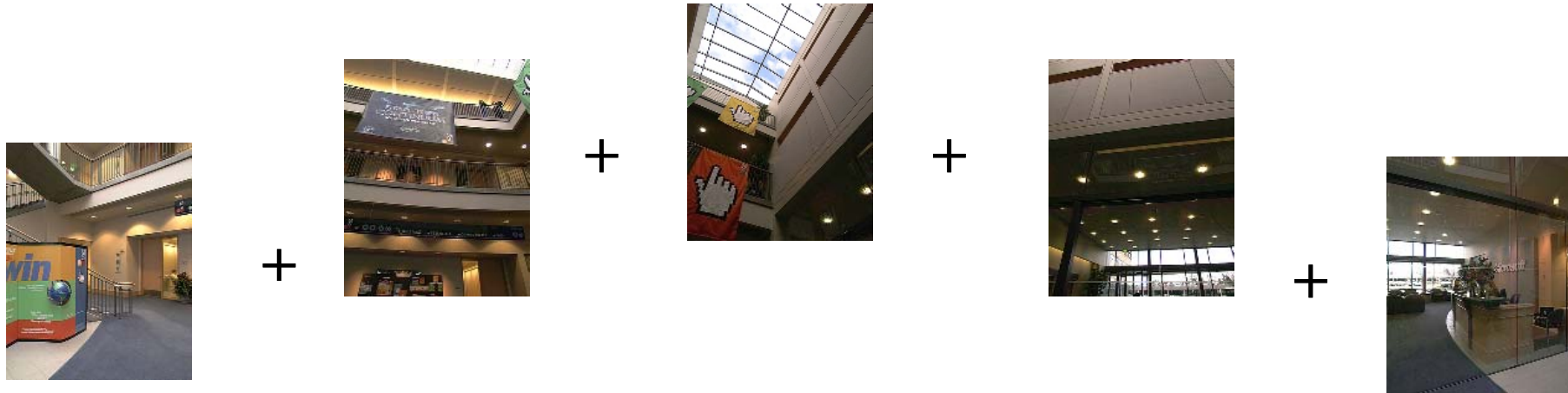
- Solution
 - add another copy of first image at the end
 - there are a bunch of ways to solve this problem
 - add displacement of $(y_1 - y_n)/(n - 1)$ to each image after the first
 - compute a global warp: $y' = y + ax$
 - run a big optimization problem, incorporating this constraint
 - best solution, but more complicated
 - known as “bundle adjustment”

- copy of first image

End-to-end alignment and crop



Viewer: panorama



example: <http://www.cs.washington.edu/education/courses/cse590ss/01wi/projects/project1/students/dougz/index.html>

Viewer: texture mapped model



example: <http://www.panoramas.dk/>

Cylindrical panorama

1. Take pictures on a tripod (or handheld)
2. Warp to cylindrical coordinate
3. Compute pairwise alignments
4. Fix up the end-to-end alignment
5. Blending
6. Crop the result and import into a viewer

Determine pairwise alignment?

- Feature-based methods: only use feature points to estimate parameters
- We will study the “Recognising panorama” paper published in ICCV 2003
- Run SIFT (or other feature algorithms) for each image, find feature matches.

Determine pairwise alignment

- $p' = Mp$, where M is a transformation matrix, p and p' are feature matches
- It is possible to use more complicated models such as affine or perspective
- For example, assume M is a 2x2 matrix

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- Find M with the least square error

$$\sum_{i=1}^n (Mp - p')^2$$

Determine pairwise alignment

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{aligned} x_1 m_{11} + y_1 m_{12} &= x'_1 \\ x_1 m_{21} + y_1 m_{22} &= y'_1 \end{aligned}$$

- Overdetermined system

$$\begin{pmatrix} x_1 & y_1 & 0 & 0 \\ 0 & 0 & x_1 & y_1 \\ x_2 & y_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 0 & 0 \\ 0 & 0 & x_n & y_n \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \end{pmatrix} = \begin{pmatrix} x'_1 \\ y'_1 \\ x'_2 \\ \vdots \\ x'_n \\ y'_n \end{pmatrix}$$

Normal equation

Given an overdetermined system

$$\mathbf{Ax} = \mathbf{b}$$

the normal equation is that which minimizes the sum of the square differences between left and right sides

$$\mathbf{A}^T \mathbf{Ax} = \mathbf{A}^T \mathbf{b}$$

Why?

Normal equation

$$E(\mathbf{x}) = (\mathbf{Ax} - \mathbf{b})^2$$

$$\begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$n \times m$, n equations, m variables

Normal equation

$$\mathbf{Ax} - \mathbf{b} = \begin{bmatrix} \sum_{j=1}^m a_{1j} x_j \\ \vdots \\ \sum_{j=1}^m a_{ij} x_j \\ \vdots \\ \sum_{j=1}^m a_{nj} x_j \end{bmatrix} - \begin{bmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} \left(\sum_{j=1}^m a_{1j} x_j \right) - b_1 \\ \vdots \\ \left(\sum_{j=1}^m a_{ij} x_j \right) - b_i \\ \vdots \\ \left(\sum_{j=1}^m a_{nj} x_j \right) - b_n \end{bmatrix}$$
$$E(\mathbf{x}) = (\mathbf{Ax} - \mathbf{b})^2 = \sum_{i=1}^n \left[\left(\sum_{j=1}^m a_{ij} x_j \right) - b_i \right]^2$$

Normal equation

$$E(\mathbf{x}) = (\mathbf{Ax} - \mathbf{b})^2 = \sum_{i=1}^n \left[\left(\sum_{j=1}^m a_{ij} x_j \right) - b_i \right]^2$$

$$0 = \frac{\partial E}{\partial x_1} = \sum_{i=1}^n 2 \left[\left(\sum_{j=1}^m a_{ij} x_j \right) - b_i \right] a_{i1}$$

$$= 2 \sum_{i=1}^n a_{i1} \sum_{j=1}^m a_{ij} x_j - 2 \sum_{i=1}^n a_{i1} b_i$$

$$0 = \frac{\partial E}{\partial \mathbf{x}} = 2(\mathbf{A}^T \mathbf{Ax} - \mathbf{A}^T \mathbf{b}) \rightarrow \mathbf{A}^T \mathbf{Ax} = \mathbf{A}^T \mathbf{b}$$

Normal equation

$$\begin{aligned} & (\mathbf{Ax} - \mathbf{b})^2 \\ &= (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b}) \\ &= \left((\mathbf{Ax})^T - \mathbf{b}^T \right) (\mathbf{Ax} - \mathbf{b}) \\ &= \left(\mathbf{x}^T \mathbf{A}^T - \mathbf{b}^T \right) (\mathbf{Ax} - \mathbf{b}) \\ &= \mathbf{x}^T \mathbf{A}^T \mathbf{Ax} - \mathbf{b}^T \mathbf{Ax} - \mathbf{x}^T \mathbf{A}^T \mathbf{b} + \mathbf{b}^T \mathbf{b} \\ &= \mathbf{x}^T \mathbf{A}^T \mathbf{Ax} - \left(\mathbf{A}^T \mathbf{b} \right)^T \mathbf{x} - \left(\mathbf{A}^T \mathbf{b} \right)^T \mathbf{x} + \mathbf{b}^T \mathbf{b} \end{aligned}$$

$$\frac{\partial E}{\partial \mathbf{x}} = 2\mathbf{A}^T \mathbf{Ax} - 2\mathbf{A}^T \mathbf{b}$$

Determine pairwise alignment

- $p' = Mp$, where M is a transformation matrix, p and p' are feature matches
- For translation model, it is easier.

$$E = \sum_{i=1}^n \left[(m_1 + x_i - x'_i)^2 + (m_2 + y_i - y'_i)^2 \right]$$

$$0 = \frac{\partial E}{\partial m_1}$$

- What if the match is false? Avoid impact of outliers.

RANSAC

- RANSAC = Random Sample Consensus
- An algorithm for robust fitting of models in the presence of many data outliers
- Compare to robust statistics
- Given N data points x_i , assume that majority of them are generated from a model with parameters Θ , try to recover Θ .

RANSAC algorithm

Run k times: ← How many times?

(1) draw n samples randomly ← How big?
Smaller is better

(2) fit parameters Θ with these n samples

(3) for each of other $N-n$ points, calculate
its distance to the fitted model, count the
number of inlier points c

Output Θ with the largest c

How to define?
Depends on the problem.

How to determine k

p : probability of real inliers

P : probability of success after k trials

$$P = 1 - (1 - p^n)^k$$



n samples are all inliers



a failure



failure after k trials

$$k = \frac{\log(1 - P)}{\log(1 - p^n)}$$

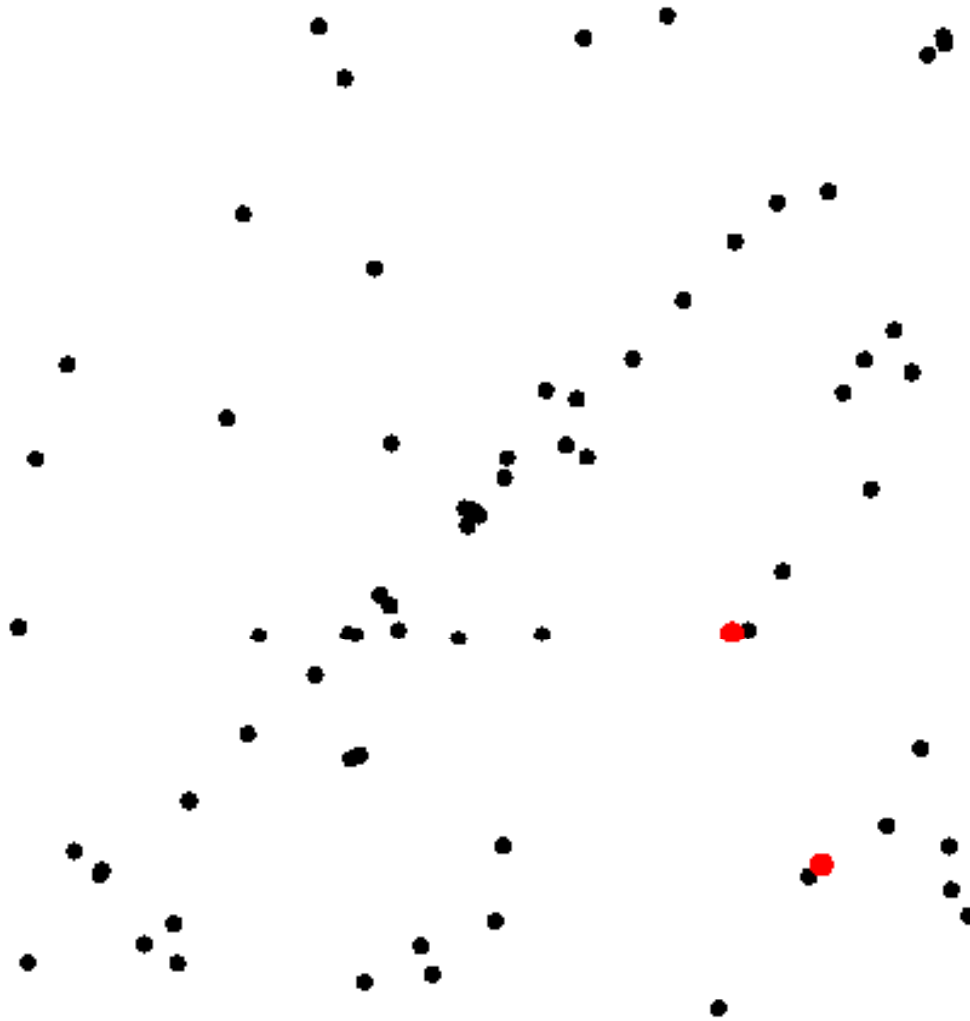
for $P=0.99$

| n | p | k |
|-----|-----|-----|
| 3 | 0.5 | 35 |
| 6 | 0.6 | 97 |
| 6 | 0.5 | 293 |

Example: line fitting

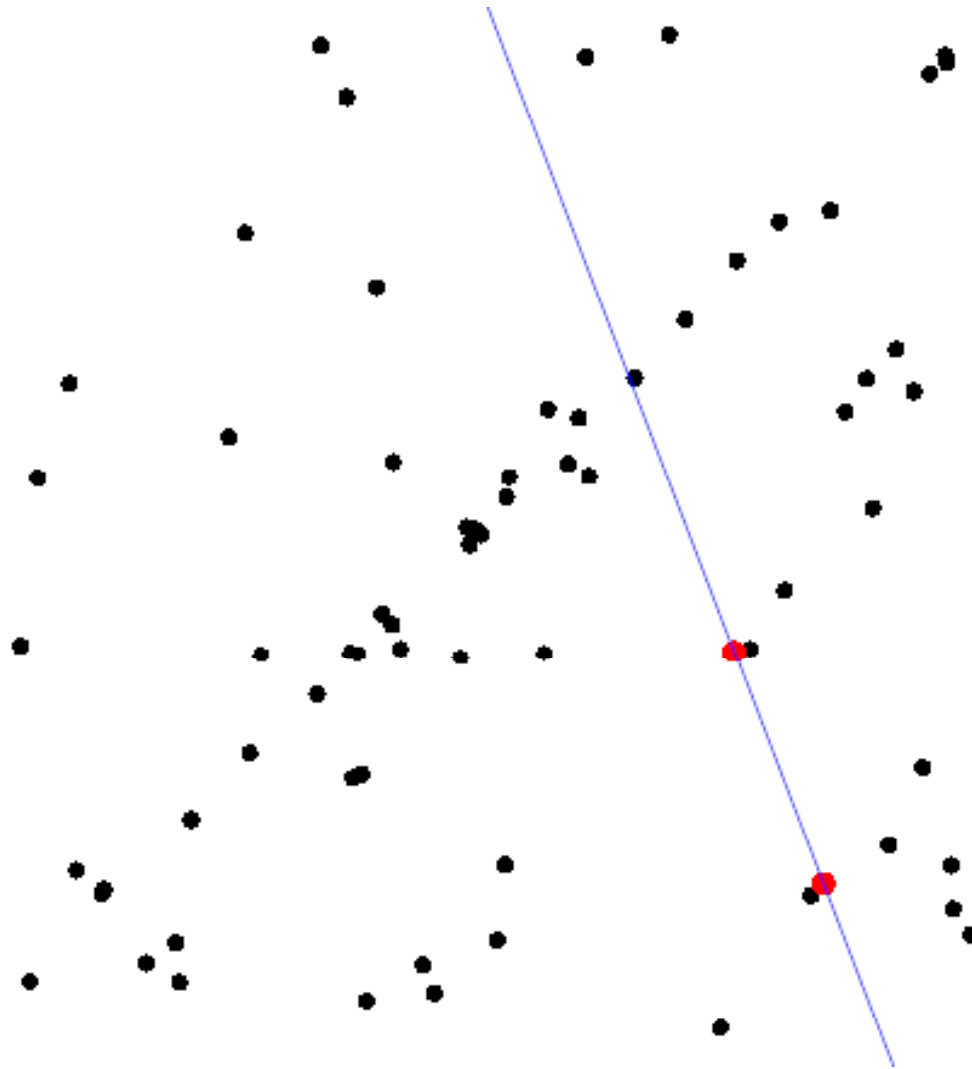


Example: line fitting

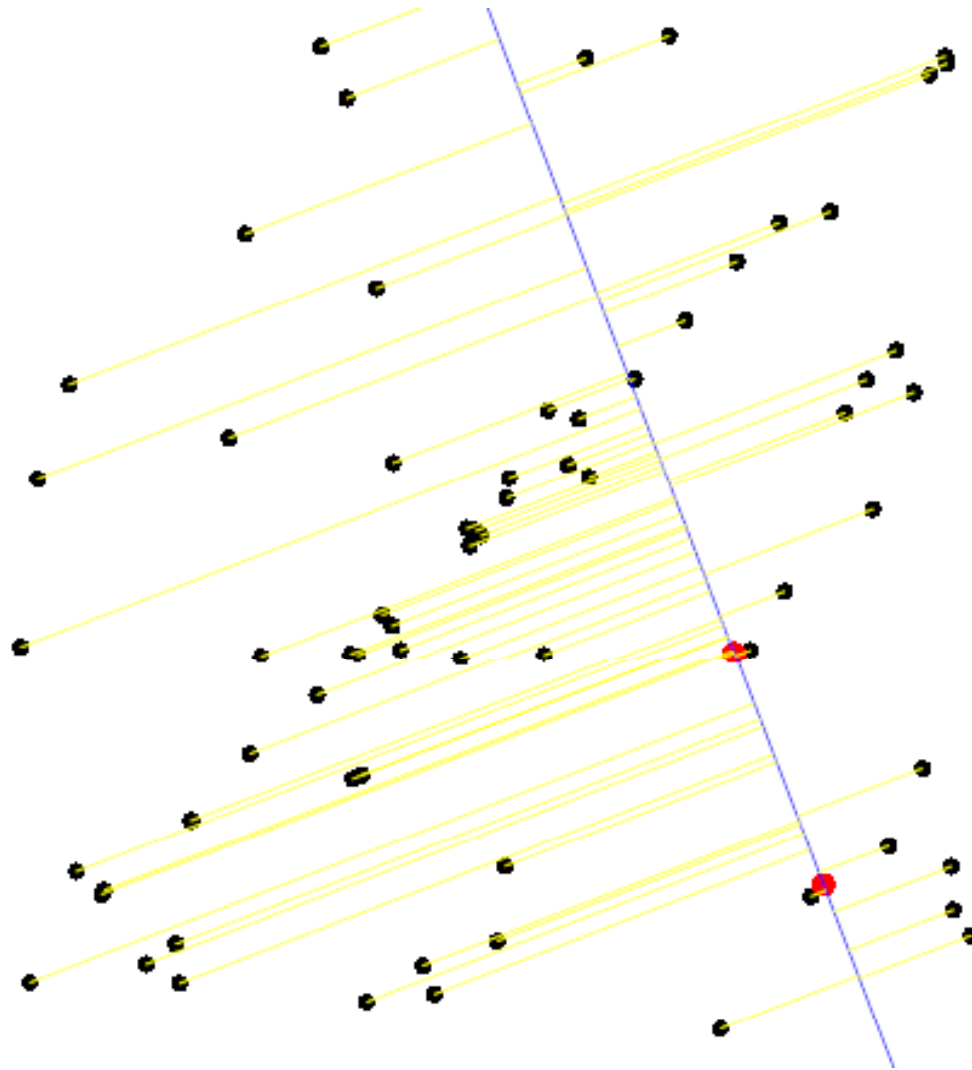


$n=2$

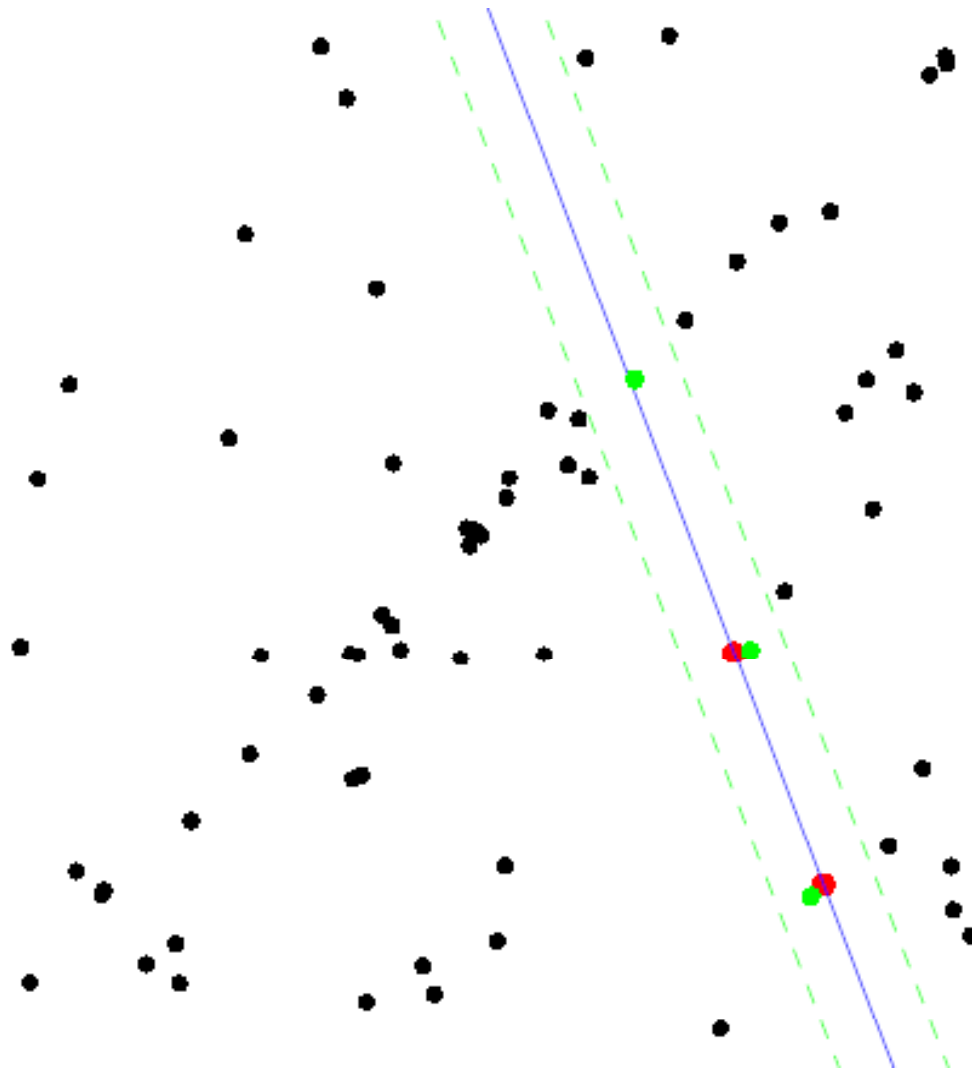
Model fitting



Measure distances

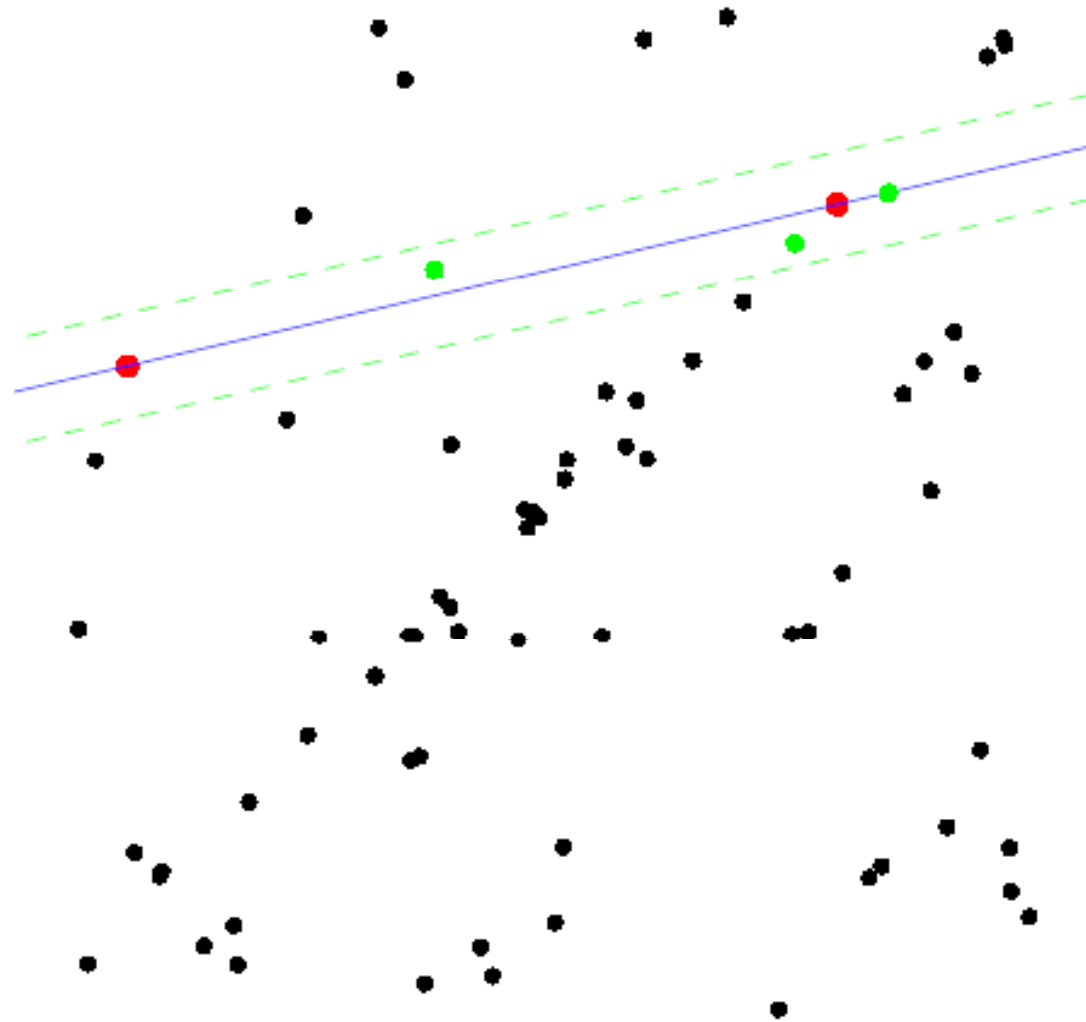


Count inliers



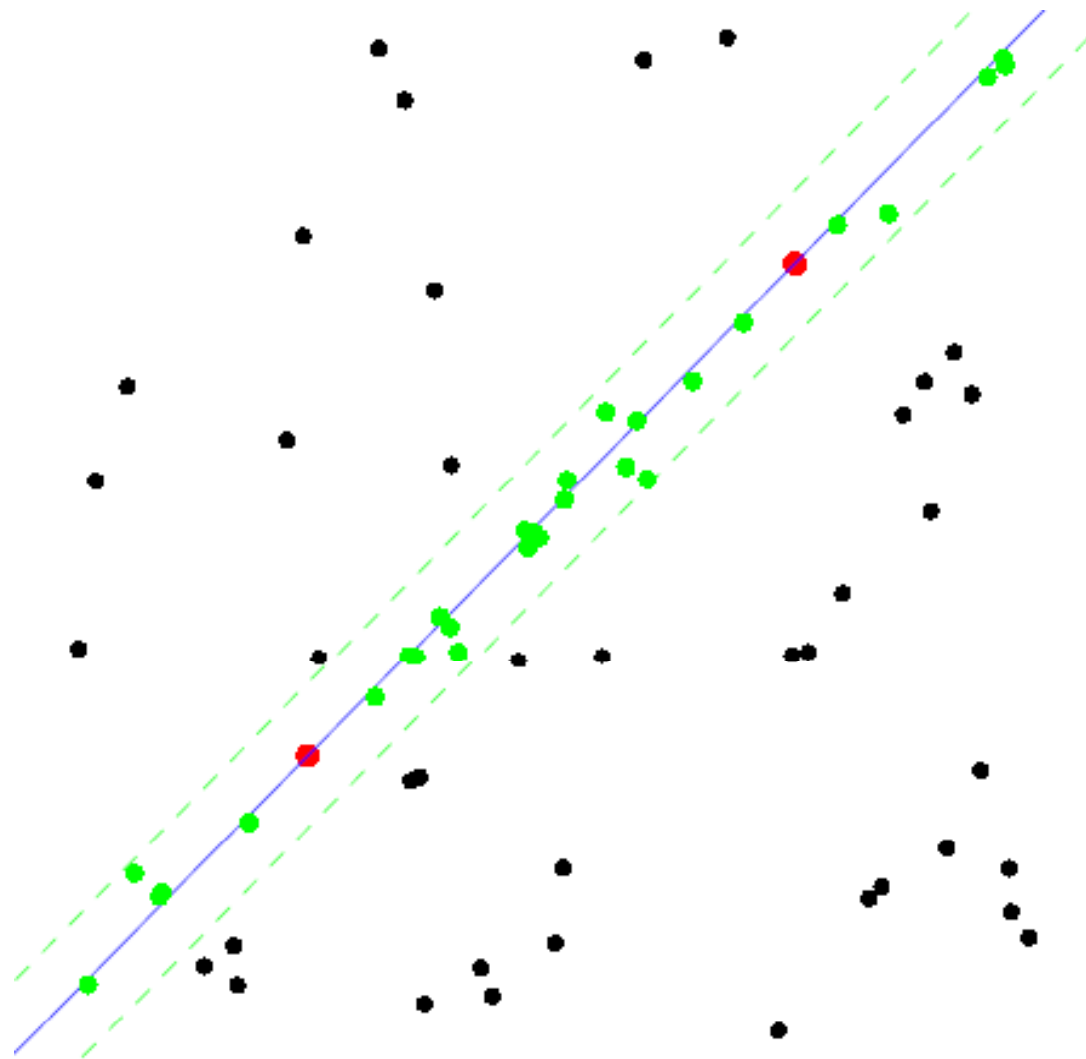
$$c=3$$

Another trial



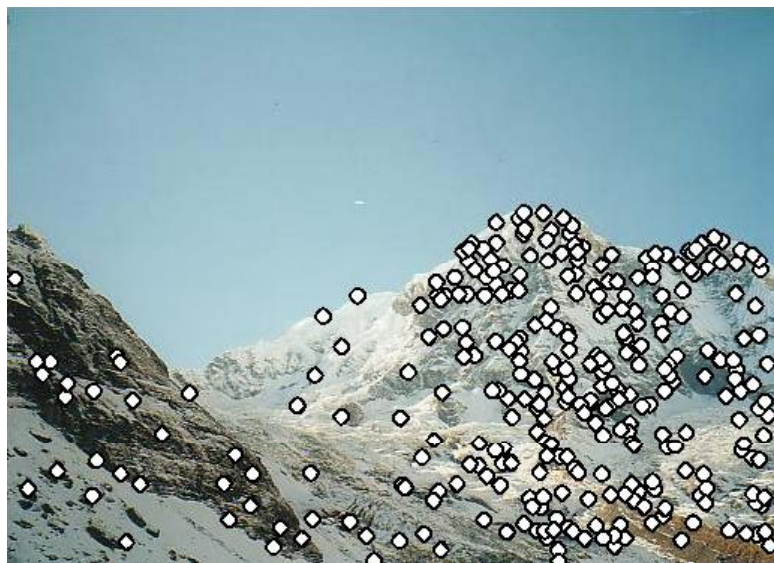
$c=3$

The best model

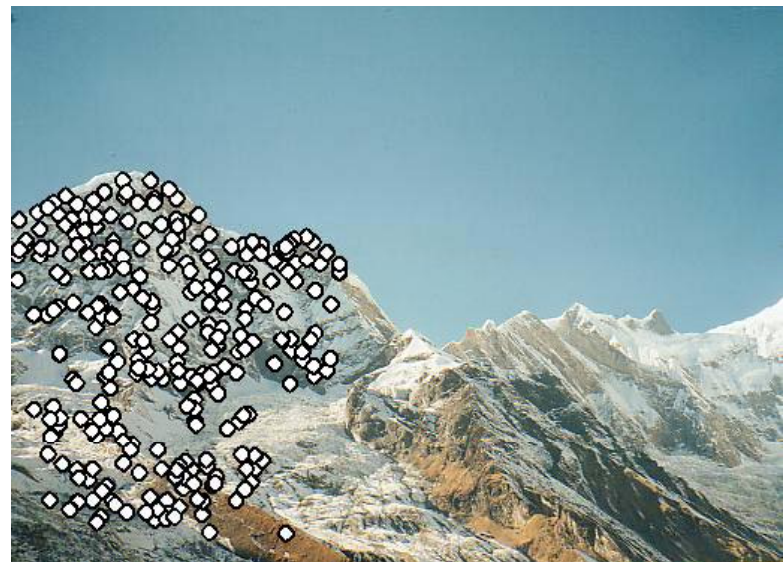
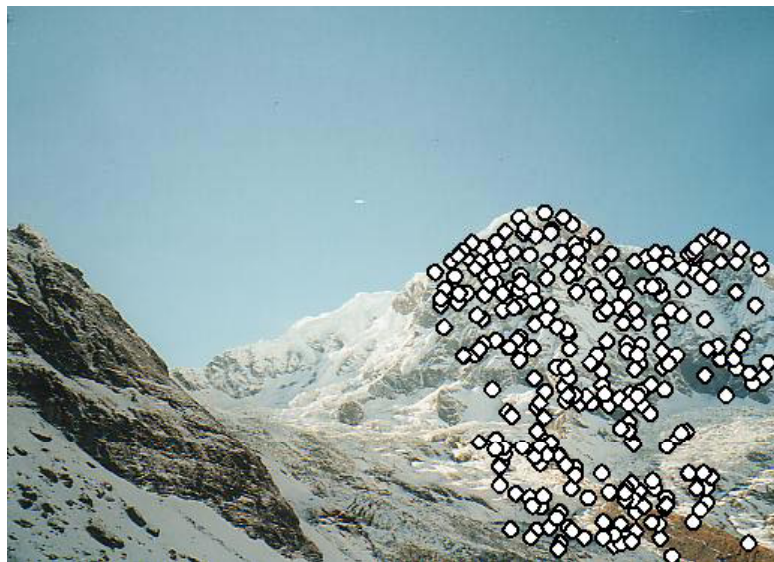


$c=15$

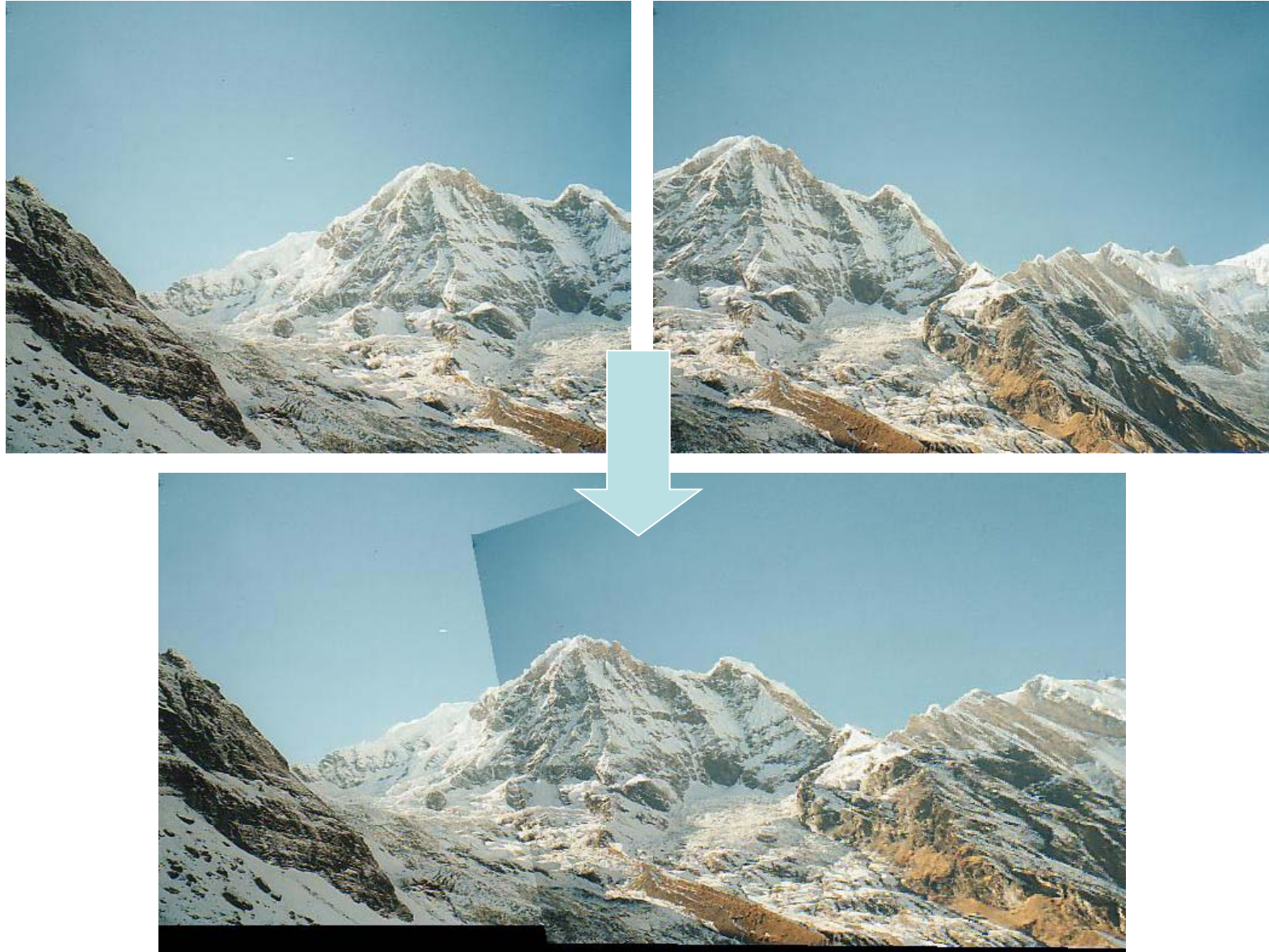
RANSAC for Homography



RANSAC for Homography



RANSAC for Homography



Applications of panorama in VFX

- Background plates
- Image-based lighting

Troy (image-based lighting)



http://www.cgnetworks.com/story_custom.php?story_id=2195&page=4

Spiderman 2 (background plate)

