# Image warping/morphing

Digital Visual Effects

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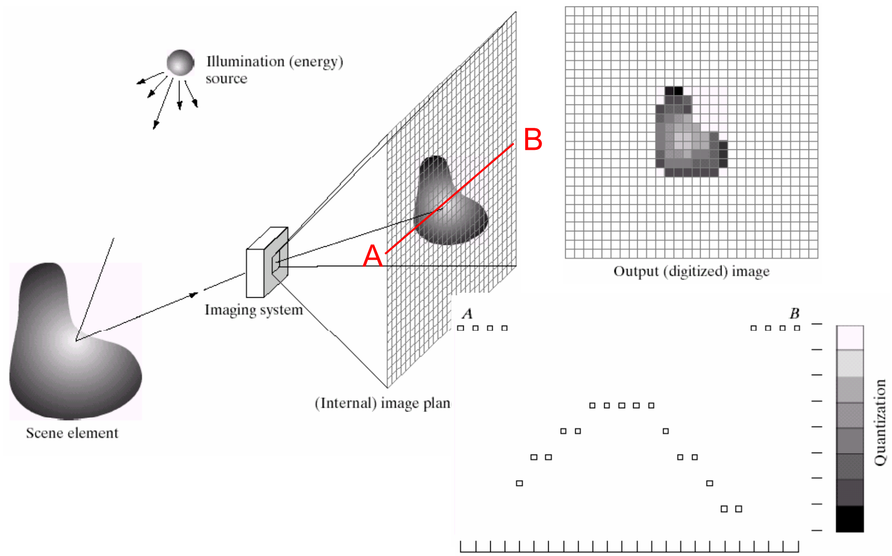
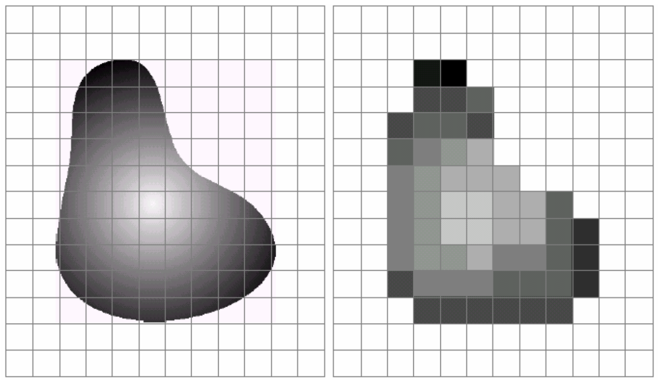
*with slides by Richard Szeliski, Steve Seitz, Tom Funkhouser and Alexei Efros*

# Image warping

## Image formation



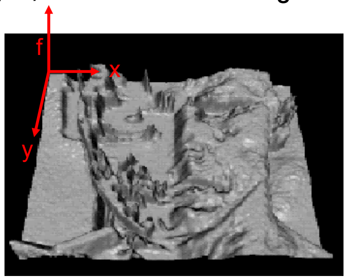

## Sampling and quantization



## What is an image



- We can think of an **image** as a function, *f*: R²→R:
  - *f*(*x, y*) gives the **intensity** at position (*x, y*)
  - defined over a rectangle, with a finite range:
    - *f*: [*a*,*b*]x[*c*,*d*] → [0,1]




- A color image

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

## A digital image



- We usually operate on **digital** (**discrete**) images:
  - **Sample** the 2D space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- If our samples are D apart, we can write this as:

  *f*[*i* ,*j*] = Quantize{ *f*(*i* D, *j* D) }
- The image can now be represented as a matrix of integer values

$j \rightarrow$, $i \downarrow$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 62 | 79 | 23 | 119 | 120 | 105 | 4 | 0 |
| 10 | 10 | 9 | 62 | 12 | 78 | 34 | 0 |
| 10 | 58 | 197 | 46 | 46 | 0 | 0 | 48 |
| 176 | 135 | 5 | 188 | 191 | 68 | 0 | 49 |
| 2 | 1 | 1 | 29 | 26 | 37 | 0 | 77 |
| 0 | 89 | 144 | 147 | 187 | 102 | 62 | 208 |
| 255 | 252 | 0 | 166 | 123 | 62 | 0 | 31 |
| 166 | 63 | 127 | 17 | 1 | 0 | 99 | 30 |

## Image warping



image filtering: change *range* of image

$$g(x) = h(f(x))$$

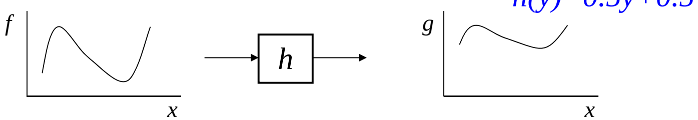
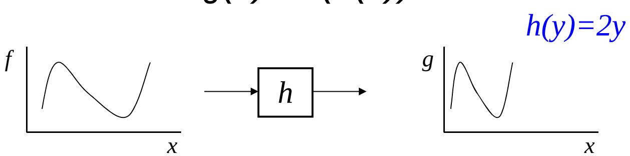



image warping: change *domain* of image

$$g(x) = f(h(x))$$




## Image warping



image filtering: change *range* of image

$$g(x) = h(f(x))$$

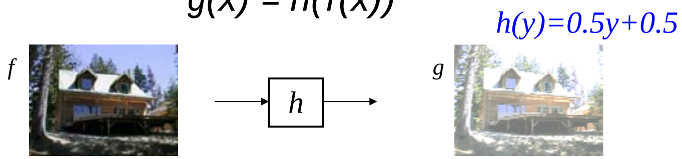
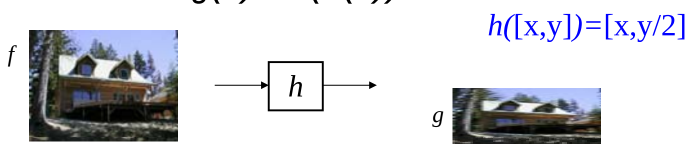


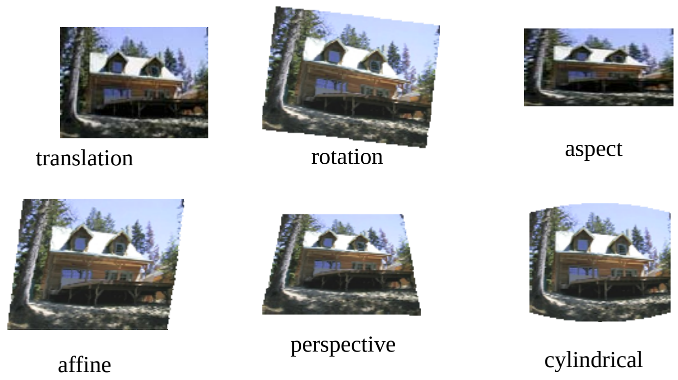

image warping: change *domain* of image

$$g(x) = f(h(x))$$

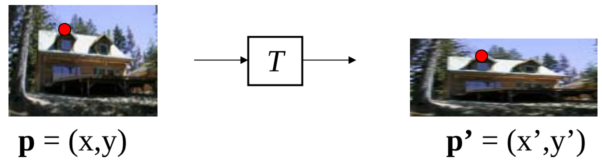


## Parametric (global) warping

Examples of parametric warps:




## Parametric (global) warping



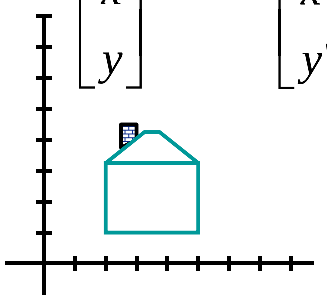

- Transformation T is a coordinate-changing machine: p' = *T*(p)
- What does it mean that *T* is global?
  - Is the same for any point p
  - can be described by just a few numbers (parameters)
- Represent *T* as a matrix: p' = **M***p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

## Scaling

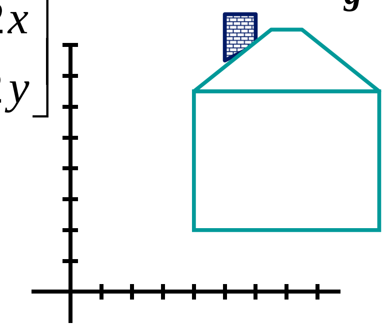
- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:

$$f \quad \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix} \quad g$$





$\times$ 2

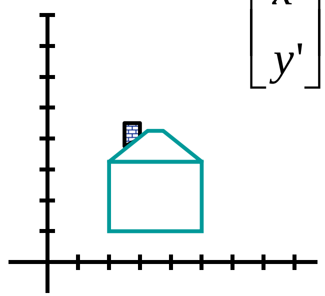


## Scaling

- *Non-uniform scaling*: different scalars per component:

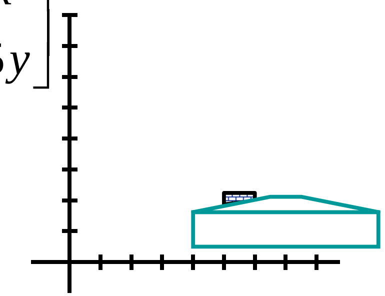
$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = g\left(\begin{bmatrix} x' \\ y' \end{bmatrix}\right)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2x \\ 0.5y \end{bmatrix}$$





x $\times$ 2,
y $\times$ 0.5



## Scaling

- Scaling operation: $x' = ax$

  $y' = by$

- Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

What's inverse of S?

## 2-D Rotation

- This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Even though sin(θ) and cos(θ) are nonlinear to θ,
  - *x' is a linear combination of x and y*
  - *y' is a linear combination of x and y*
- What is the inverse transformation?
  - Rotation by –θ
  - For rotation matrices, det(R) = 1 so $\mathbf{R}^{-1} = \mathbf{R}^{T}$

## 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$x' = x \qquad y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\boldsymbol{x' = s_x * x} \qquad \boldsymbol{y' = s_y * y}$$

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 \\ 0 & \boldsymbol{s_y} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}$$

## 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$x' = \cos\theta * x - \sin\theta * y$$
$$y' = \sin\theta * x + \cos\theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$\boldsymbol{x' = x + sh_x * y}$$
$$\boldsymbol{y' = sh_y * x + y}$$

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} \\ \boldsymbol{sh_y} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}$$

## 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x \qquad y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x \qquad y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

## All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror
- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

## 2x2 Matrices

- What types of transformations can not be represented with a 2x2 matrix?

2D Translation?

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{t}_x$$
$$\boldsymbol{y}' = \boldsymbol{y} + \boldsymbol{t}_y$$

NO!

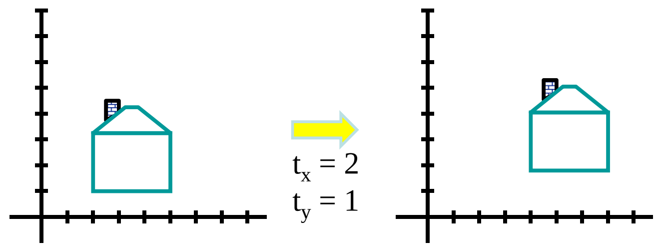
$$\boxed{\text{Only linear 2D transformations can be represented with a 2x2 matrix}}$$

## Translation

- Example of translation

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



## Affine Transformations

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations
- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition
  - Models change of basis

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

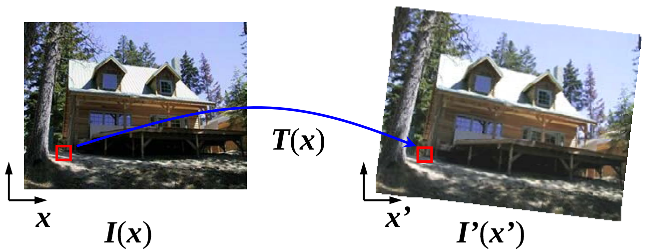
## Projective Transformations

- Projective transformations …
  - Affine transformations, and
  - Projective warps
- Properties of projective transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines do not necessarily remain parallel
  - Ratios are not preserved
  - Closed under composition
  - Models change of basis

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

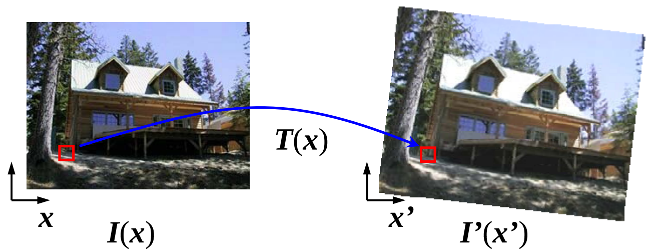
## Image warping

- Given a coordinate transform $x' = T(x)$ and a source image $I(x)$, how do we compute a transformed image $I'(x') = I(T(x))$?



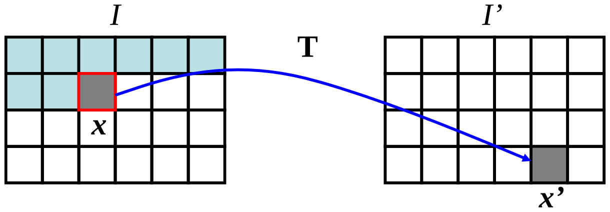

## Forward warping

- Send each pixel $I(x)$ to its corresponding location $x' = T(x)$ in $I'(x')$

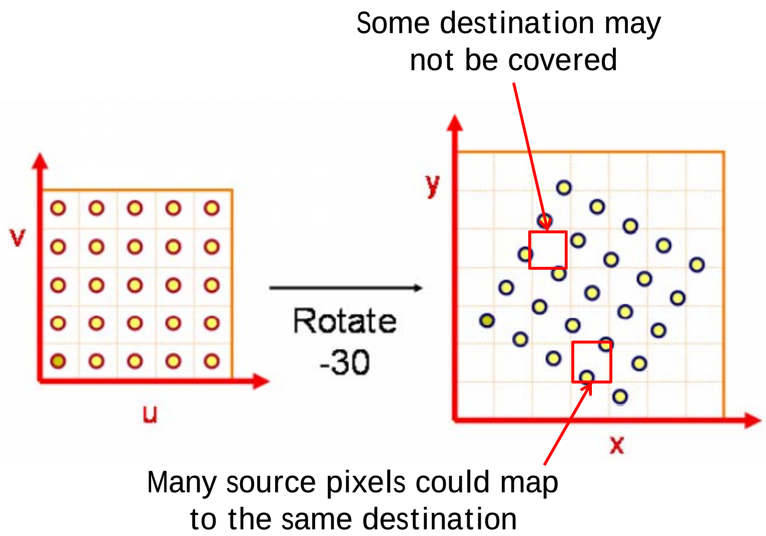


## Forward warping

```
fwarp(I, I', T)
{
  for (y=0; y<I.height; y++)
    for (x=0; x<I.width; x++) {
      (x',y')=T(x,y);
      I'(x',y')=I(x,y);
    }
}
```

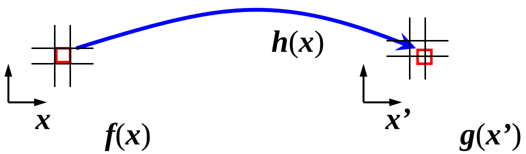



## Forward warping



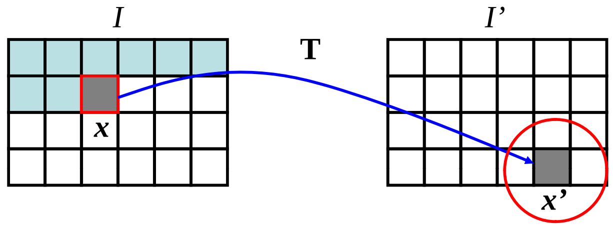

## Forward warping

- Send each pixel $I(x)$ to its corresponding location $x' = T(x)$ in $I'(x')$
  - What if pixel lands "between" two pixels?
  - Will be there holes?
  - Answer: add "contribution" to several pixels, normalize later (*splatting*)



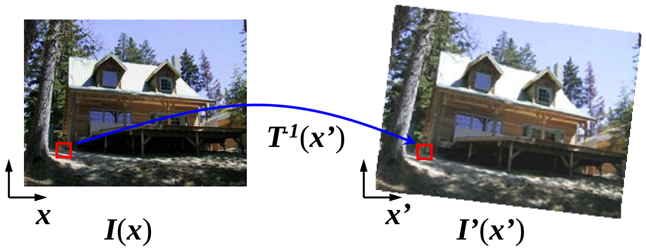

## Forward warping

```
fwarp(I, I', T)
{
  for (y=0; y<I.height; y++)
    for (x=0; x<I.width; x++) {
      (x',y')=T(x,y);
      Splatting(I',x',y',I(x,y),kernel);
    }
}
```



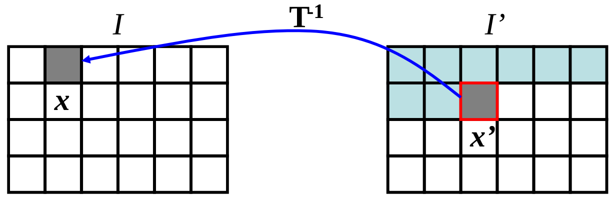
## Inverse warping

- Get each pixel $I'(x')$ from its corresponding location $x = T^{-1}(x')$ in $I(x)$




## Inverse warping

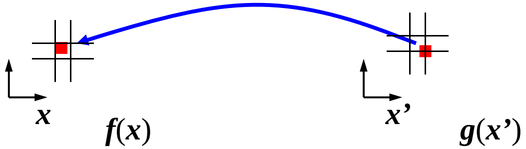
```
iwarp(I, I', T)
{
  for (y=0; y<I'.height; y++)
    for (x=0; x<I'.width; x++) {
      (x,y)=T-1(x',y');
      I'(x',y')=I(x,y);
    }
}
```




## Inverse warping

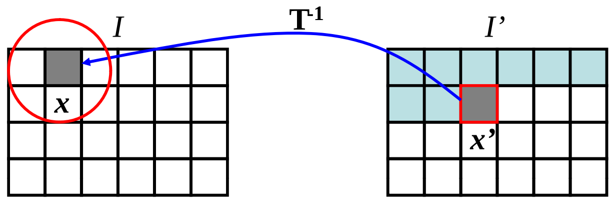
- Get each pixel $I'(x')$ from its corresponding location $x = T^{-1}(x')$ in $I(x)$

- What if pixel comes from “between” two pixels?
- Answer: *resample* color value from *interpolated* (*prefiltered*) source image



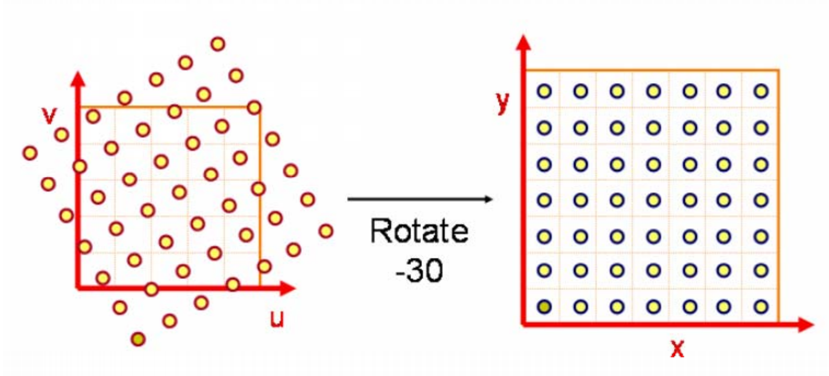

## Inverse warping

```
iwarp(I, I', T)
{
  for (y=0; y<I'.height; y++)
    for (x=0; x<I'.width; x++) {
      (x,y)=T-1(x',y');
      I'(x',y')=Reconstruct(I,x,y,kernel);
    }
}
```



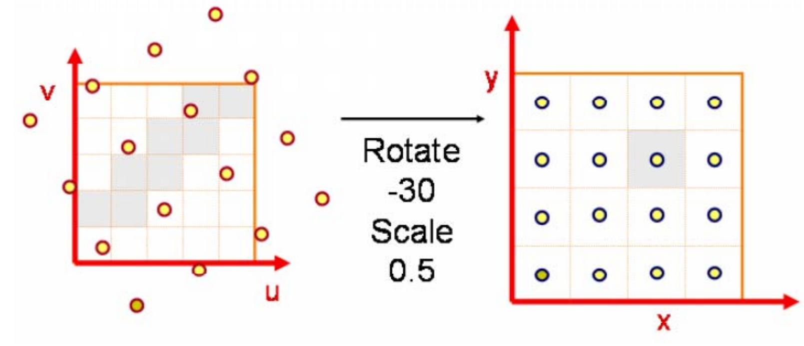
## Inverse warping

- No hole, but must resample
- What value should you take for non-integer coordinate? Closest one?



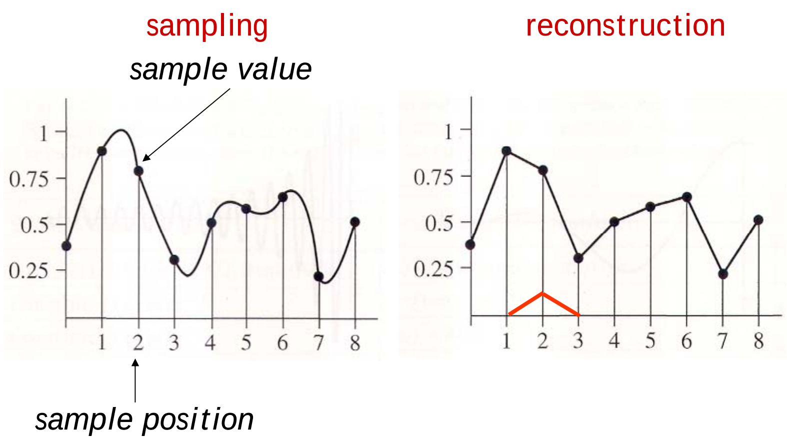

## Inverse warping

- It could cause aliasing




## Reconstruction

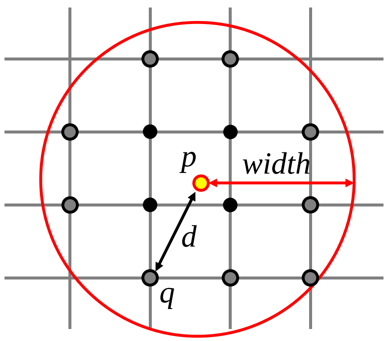
- Reconstruction generates an approximation to the original function. Error is called aliasing.




## Reconstruction

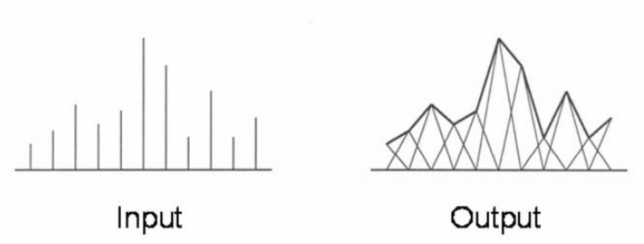
- Computed weighted sum of pixel neighborhood; output is weighted average of input, where weights are normalized values of filter kernel k

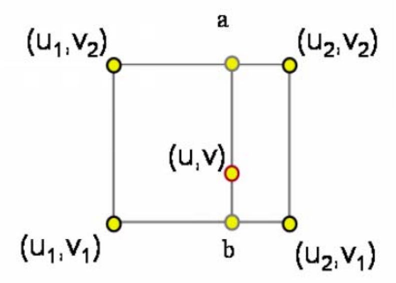
$$p = \frac{\sum_i k(q_i) q_i}{\sum_i k(q_i)}$$



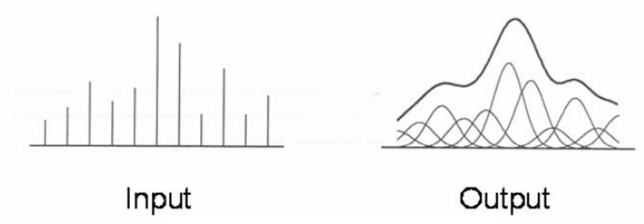

```
color=0;
weights=0;
for all q's dist < width
  d = dist(p, q);
  w = kernel(d);
  color += w*q.color;
  weights += w;
p.Color = color/weights;
```

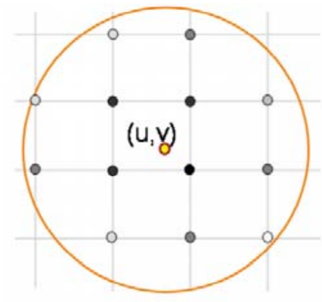
## Triangle filter



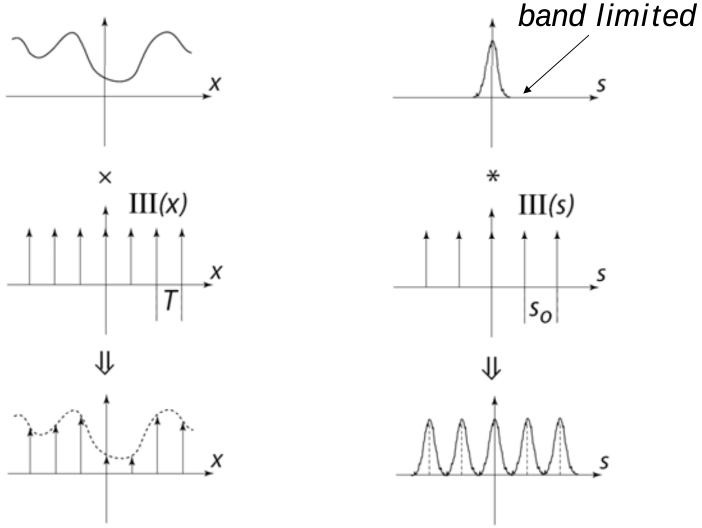




## Gaussian filter

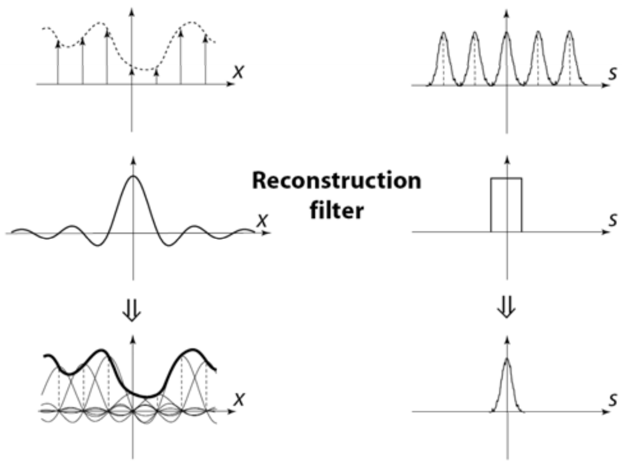



## Sampling




## Reconstruction



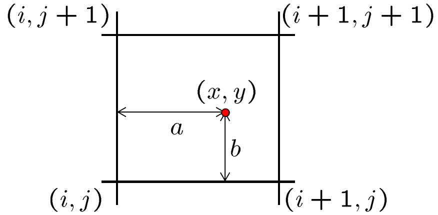

*The reconstructed function is obtained by interpolating among the samples in some manner*

## Reconstruction (interpolation)

- Possible reconstruction filters (kernels):
  - nearest neighbor
  - bilinear
  - bicubic
  - sinc (optimal reconstruction)

## Bilinear interpolation (triangle filter)

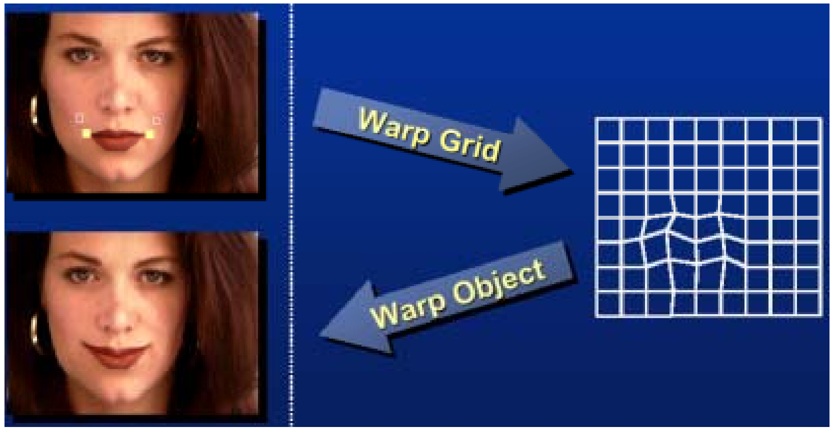
- A simple method for resampling images




$$
\begin{aligned}
f(x,y) = \quad & (1-a)(1-b) & f[i,j] \\
& +a(1-b) & f[i+1,j] \\
& +ab & f[i+1,j+1] \\
& +(1-a)b & f[i,j+1]
\end{aligned}
$$

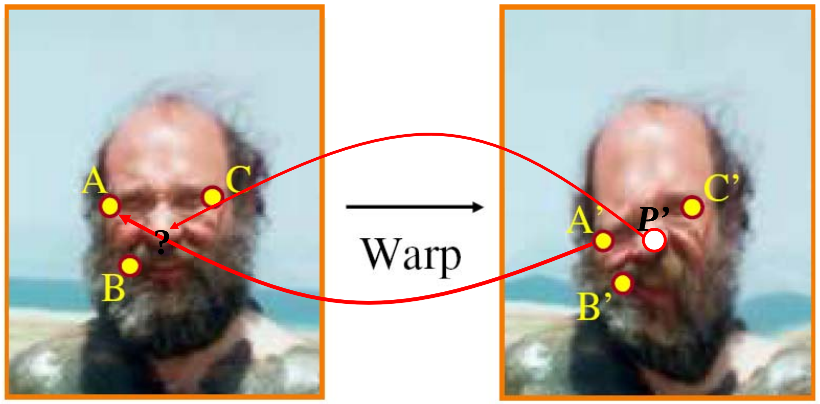
## Non-parametric image warping

- Specify a more detailed warp function
- Splines, meshes, optical flow (per-pixel motion)




## Non-parametric image warping

- Mappings implied by correspondences
- Inverse warping

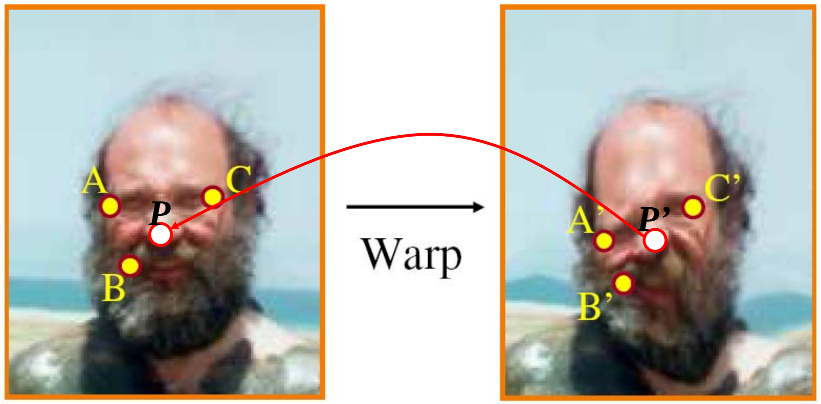


## Non-parametric image warping

$$P = w_A A + w_B B + w_C C$$

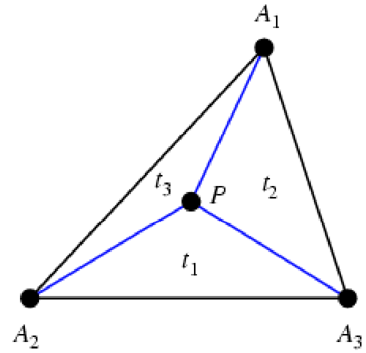
$$P' = w_A A' + w_B B' + w_C C'$$

*Barycentric coordinate*






## Barycentric coordinates




$$P = t_1 A_1 + t_2 A_2 + t_3 A_3$$

$$t_1 + t_2 + t_3 = 1$$

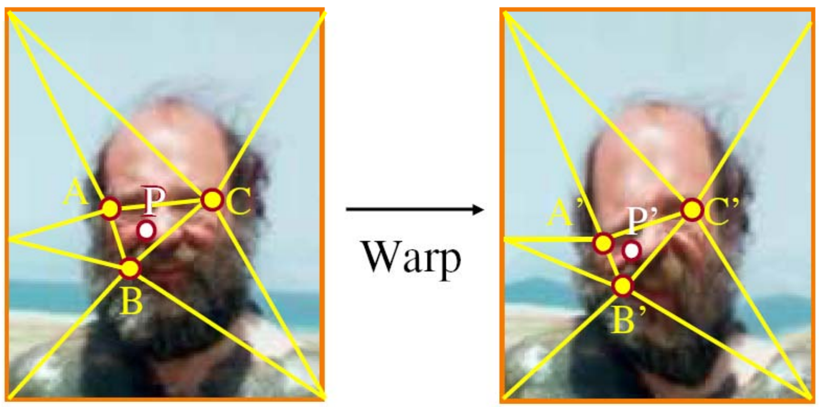


## Non-parametric image warping

$$P = w_A A + w_B B + w_C C$$

$$P' = w_A A' + w_B B' + w_C C'$$

*Barycentric coordinate*





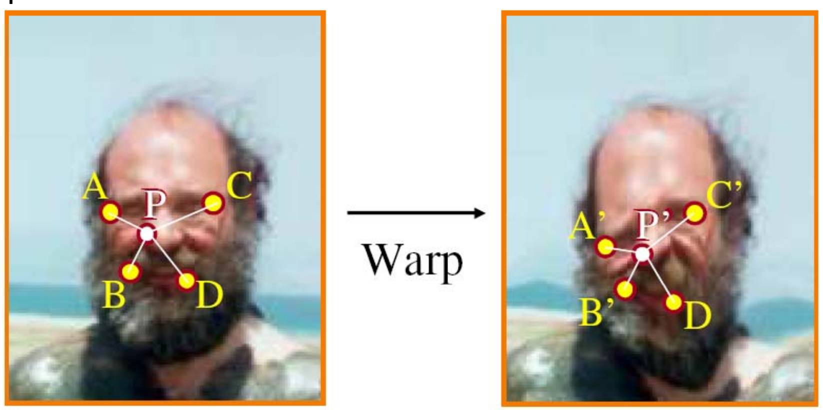

## Non-parametric image warping

Gaussian $\rho(r) = e^{-\beta r^2}$

thin plate spline $\rho(r) = r^2 \log(r)$

$$\Delta P = \frac{1}{K} \sum_i \boxed{k_{X_i}} (P') \Delta X_i$$

*radial basis function*



## Image warping

- Warping is a useful operation for mosaics, video matching, view interpolation and so on.

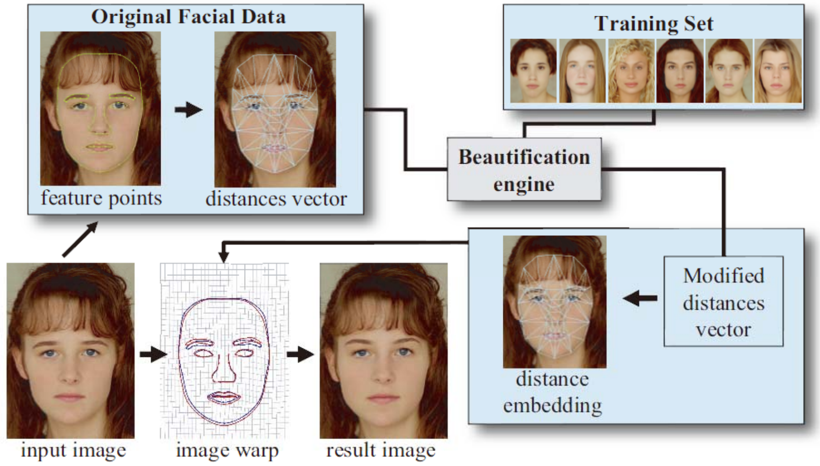
# An application of image warping: face beautification

## Data-driven facial beautification

## Facial beautification

## Facial beautification

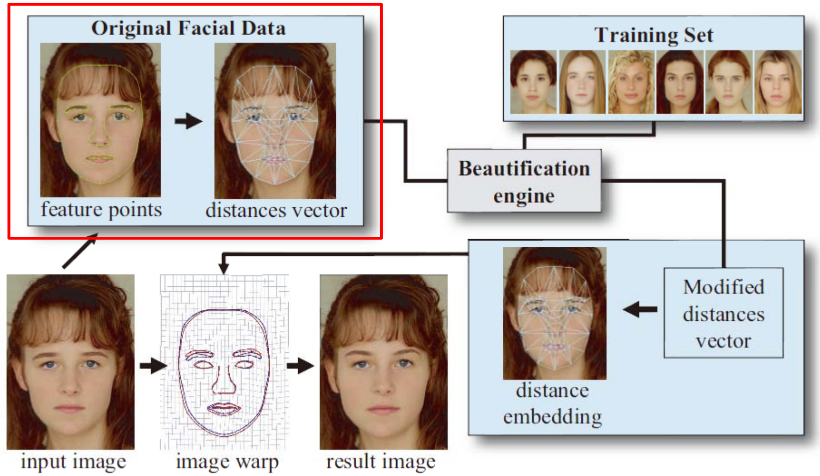
## Facial beautification




## Training set

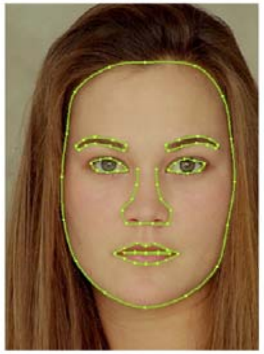
- Face images
  - 92 young Caucasian female
  - 33 young Caucasian male

## Feature extraction

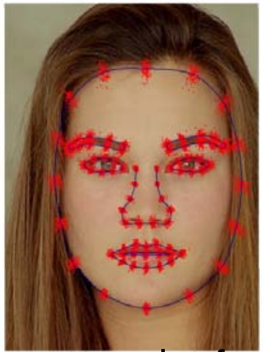


## Feature extraction

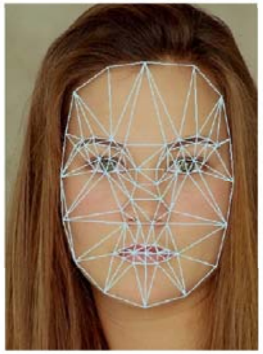
- Extract 84 feature points by BTSM
- Delaunay triangulation -> 234D distance vector (normalized by the square root of face area)



BTSM

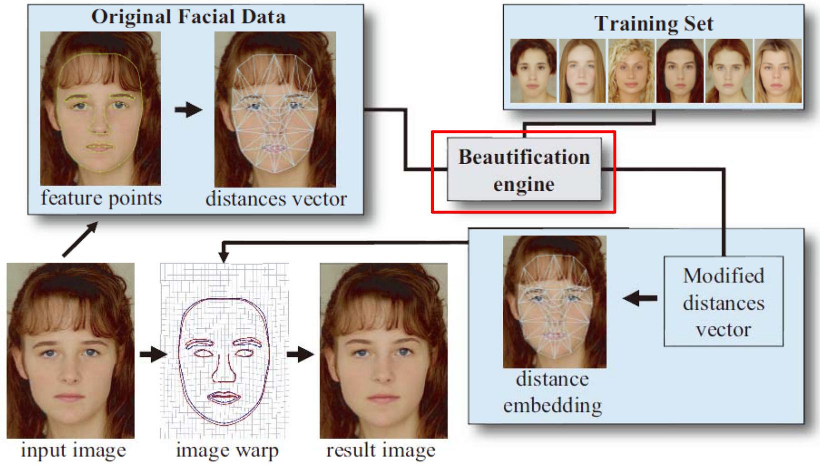


scatter plot for all training faces



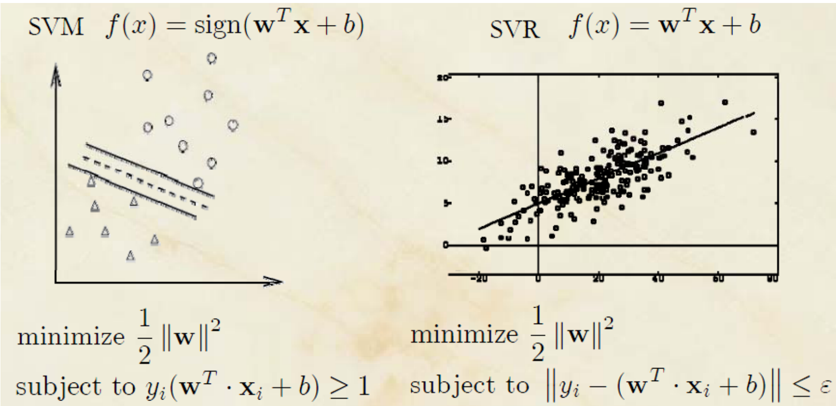
234D vector

## Beautification engine




## Support vector regression (SVR)

- Similar concept to SVM, but for regression
- RBF kernels
- $f_b(v)$




## Beautification process

- Given the normalized distance vector $v$, generate a nearby vector $v'$ so that

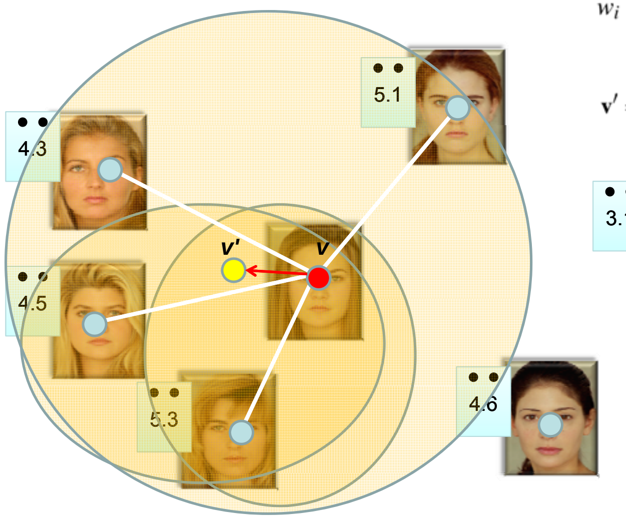
$$f_b(v') > f_b(v)$$

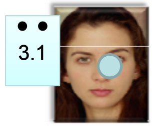
- Two options
  - KNN-based
  - SVR-based

## KNN-based beautification

$$w_i = \frac{b_i}{\|\mathbf{v} - \mathbf{v}_i\|}$$

$$\mathbf{v}' = \frac{\sum_{i=1}^{K} w_i \mathbf{v}_i}{\sum_{i=1}^{K} w_i}$$




## SVR-based beautification

- Directly use $f_b$ to seek $v'$

$$\mathbf{v}' = \underset{\mathbf{u}}{\operatorname{argmin}}\ E(\mathbf{u}),\ \text{where } E(\mathbf{u}) = -f_b(\mathbf{u})$$

- Use standard no-derivative direction set method for minimization
- Features were reduced to 35D by PCA

## SVR-based beautification

- Problems: it sometimes yields distance vectors corresponding to invalid human face
- Solution: add log-likelihood term (LP)

$$E(\mathbf{u}) = (\alpha - 1) f_b(\mathbf{u}) - \alpha LP(\mathbf{u})$$

- LP is approximated by modeling face space as a multivariate Gaussian distribution

$\hat{\mathbf{u}}$'s i-th component

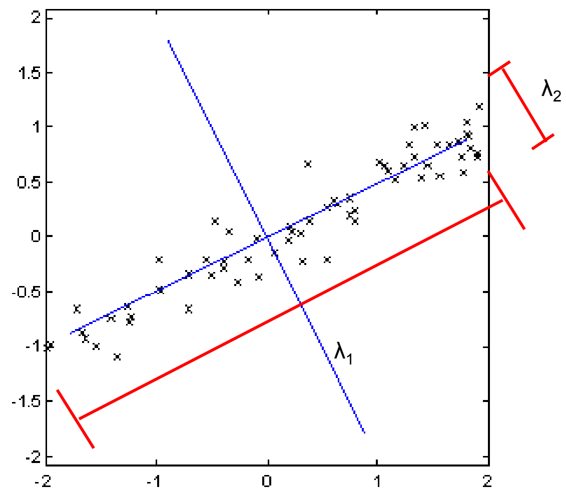
$$P(\hat{\mathbf{u}}) = \frac{1}{(2\pi)^{N/2}\sqrt{\prod_i \lambda_i}} \prod_i \exp\left(\frac{-\beta_i^2}{2\lambda_i}\right)$$

$\mathbf{u}$'s projection in PCA space

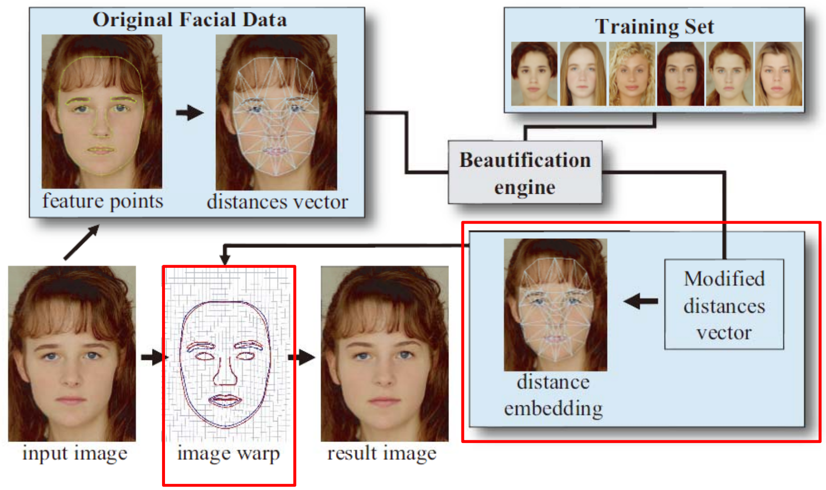
i-th eigenvalue

$$LP(\hat{\mathbf{u}}) = \sum \frac{-\beta_i^2}{2\lambda_i} + \text{const}$$

## PCA



## Embedding and warping




## Distance embedding

- Convert modified distance vector *v'* to a new face landmark

$$E(q_1,\ldots,q_N)=\sum_{e_{ij}}\alpha_{ij}\left(\|q_i-q_j\|^2-d_{ij}^2\right)^2$$

1 if i and j belong to different facial features
10 otherwise

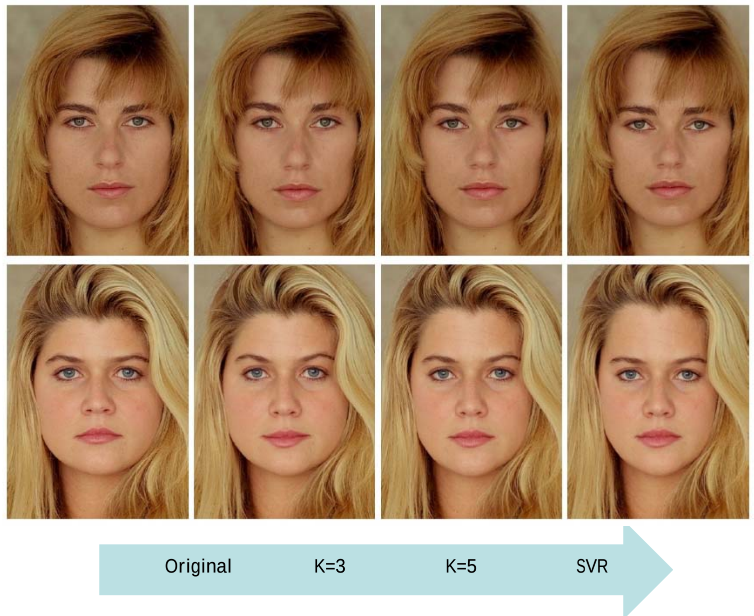
- A graph drawing problem referred to as a stress minimization problem, solved by LM algorithm for non-linear minimization

## Distance embedding

- Post processing to enforce similarity transform for features on eyes by minimizing

$$\sum\|Sp_i-q_i\|^2$$

$$S=\begin{pmatrix} a & b & t_x \\ -b & a & t_y \\ 0 & 0 & 1 \end{pmatrix}$$



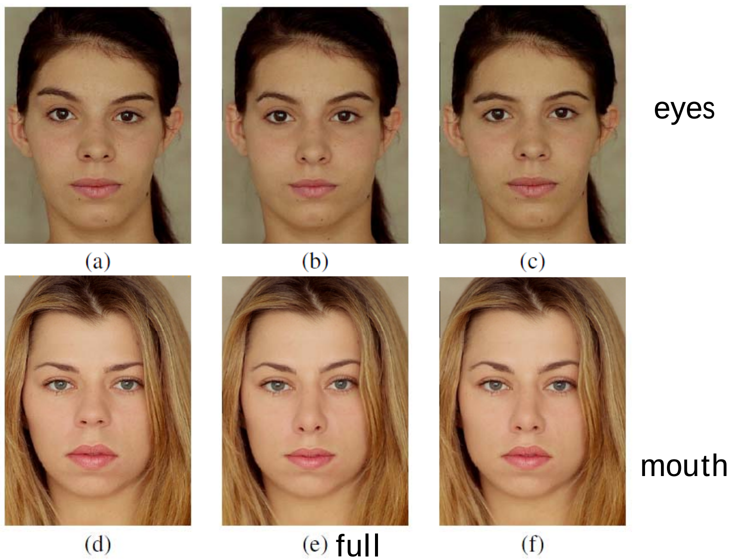
## Results (in training set)

## User study

| | |
|---|---|
| Original portrait | 3.37 (0.49) |
| Warped to mean | 3.75 (0.49) |
| KNN-beautified (best) | 4.14 (0.51) |
| SVR-beautified | 4.51 (0.49) |

## Results (not in training set)

## By parts



## Different degrees

50%

100%

## Facial collage

# Image morphing

## Image morphing

- The goal is to synthesize a fluid transformation from one image to another.
- Cross dissolving is a common transition between cuts, but it is not good for morphing because of the ghosting effects.

image #1 dissolving image #2

## Artifacts of cross-dissolving

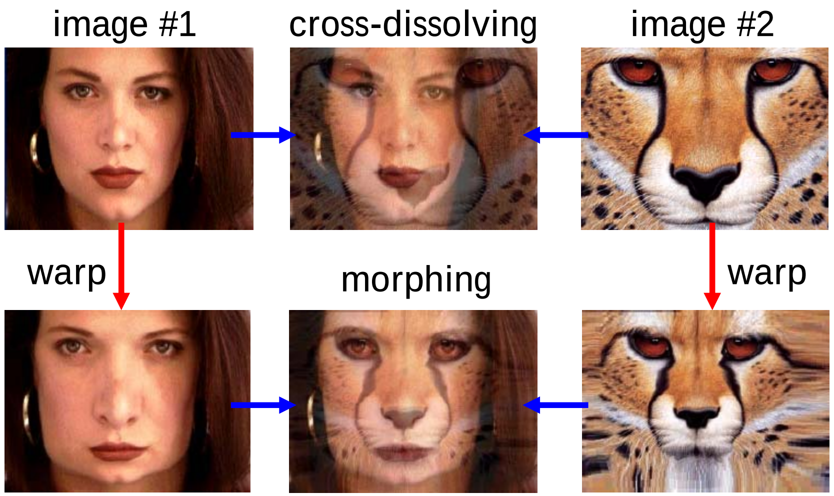
http://www.salavon.com/

## Image morphing

- Why ghosting?
- Morphing = warping + cross-dissolving

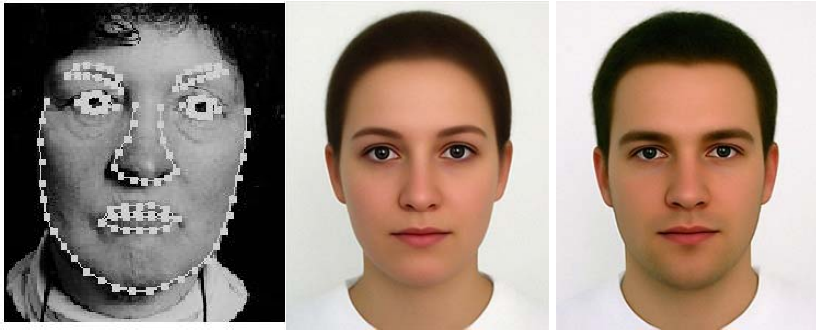
shape (geometric) — color (photometric)

## Image morphing




## Morphing sequence

## Face averaging by morphing

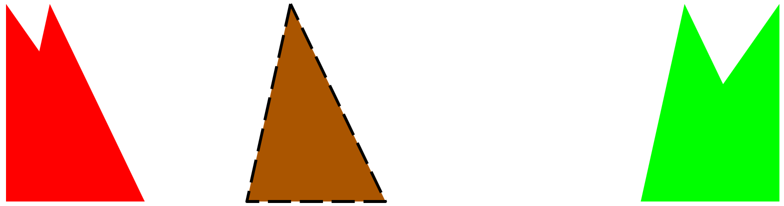


average faces

## Image morphing

create a morphing sequence: for each time t

1. Create an intermediate warping field (by interpolation)
2. Warp both images towards it
3. Cross-dissolve the colors in the newly warped images



*t=0* *t=0.33* *t=1*

## An ideal example (in 2004)

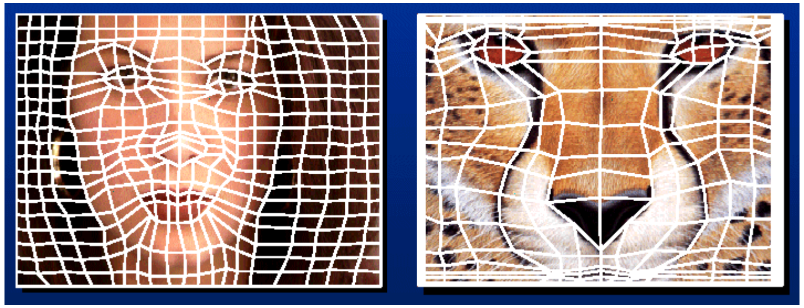

t=0 morphing t=1

## An ideal example

t=0 middle face (t=0.5) t=1

## Warp specification (mesh warping)

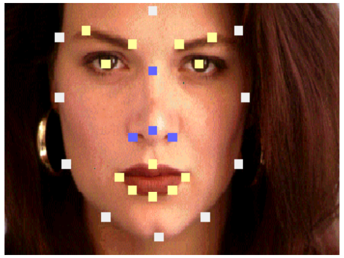
- How can we specify the warp?
  1. Specify corresponding *spline control points*
     *interpolate* to a complete warping function

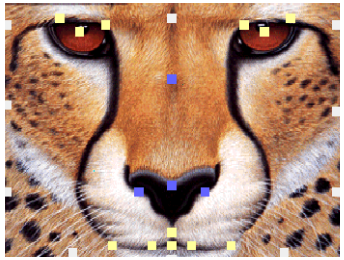


easy to implement, but less expressive

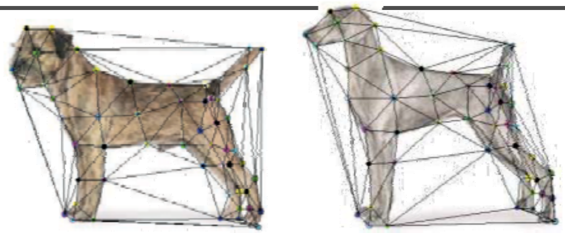
## Warp specification

- How can we specify the warp
  2. Specify corresponding *points*
     - *interpolate* to a complete warping function





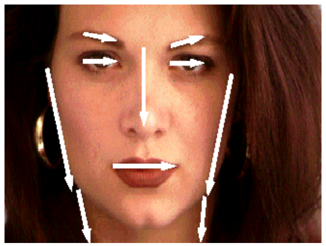
## Solution: convert to mesh warping

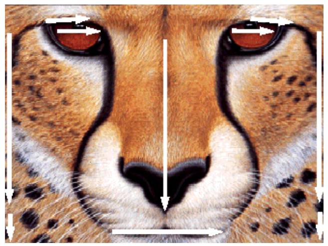


1. Define a triangular mesh over the points
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences
2. Warp each triangle separately from source to destination
   - How do we warp a triangle?
   - 3 points = affine warp!
   - Just like texture mapping

## Warp specification (field warping)

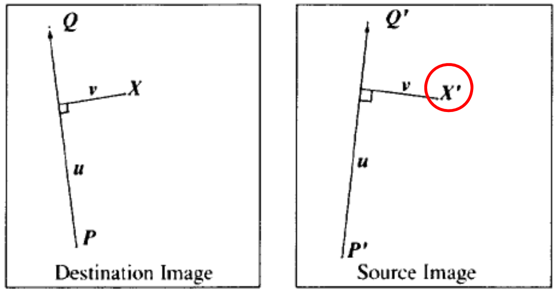
- How can we specify the warp?
  3. Specify corresponding *vectors*
     - *interpolate* to a complete warping function
     - The Beier & Neely Algorithm





## Beier&Neely (SIGGRAPH 1992)

- Single line-pair PQ to P'Q':



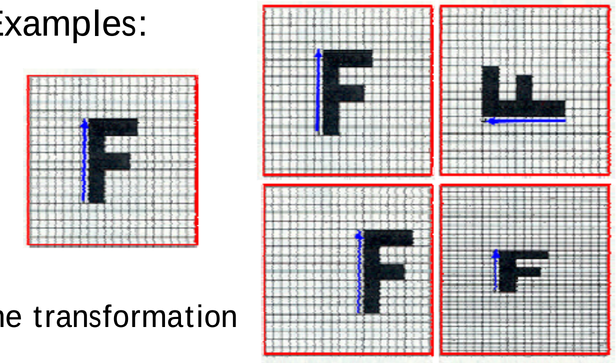

$$u = \frac{(X-P)\cdot(Q-P)}{\|Q-P\|^2} \quad (1)$$

$$v = \frac{(X-P)\cdot Perpendicular(Q-P)}{\|Q-P\|} \quad (2)$$

$$X' = P' + u\cdot(Q'-P') + \frac{v\cdot Perpendicular(Q'-P')}{\|Q'-P'\|} \quad (3)$$

## Algorithm (single line-pair)

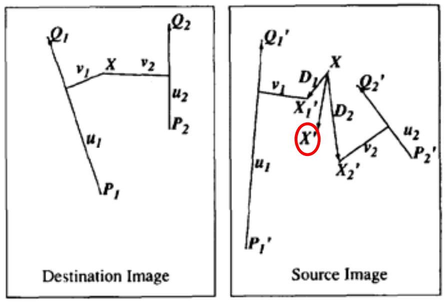
- For each X in the destination image:
  1. Find the corresponding u,v
  2. Find X' in the source image for that u,v
  3. destinationImage(X) = sourceImage(X')
- Examples:



Affine transformation

## Multiple Lines

$$D_i = X_i' - X_i$$



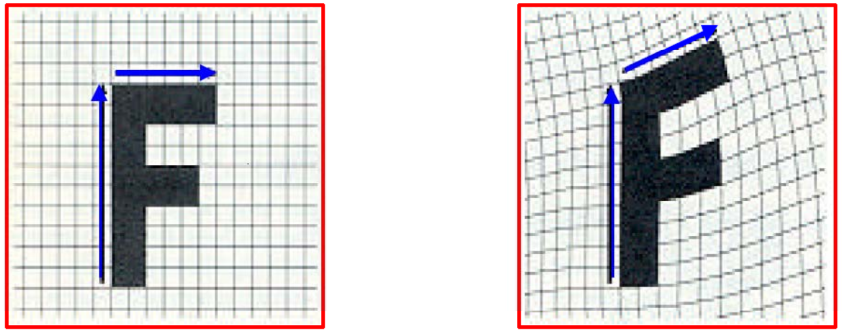

$$weight[i] = \left(\frac{length[i]^p}{a + dist[i]}\right)^b$$

*length* = length of the line segment,
*dist* = distance to line segment
The influence of $a, p, b$. The same as the average of $X_i$'

## Full Algorithm

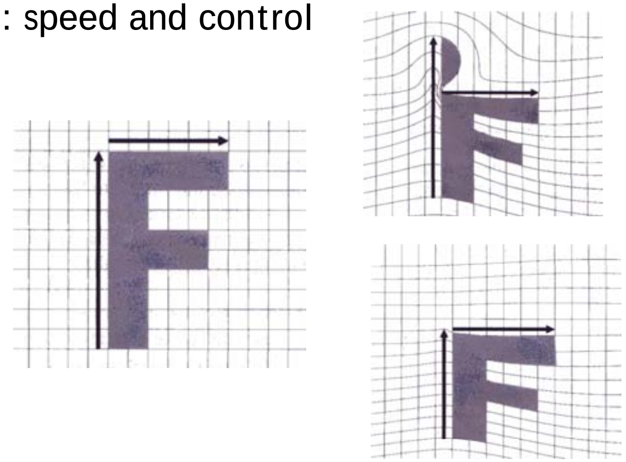
```
WarpImage(SourceImage, L'[...], L[...])
begin
    foreach destination pixel X do
        XSum = (0,0)
        WeightSum = 0
        foreach line L[i] in destination do
            X'[i]= X transformed by (L[i],L'[i])
            weight[i] = weight assigned to X'[i]
            XSum = Xsum + X'[i] * weight[i]
            WeightSum += weight[i]
        end
        X' = XSum/WeightSum
        DestinationImage(X) = SourceImage(X')
    end
    return Destination
end
```

## Resulting warp



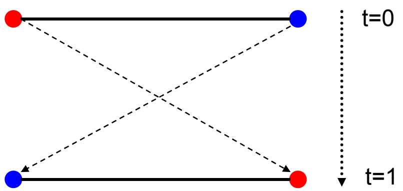
## Comparison to mesh morphing

- Pros: more expressive
- Cons: speed and control



## Warp interpolation

- How do we create an intermediate warp at time t?
  - linear interpolation for line end-points
  - But, a line rotating 180 degrees will become 0 length in the middle
  - One solution is to interpolate line mid-point and orientation angle




## Animation

GenerateAnimation(Image$_0$, L$_0$[...],Image$_1$, L$_1$[...])
**begin**
  **foreach** intermediate frame time t **do**
    **for** i=1 to number of line-pairs **do**
      L[i] = line t-th of the way from L$_0$[i] to L$_1$[i].
    **end**
    Warp$_0$ = WarpImage( Image$_0$, L$_0$[...], L[...])
    Warp$_1$ = WarpImage( Image$_1$, L$_1$[...], L[...])
    **foreach** pixel p in FinalImage **do**
      FinalImage(p) = (1-t) Warp$_0$(p) + t Warp$_1$(p)
    **end**
  **end**
**end**

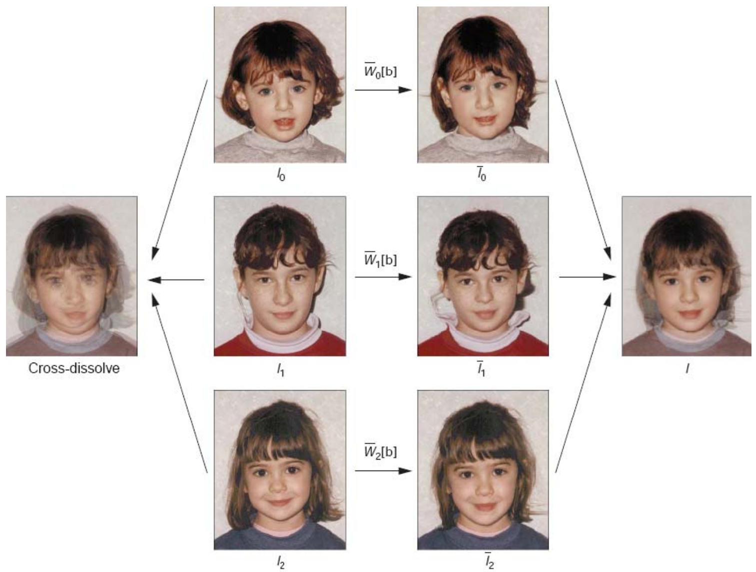
## Animated sequences

- Specify keyframes and interpolate the lines for the inbetween frames
- Require a lot of tweaking

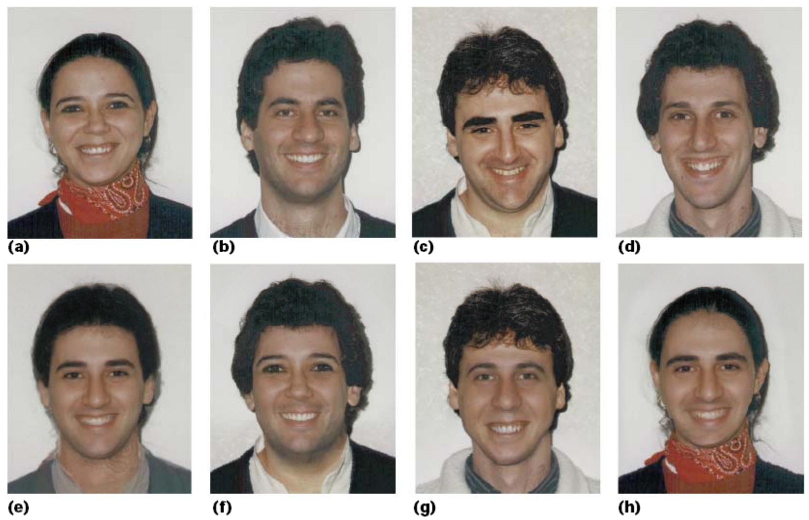
## Results

*Michael Jackson's MTV "Black or White"*

## Multi-source morphing




## Multi-source morphing



## Woman in arts

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