# Features

Digital Visual Effects, Spring 2007

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## Outline

- Features
- Harris corner detector
- SIFT
- Applications

# Features

## Features

- Properties of features
- Detector: locates feature
- Descriptor and matching metrics: describes and matches features

## Desired properties for features

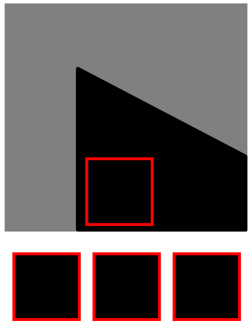
- Distinctive: a single feature can be correctly matched with high probability.
- Invariant: invariant to scale, rotation, affine, illumination and noise for robust matching across a substantial range of affine distortion, viewpoint change and so on. That is, it is repeatable.

# Harris corner detector

## Moravec corner detector (1980)

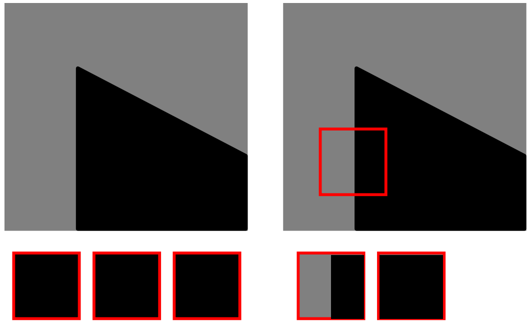
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity

## Moravec corner detector



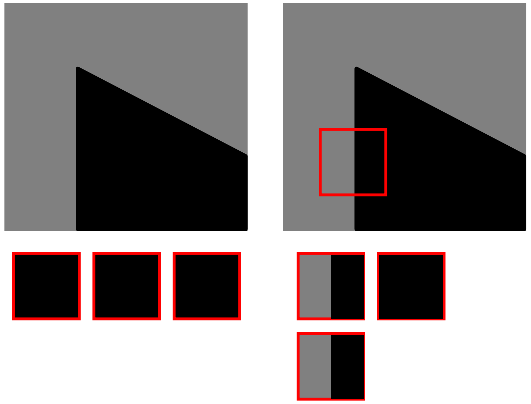
flat

## Moravec corner detector



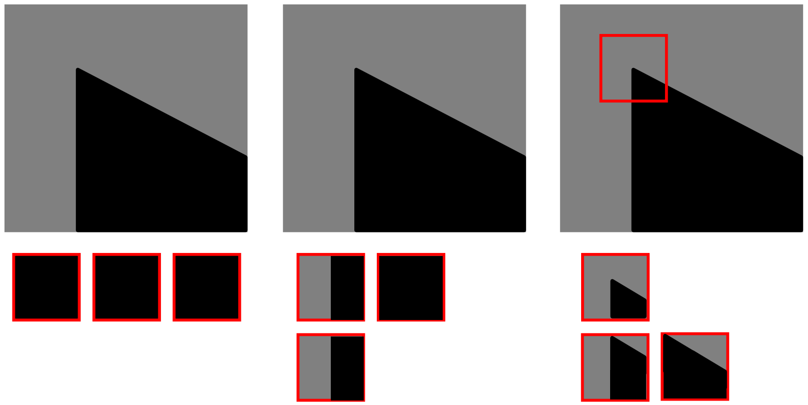
flat

## Moravec corner detector



flat edge

## Moravec corner detector



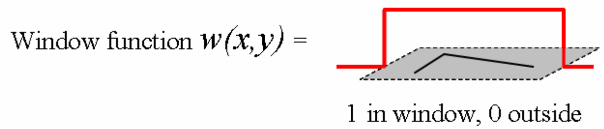
flat edge corner isolated point

## Moravec corner detector

Change of intensity for the shift [*u*,*v*]:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function — Shifted intensity — Intensity




Four shifts: (u,v) = (1,0), (1,1), (0,1), (-1, 1)
Look for local maxima in *min{E}*

## Problems of Moravec detector

- Noisy response due to a binary window function
- Only a set of shifts at every 45 degree is considered
- Only minimum of E is taken into account

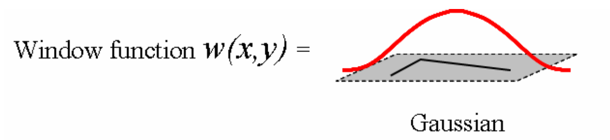
⇨ Harris corner detector (1988) solves these problems.

## Harris corner detector

Noisy response due to a binary window function

➢ Use a Gaussian function

$$w(x,y) = \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$




## Harris corner detector

Only a set of shifts at every 45 degree is considered

➢ Consider all small shifts by Taylor's expansion

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$
$$= \sum_{x,y} w(x,y)\left[I_x u + I_y v + O(u^2,v^2)\right]^2$$

$$E(u,v) = Au^2 + 2Cuv + Bv^2$$

$$A = \sum_{x,y} w(x,y) I_x^2(x,y)$$

$$B = \sum_{x,y} w(x,y) I_y^2(x,y)$$

$$C = \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y)$$

## Harris corner detector

Equivalently, for small shifts [*u*,*v*] we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \;\; M \begin{bmatrix} u \\ v \end{bmatrix}$$

, where *M* is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

## Harris corner detector

Only minimum of E is taken into account

➢A new corner measurement

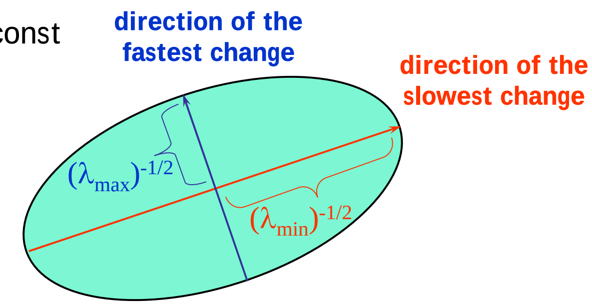
## Harris corner detector

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

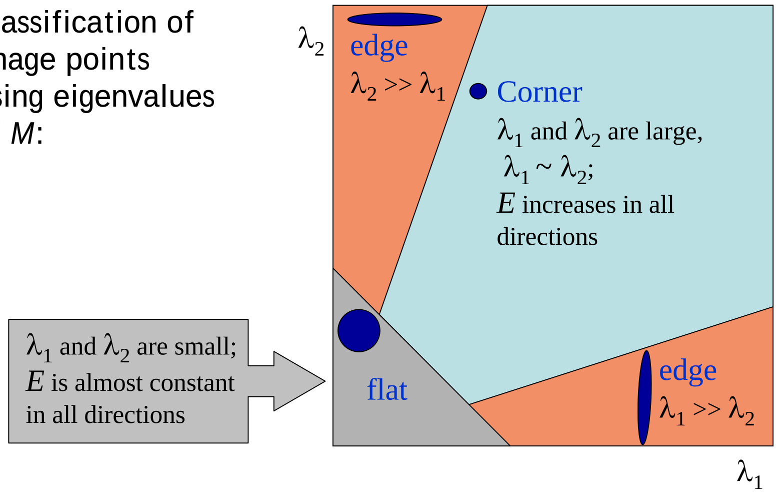
$\lambda_1, \lambda_2$ – eigenvalues of $M$

Ellipse $E(u,v)$ = const




## Harris corner detector

Classification of image points using eigenvalues of *M*:




## Harris corner detector

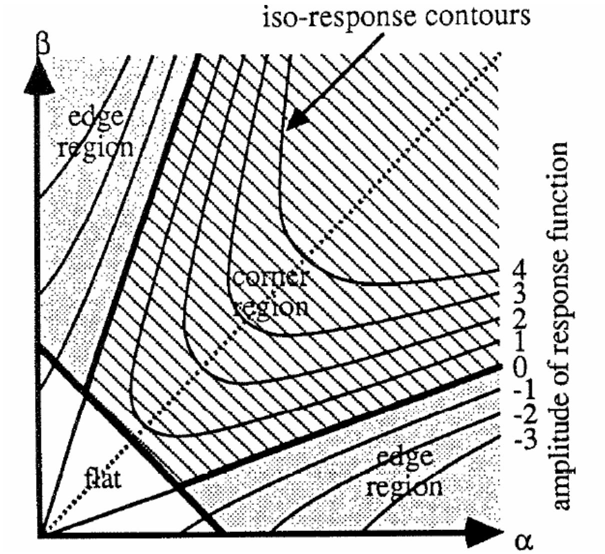
Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

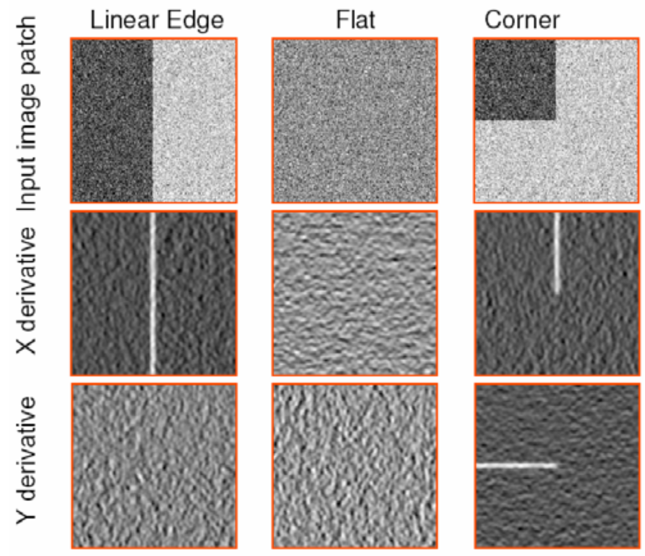
$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

($k$ – empirical constant, $k$ = 0.04-0.06)

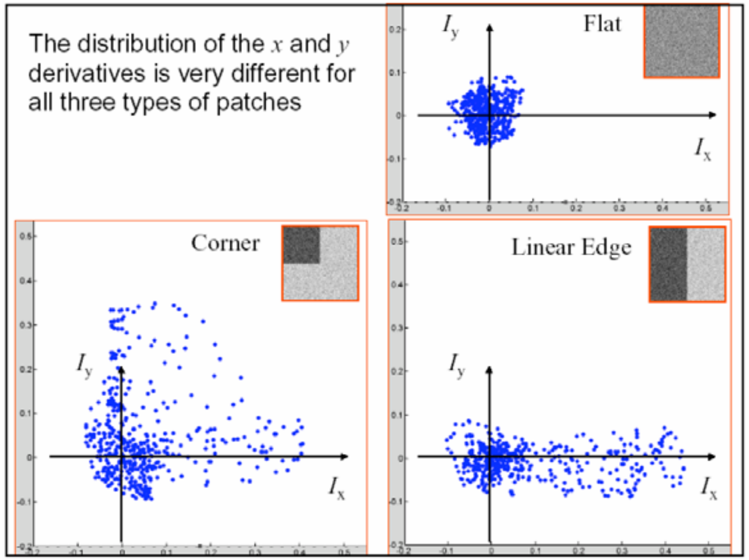
## Harris corner detector



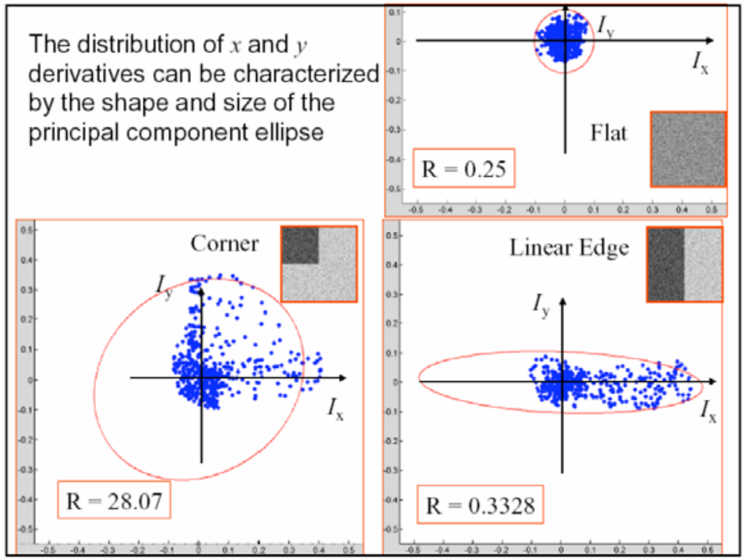

## Another view




## Another view




## Another view



## Summary of Harris detector

1. Compute $x$ and $y$ derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x.I_x \quad I_{y2} = I_y.I_y \quad I_{xy} = I_x.I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma'} * I_{x2} \quad S_{y2} = G_{\sigma'} * I_{y2} \quad S_{xy} = G_{\sigma'} * I_{xy}$$

4. Define at each pixel $(x, y)$ the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

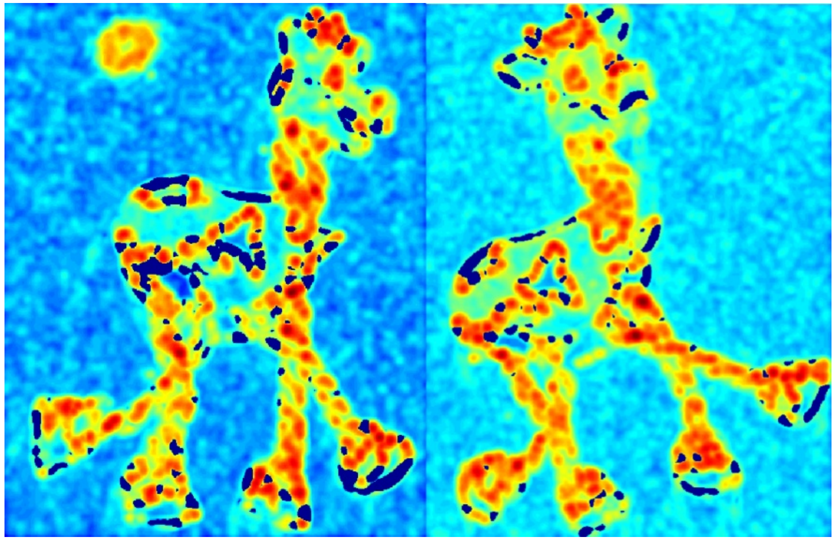
5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

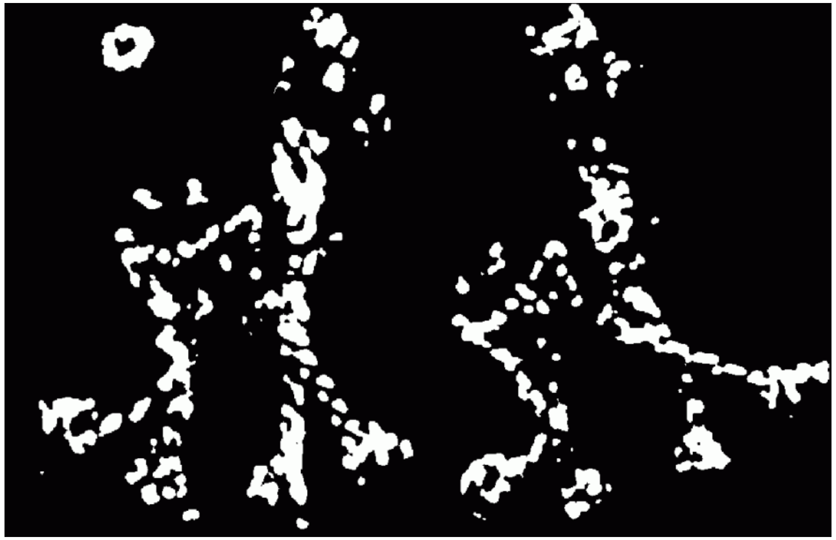
6. Threshold on value of R. Compute nonmax suppression.

## Harris corner detector (input)

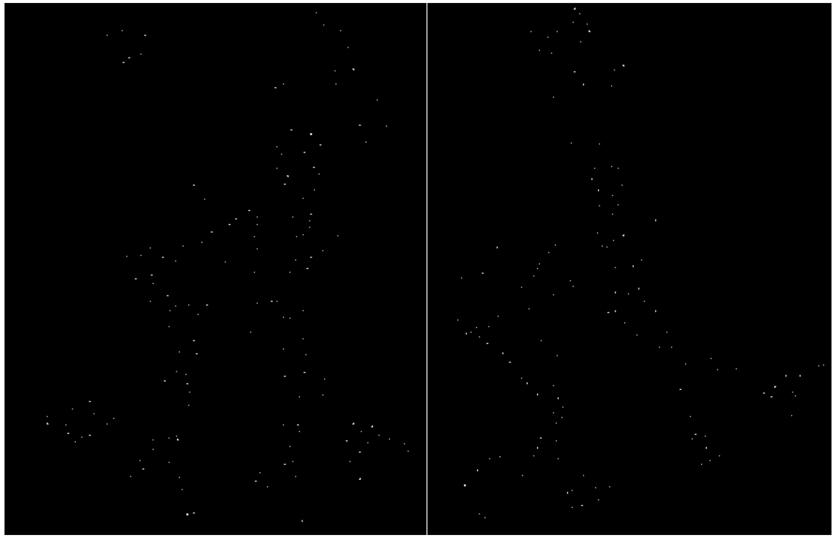
## Corner response R



## Threshold on R



## Local maximum of R



## Harris corner detector

## Harris detector: summary

- Average intensity change in direction $[u,v]$ can be expressed as a bilinear form:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

- Describe a point in terms of eigenvalues of $M$: *measure of corner response*

$$R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$

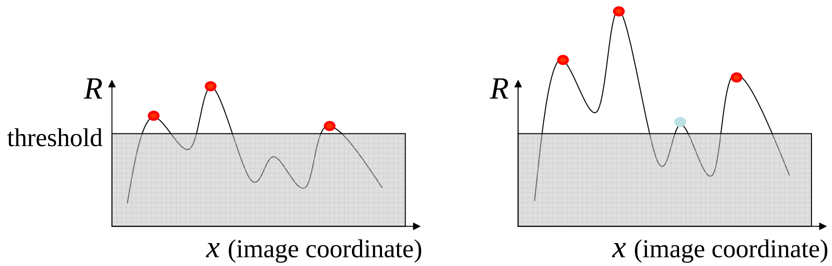
- A good (corner) point should have a *large intensity change* in *all directions*, i.e. $R$ should be large positive

## Harris detector: some properties

- Partial invariance to *affine intensity* change

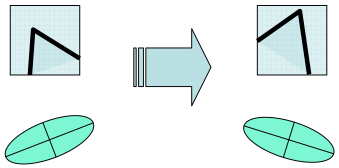
  ✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

  ✓ Intensity scale: $I \rightarrow a\,I$



## Harris Detector: Some Properties

- Rotation invariance



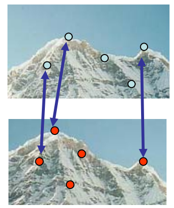
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

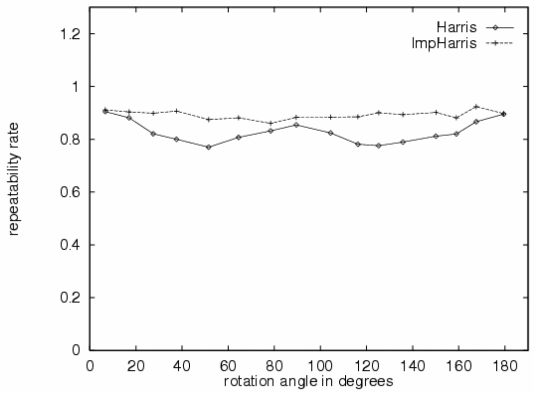
*Corner response R* is invariant to image rotation

## Harris Detector is rotation invariant

Repeatability rate:

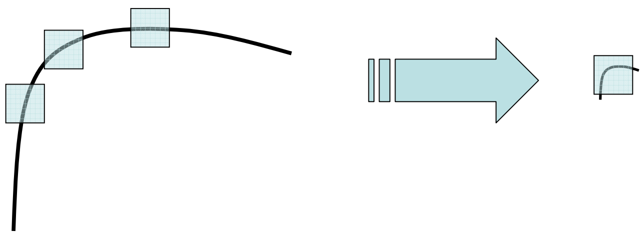
$$\frac{\#\text{ correspondences}}{\#\text{ possible correspondences}}$$






## Harris Detector: Some Properties

- But: non-invariant to *image scale*!



All points will be classified as edges

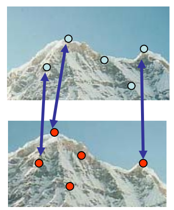
**Corner !**

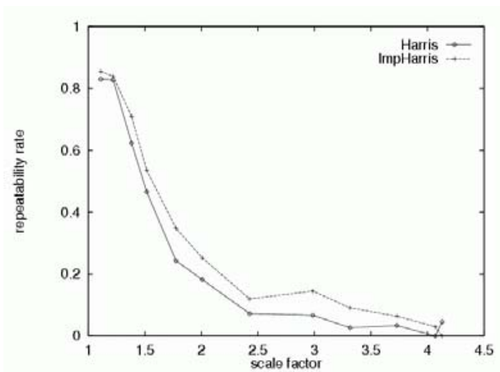
## Harris detector: some properties

- Quality of Harris detector for different scale changes

Repeatability rate:

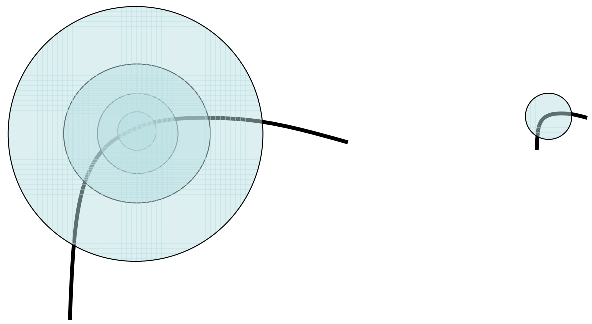
$$\frac{\#\text{ correspondences}}{\#\text{ possible correspondences}}$$





## Scale invariant detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



## Scale invariant detection

- The problem: how do we choose corresponding circles *independently* in each image?
- Aperture problem