

# Motion estimation

Digital Visual Effects, Spring 2005

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2005/3/23

*with slides by Michael Black and P. Anandan*

# Announcements

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- Project #1 is due on next Tuesday, submission mechanism will be announced later this week.
- grading: report is important, results (good/bad), discussions on implementation, interface, features, etc.

# Outline

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- Motion estimation
- Lucas-Kanade algorithm
- Tracking
- Optical flow

# Motion estimation

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- Parametric motion (image alignment)
- Tracking
- Optical flow

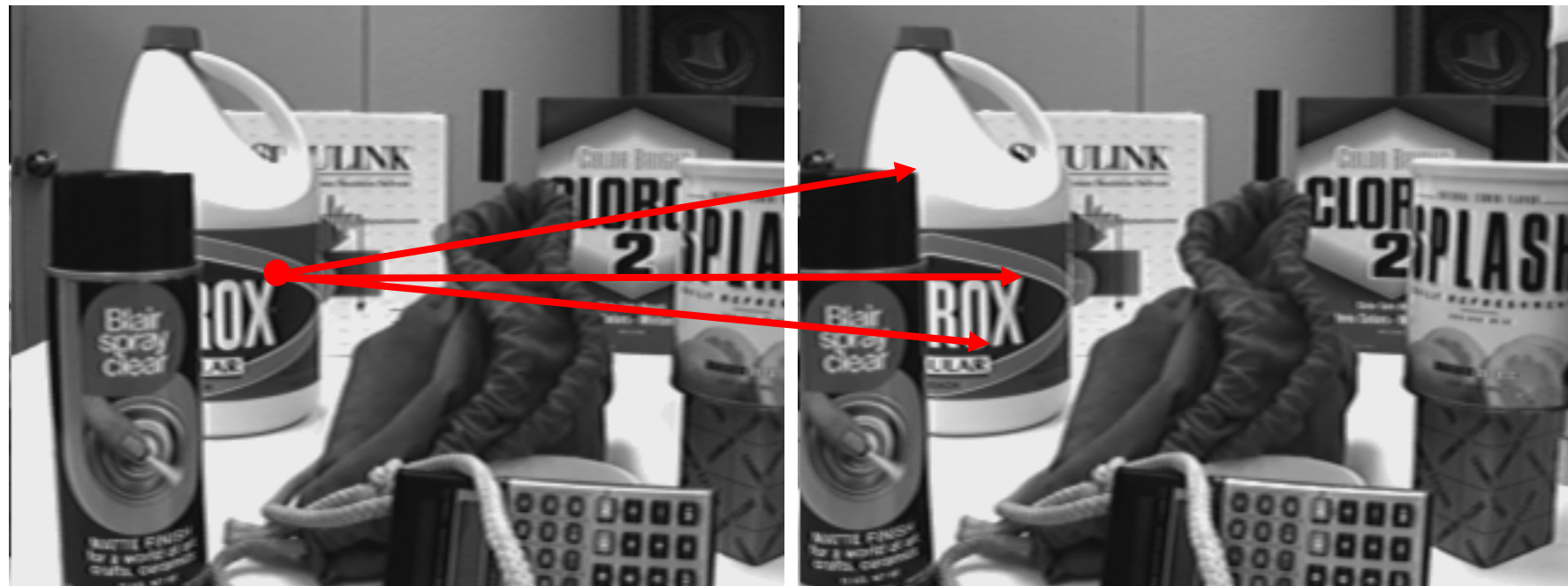
# Parametric motion

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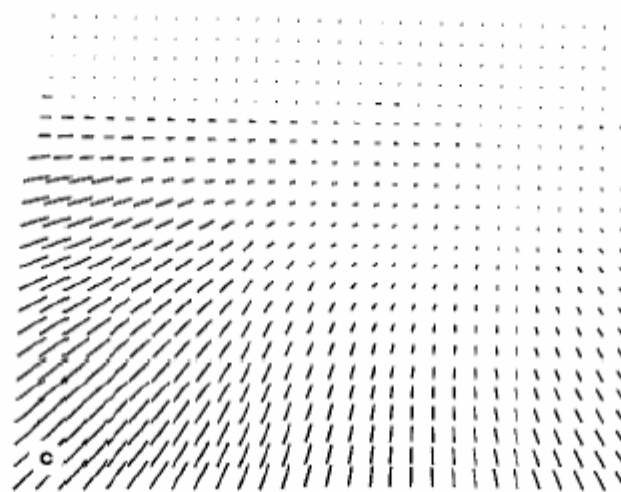
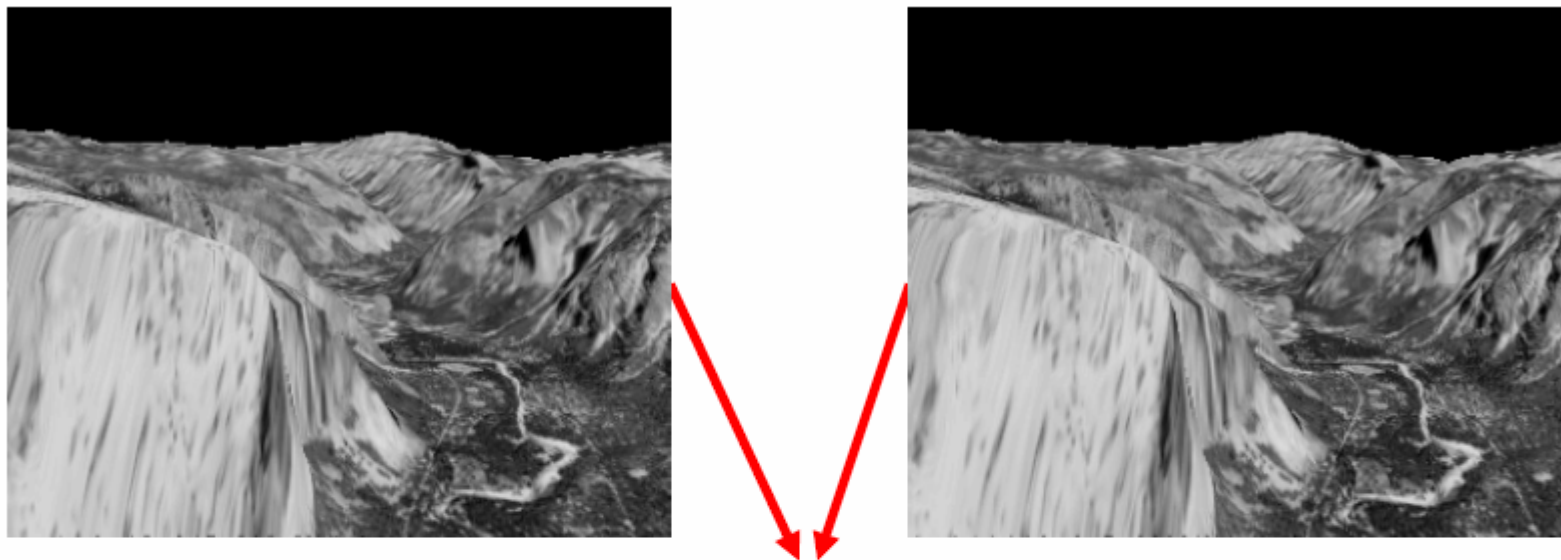


# Tracking

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# Optical flow



# Three assumptions

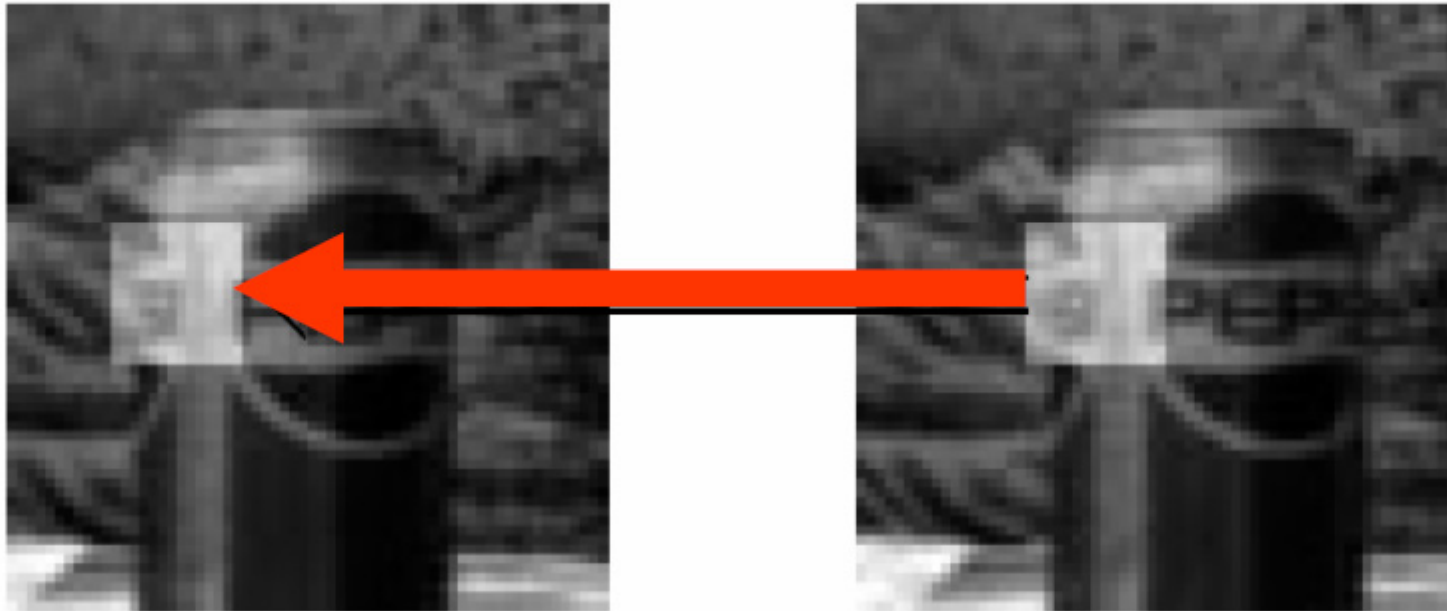
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- Brightness consistency
- Spatial coherence
- Temporal persistence



# Brightness consistency

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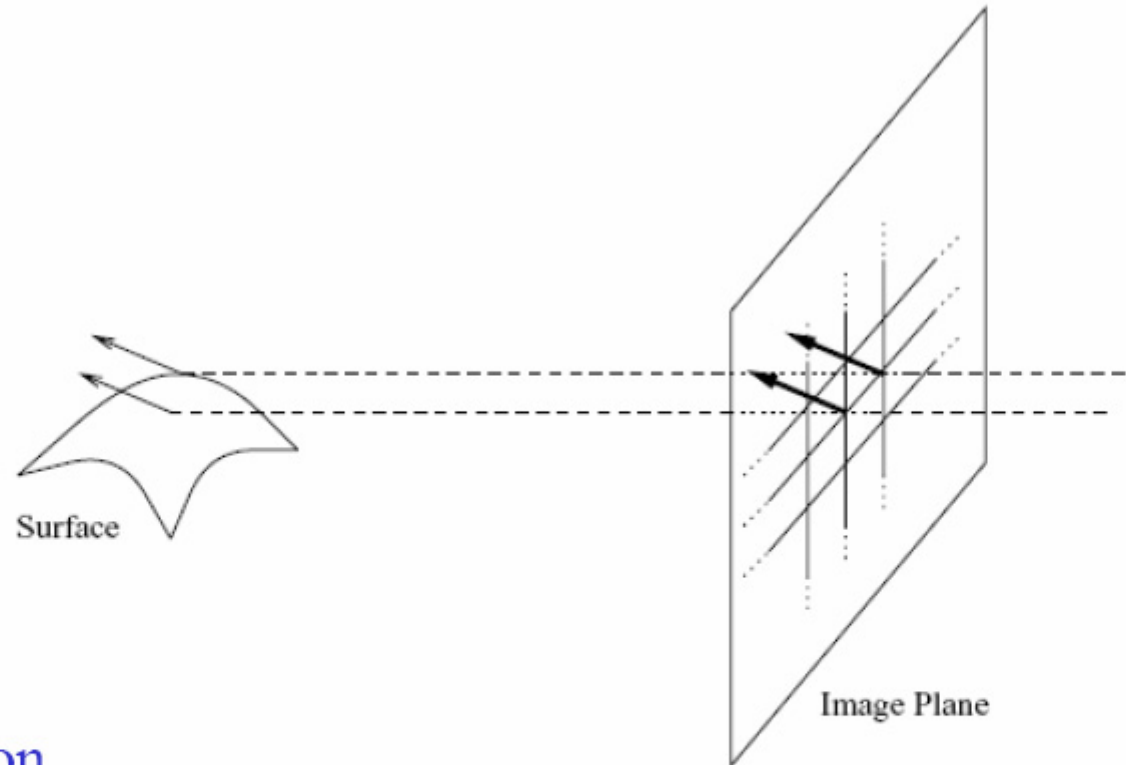


## Assumption

Image measurements (e.g. brightness) in a small region remain the same although their location may change.

# Spatial coherence

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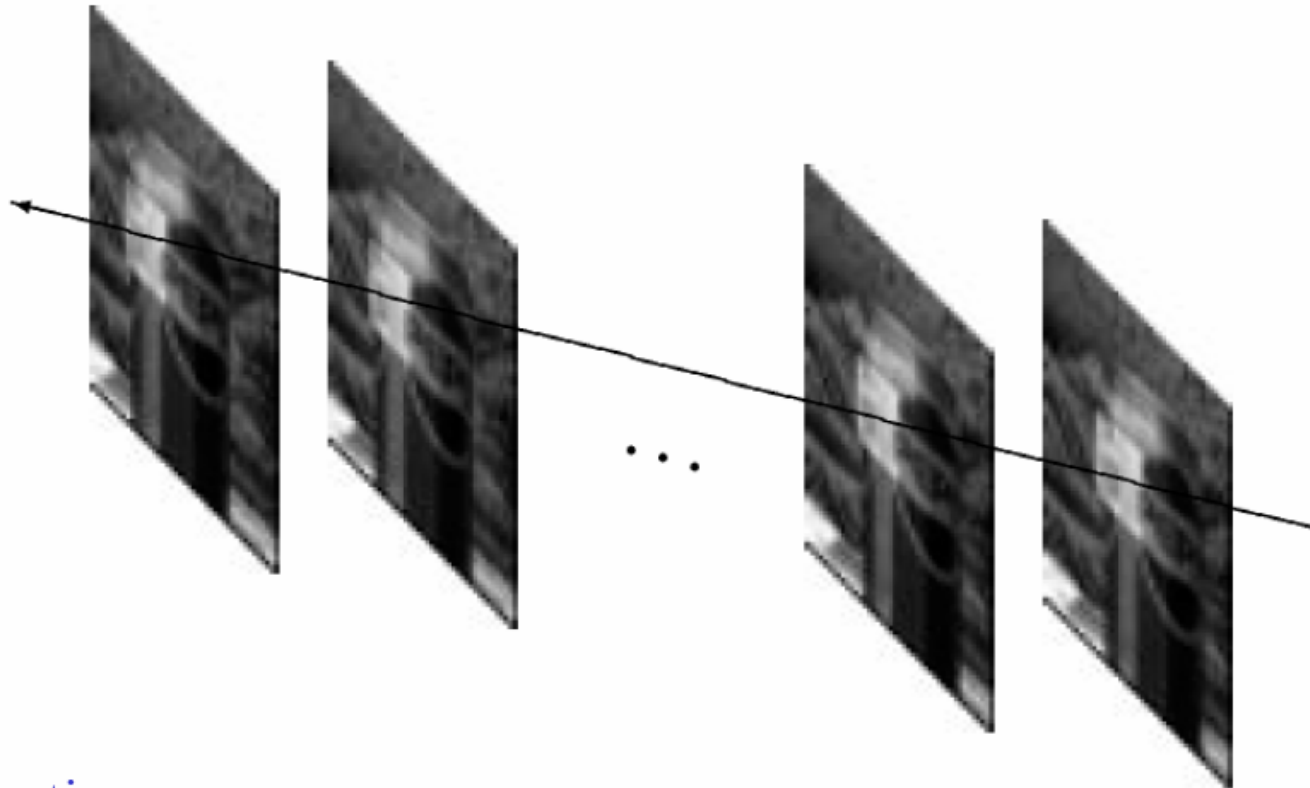


## Assumption

- \* Neighboring points in the scene typically belong to the same surface and hence typically have similar motions.
- \* Since they also project to nearby points in the image, we expect spatial coherence in image flow.

# Temporal persistence

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Assumption:

The image motion of a surface patch changes gradually over time.

# Image registration

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Goal: register a template image  $J(x)$  and an input image  $I(x)$ , where  $x=(x,y)^T$ .

Image alignment:  $I(x)$  and  $J(x)$  are two images

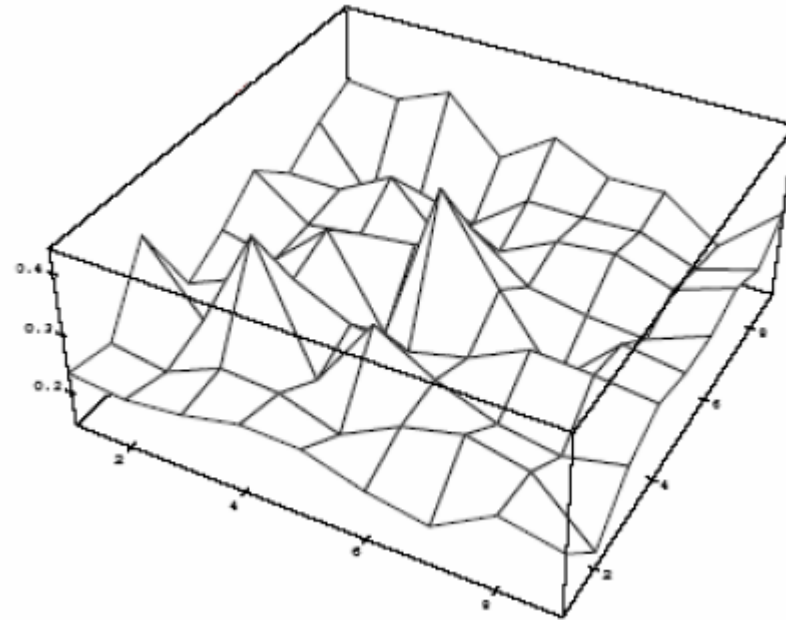
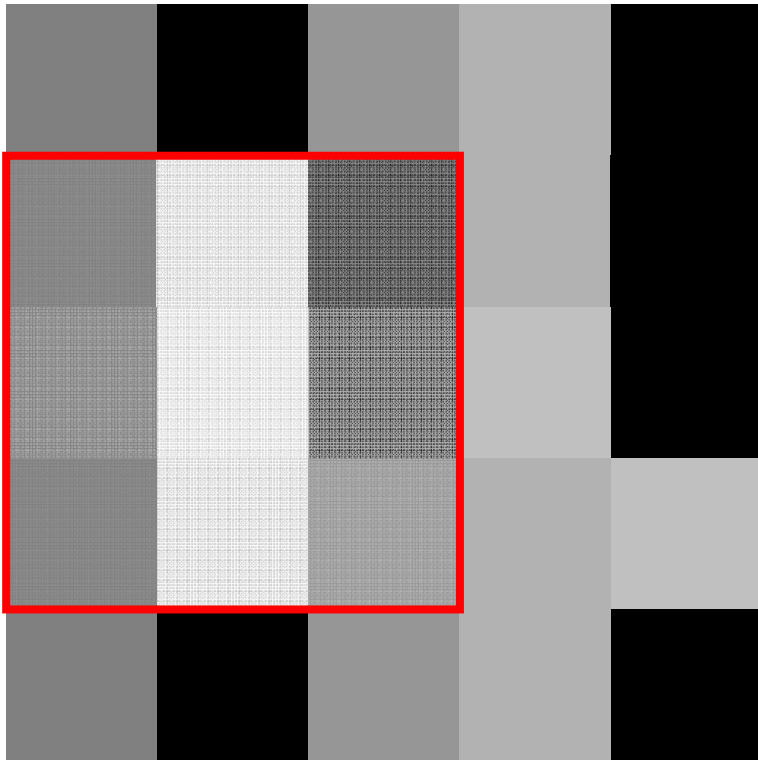
Tracking:  $I(x)$  is the image at time  $t$ .  $J(x)$  is a small patch around the point  $p$  in the image at  $t+1$ .

Optical flow:  $I(x)$  and  $J(x)$  are images of  $t$  and  $t+1$ .

# Simple approach

- Minimize brightness difference

$$E(u, v) = \sum_{x, y} (I(x + u, y + v) - J(x, y))^2$$



# Simple SSD algorithm

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For each offset  $(u, v)$

    compute  $E(u, v)$ ;

Choose  $(u, v)$  which minimizes  $E(u, v)$ ;

Problems:

- Not efficient
- No sub-pixel accuracy

# Lucas-Kanade algorithm

# Newton's method

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- Root finding for  $f(x)=0$

Taylor's expansion:

$$f(x_0 + \epsilon) = f(x_0) + f'(x_0)\epsilon + \frac{1}{2}f''(x_0)\epsilon^2 + \dots$$

$$f(x_0 + \epsilon) \approx f(x_0) + f'(x_0)\epsilon.$$

$$\epsilon_n = -\frac{f(x_n)}{f'(x_n)}.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



# Lucas-Kanade algorithm

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$$E(u, v) = \sum_{x,y} (I(x+u, y+v) - J(x, y))^2$$

$$I(x+u, y+v) \approx I(x, y) + uI_x + vI_y$$

$$= \sum_{x,y} (I(x, y) - J(x, y) + uI_x + vI_y)^2$$

$$0 = \frac{\partial E}{\partial u} = \sum_{x,y} 2I_x (I(x, y) - J(x, y) + uI_x + vI_y)$$

$$0 = \frac{\partial E}{\partial v} = \sum_{x,y} 2I_y (I(x, y) - J(x, y) + uI_x + vI_y)$$

# Lucas-Kanade algorithm

$$0 = \frac{\partial E}{\partial u} = \sum_{x,y} 2I_x (I(x,y) - J(x,y) + uI_x + vI_y)$$

$$0 = \frac{\partial E}{\partial v} = \sum_{x,y} 2I_y (I(x,y) - J(x,y) + uI_x + vI_y)$$

$$\rightarrow \begin{cases} \sum_{x,y} I_x^2 u + I_x I_y v = \sum_{x,y} I_x (J(x,y) - I(x,y)) \\ \sum_{x,y} I_x I_y u + I_y^2 v = \sum_{x,y} I_y (J(x,y) - I(x,y)) \end{cases}$$

$$\rightarrow \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_{x,y} I_x (J(x,y) - I(x,y)) \\ \sum_{x,y} I_y (J(x,y) - I(x,y)) \end{bmatrix}$$

# Lucas-Kanade algorithm

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iterate

shift  $I(x,y)$  with  $(u,v)$

compute gradient image  $I_x, I_y$

compute error image  $J(x,y)-I(x,y)$

compute Hessian matrix

solve the linear system

$$(u,v)=(u,v)+(\Delta u,\Delta v)$$

until converge

$$\begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_{x,y} I_x (J(x,y) - I(x,y)) \\ \sum_{x,y} I_y (J(x,y) - I(x,y)) \end{bmatrix}$$

# Parametric model

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$$E(u, v) = \sum_{x, y} (I(x + u, y + v) - J(x, y))^2$$

$$\longrightarrow E(\mathbf{p}) = \sum_{\mathbf{x}} (I(\mathbf{W}(\mathbf{x}; \mathbf{p})) - J(\mathbf{x}))^2$$

**translation**      $\mathbf{W}(\mathbf{x}; \mathbf{p}) = \begin{pmatrix} x + d_x \\ y + d_y \end{pmatrix}, p = (d_x, d_y)^T$

**affine**      $\mathbf{W}(\mathbf{x}; \mathbf{p}) = \mathbf{A}\mathbf{x} + \mathbf{d} = \begin{pmatrix} 1 + d_{xx} & d_{xy} & d_x \\ d_{yx} & 1 + d_{yy} & d_y \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix},$

$$p = (d_{xx}, d_{xy}, d_{yx}, d_{yy}, d_x, d_y)^T$$

# Parametric model

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$$\text{minimize } \sum_{\mathbf{x}} (I(\mathbf{W}(\mathbf{x}; \mathbf{p} + \Delta \mathbf{p})) - J(\mathbf{x}))^2$$

with respect to  $\Delta \mathbf{p}$

$$\mathbf{W}(\mathbf{x}; \mathbf{p} + \Delta \mathbf{p}) \approx \mathbf{W}(\mathbf{x}; \mathbf{p}) + \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p}$$

$$I(\mathbf{W}(\mathbf{x}; \mathbf{p} + \Delta \mathbf{p})) \approx I(\mathbf{W}(\mathbf{x}; \mathbf{p}) + \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p})$$

$$\approx I(\mathbf{W}(\mathbf{x}; \mathbf{p})) + \frac{\partial I}{\partial \mathbf{x}} \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p}$$

$$\longrightarrow \text{minimize } \sum_{\mathbf{x}} \left( I(\mathbf{W}(\mathbf{x}; \mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - J(\mathbf{x}) \right)^2$$

# Parametric model

warped image

image gradient

$$\sum_{\mathbf{x}} \left( I(\mathbf{W}(\mathbf{x}; \mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - J(\mathbf{x}) \right)^2$$

*Jacobian of the warp*

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_2} & \dots & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_n} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_2} & \dots & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_n} \end{pmatrix}$$

# Jacobian of the warp

---

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_2} & \dots & \frac{\partial \mathbf{W}_x}{\partial \mathbf{p}_n} \\ \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_1} & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_2} & \dots & \frac{\partial \mathbf{W}_y}{\partial \mathbf{p}_n} \end{pmatrix}$$

For example, for affine

$$\mathbf{W}(\mathbf{x}; \mathbf{p}) = \begin{pmatrix} 1 + d_{xx} & d_{xy} & d_x \\ d_{yx} & 1 + d_{yy} & d_y \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} (1 + d_{xx})x + d_{xy}y + d_x \\ d_{yx}x + (1 + d_{yy})y + d_y \end{pmatrix}$$

$$\rightarrow \frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{pmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{pmatrix}$$

# Parametric model

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$$\text{minimize } \sum_{\mathbf{x}} \left( I(\mathbf{W}(\mathbf{x}; \mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - J(\mathbf{x}) \right)^2$$

$$\rightarrow 0 = \sum_{\mathbf{x}} \left[ \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[ I(\mathbf{W}(\mathbf{x}; \mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - J(\mathbf{x}) \right]$$

$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[ \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T [J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x}; \mathbf{p}))]$$

$$\text{Hessian } \mathbf{H} = \sum_{\mathbf{x}} \left[ \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[ \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$



# Lucas-Kanade algorithm

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iterate

warp I with  $W(x;p)$

compute error image  $J(x,y)-I(W(x,p))$

compute gradient image

evaluate Jacobian  $\frac{\partial W}{\partial p}$  at  $(x;p)$

compute  $\nabla I \frac{\partial W}{\partial p}$

compute Hessian

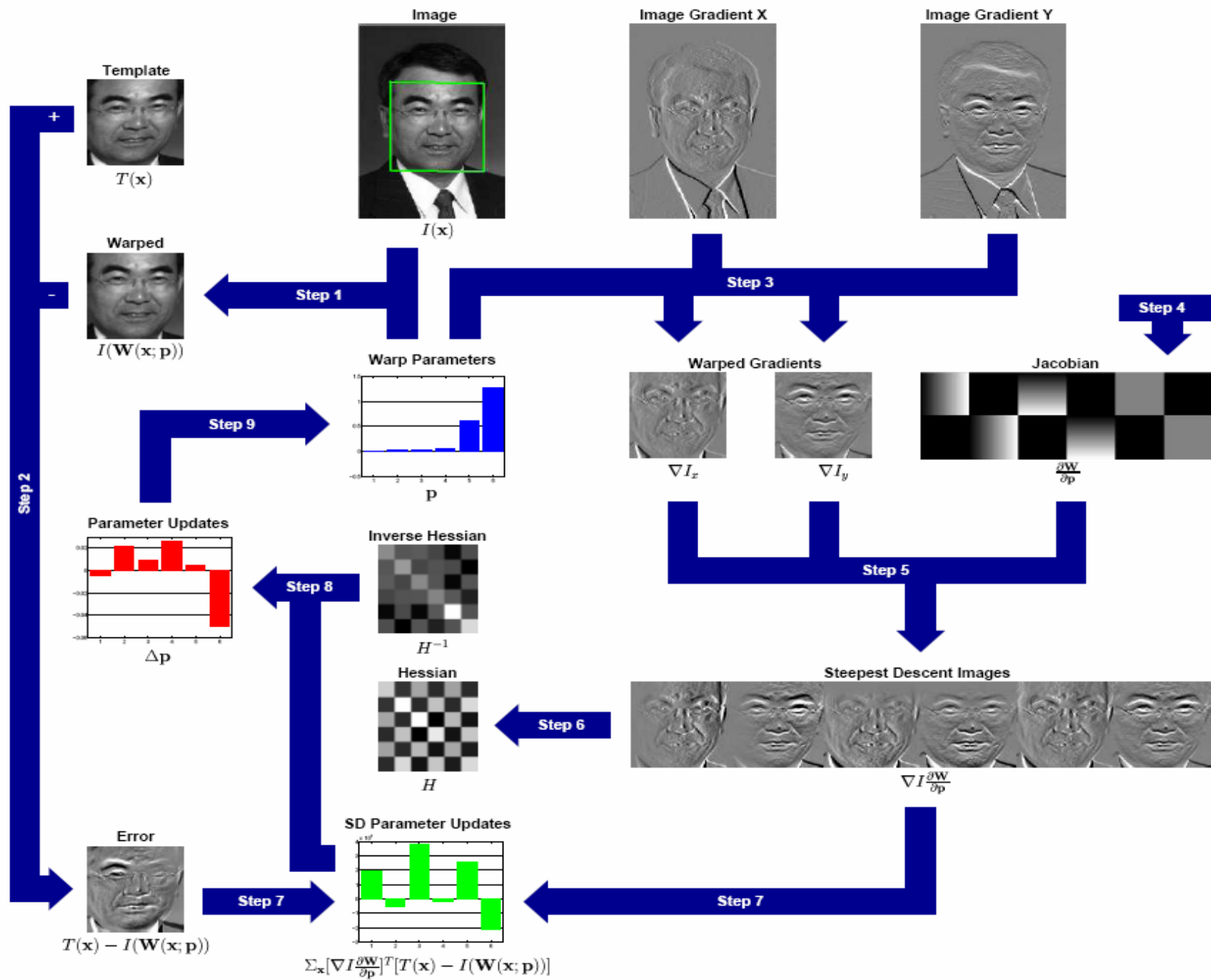
compute  $\sum_x \left[ \nabla I \frac{\partial W}{\partial p} \right]^T [J(x) - I(W(x;p))]$

solve  $\Delta p$

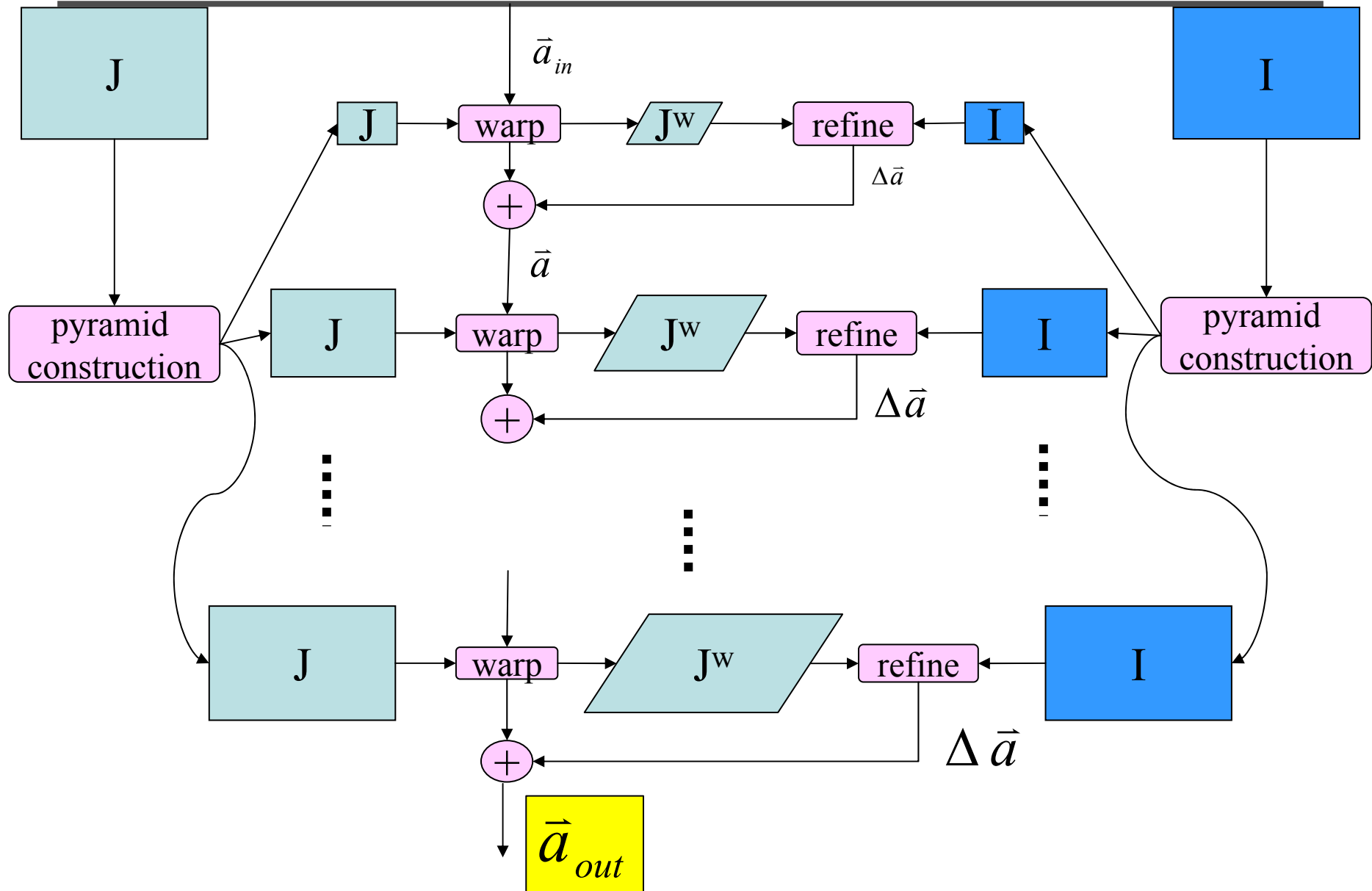
update  $p$  by  $p + \Delta p$

until converge

$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_x \left[ \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T [J(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$$



# Coarse-to-fine strategy



# Application of image alignment



# Tracking

# Tracking

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# Tracking

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brightness constancy  $I(x + u, y + v, t + 1) - I(x, y, t) = 0$

$$I(x, y, t) + uI_x(x, y, t) + vI_y(x, y, t) + I_t(x, y, t) - I(x, y, t) \approx 0$$

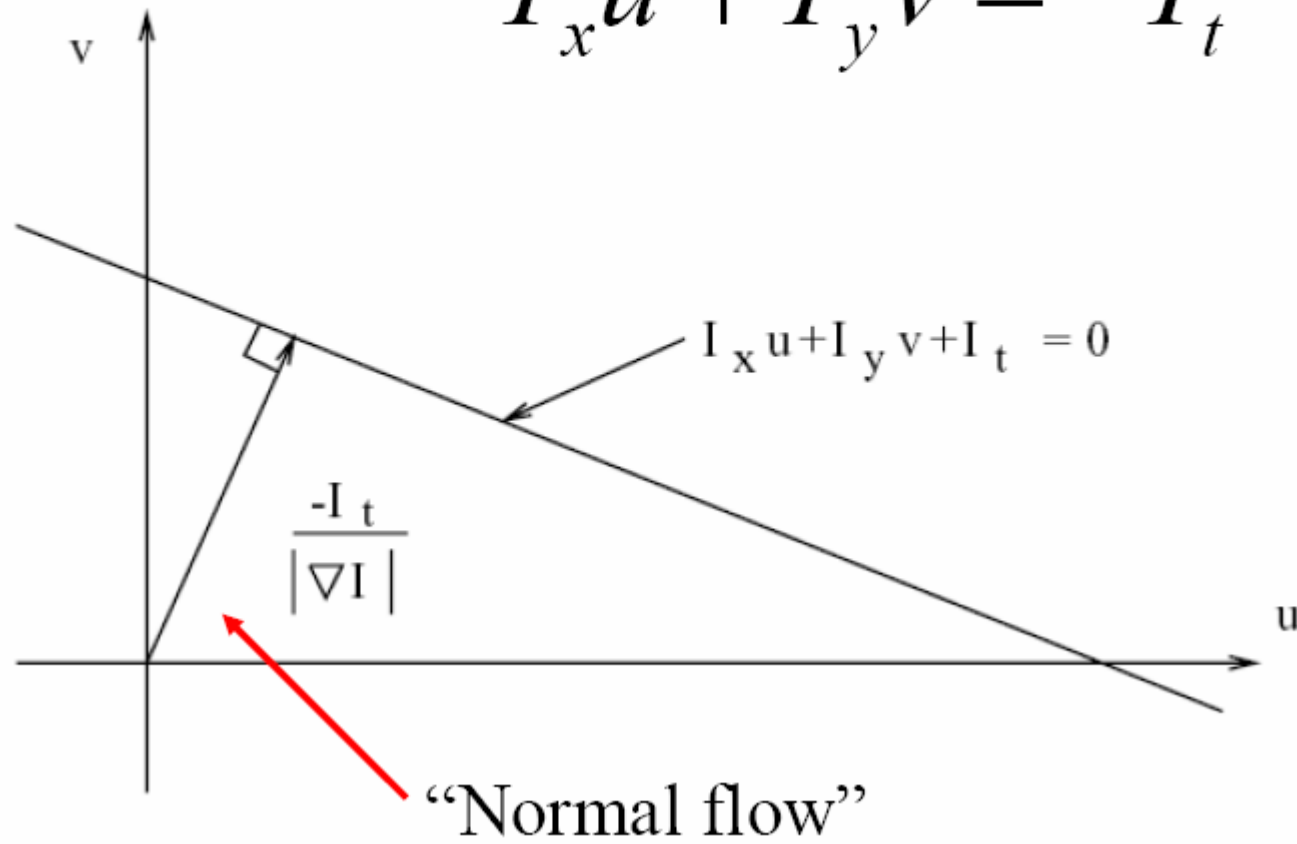
$$uI_x(x, y, t) + vI_y(x, y, t) + I_t(x, y, t) = 0$$

$$I_x u + I_y v + I_t = 0 \quad \text{optical flow constraint equation}$$

# Optical flow constraint equation

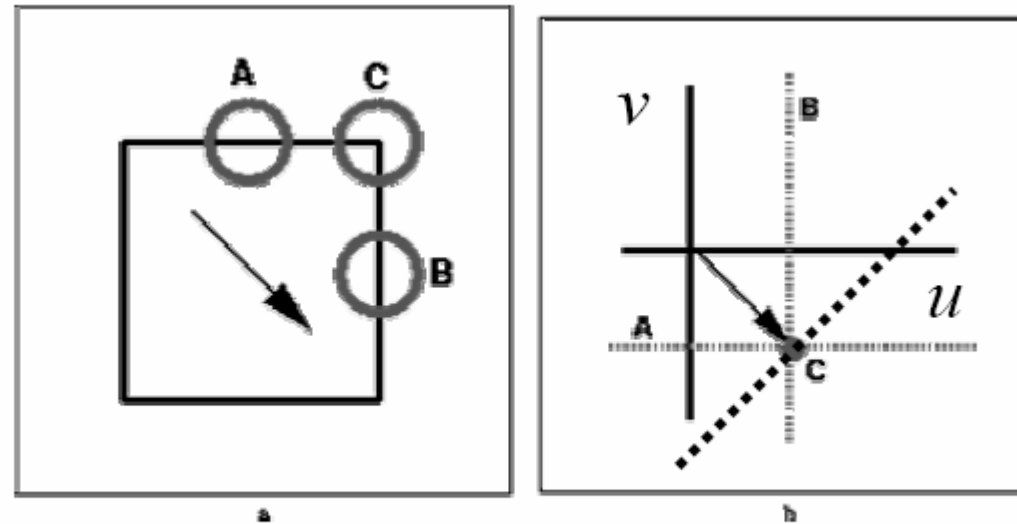
At a single image pixel, we get a line:

$$I_x u + I_y v = -I_t$$





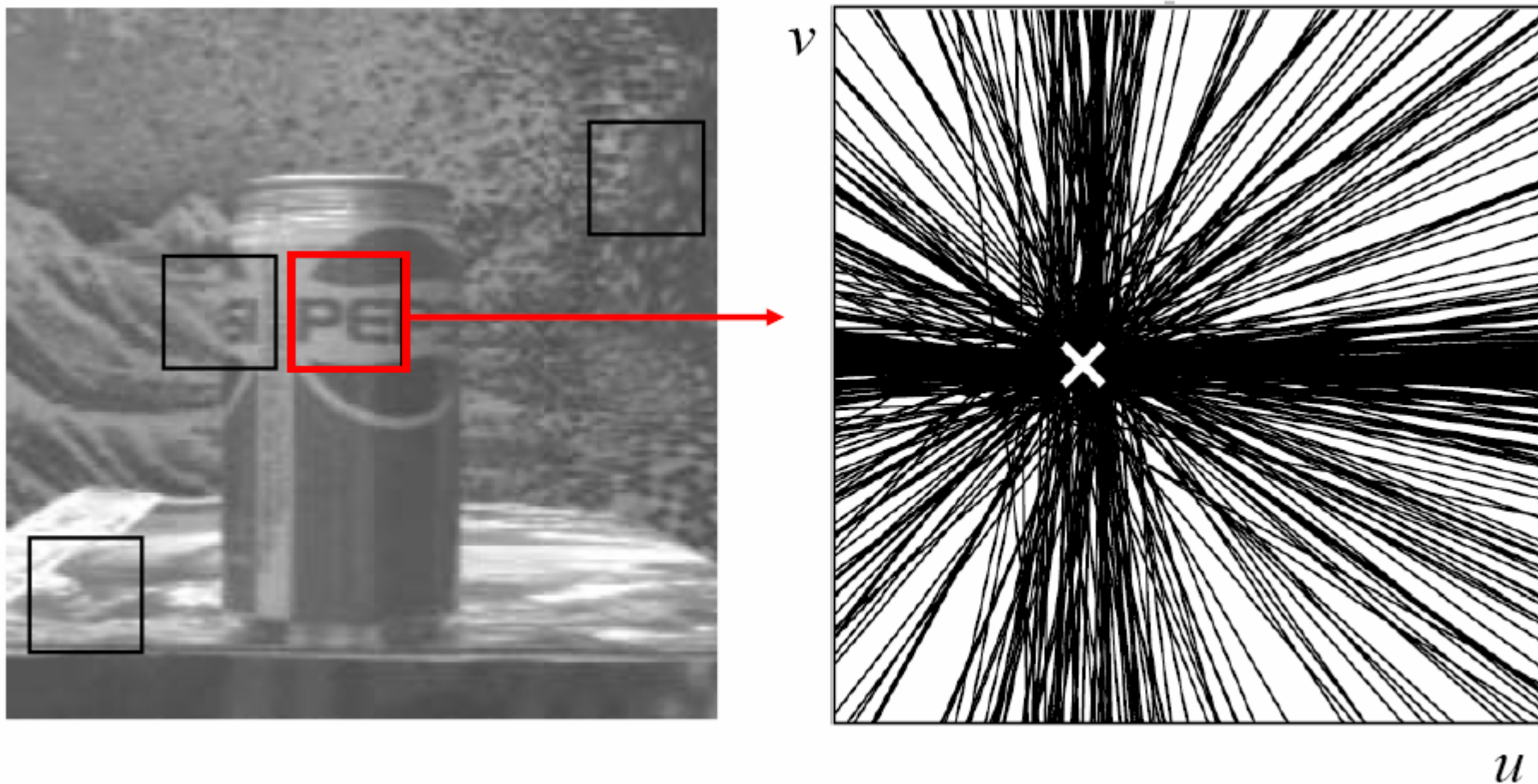
# Multiple constraint



Combine constraints to get an estimate of velocity.

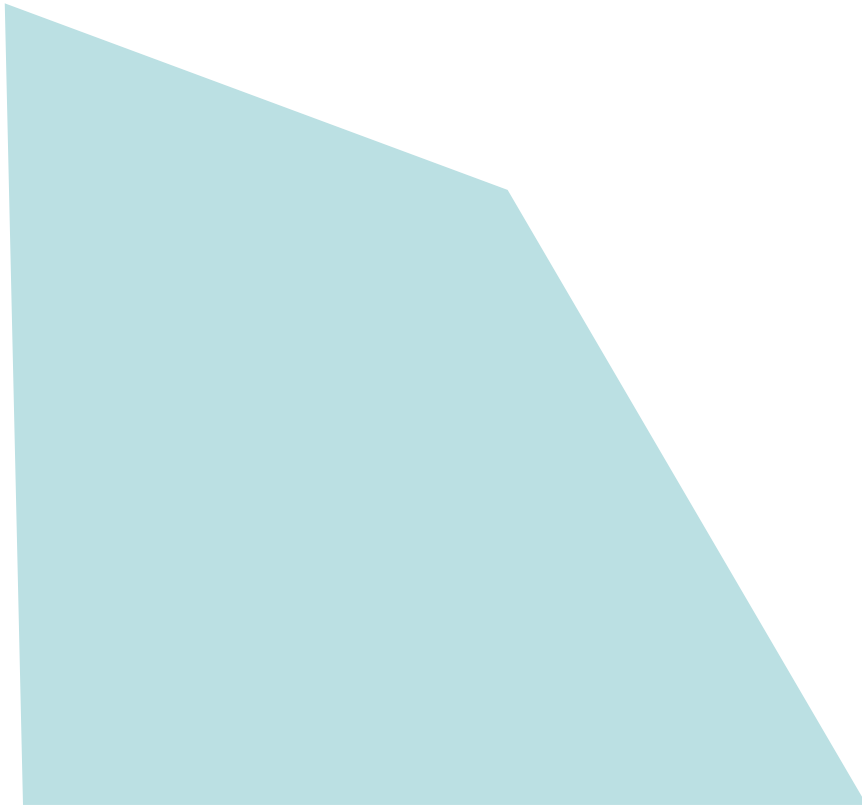
# Area-based method

- Assume spatial smoothness



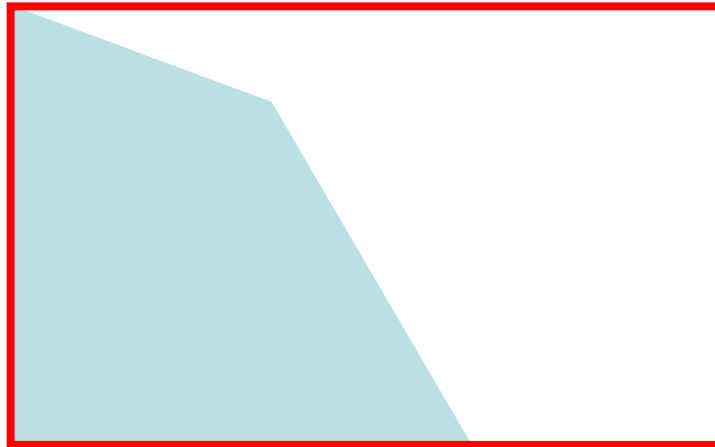
# Aperture problem

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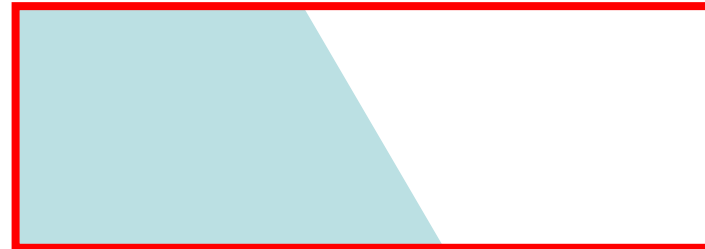
# Aperture problem

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# Aperture problem

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# Demo for aperture problem

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- [http://www.sandlotscience.com/Distortions/Breathing\\_objects.htm](http://www.sandlotscience.com/Distortions/Breathing_objects.htm)
- <http://www.sandlotscience.com/Ambiguous/barberpole.htm>

# Aperture problem

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- Larger window reduces ambiguity, but easily violates spatial smoothness assumption

# Area-based method

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- Assume spatial smoothness

$$E(u, v) = \sum_{x,y} (I_x u + I_y v + I_t)^2$$

$$\frac{\partial E}{\partial u} = \sum_R (I_x u + I_y v + I_t) I_x = 0$$

$$\frac{\partial E}{\partial v} = \sum_R (I_x u + I_y v + I_t) I_y = 0$$



# Area-based method

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$$\left[ \sum_R I_x^2 \right] u + \left[ \sum_R I_x I_y \right] v = - \sum_R I_x I_t$$

$$\left[ \sum_R I_x I_y \right] u + \left[ \sum_R I_y^2 \right] v = - \sum_R I_y I_t$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} - \sum I_x I_t \\ - \sum I_y I_t \end{bmatrix}$$

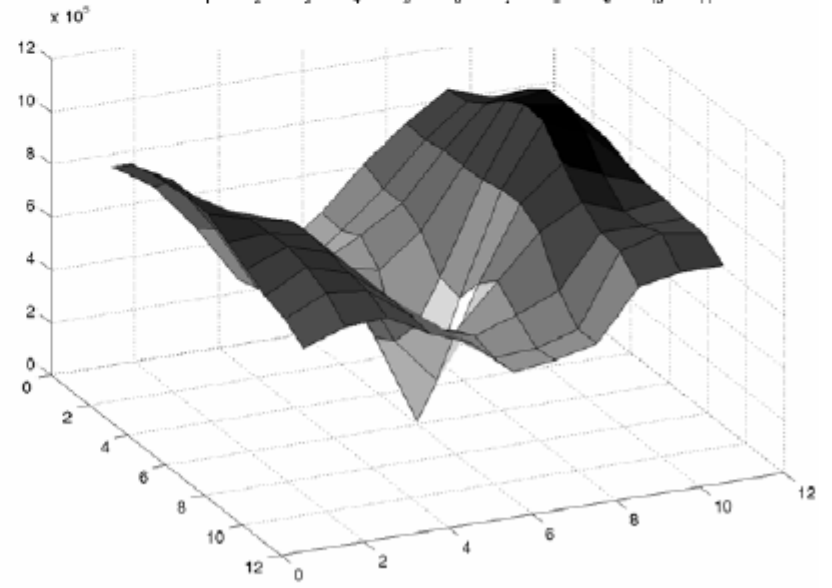
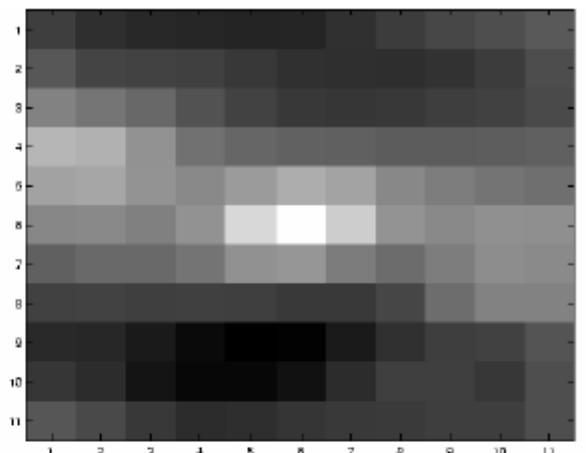
**must be invertible**

# Area-based method

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- The eigenvalues tell us about the local image structure.
- They also tell us how well we can estimate the flow in both directions
- Link to Harris corner detector

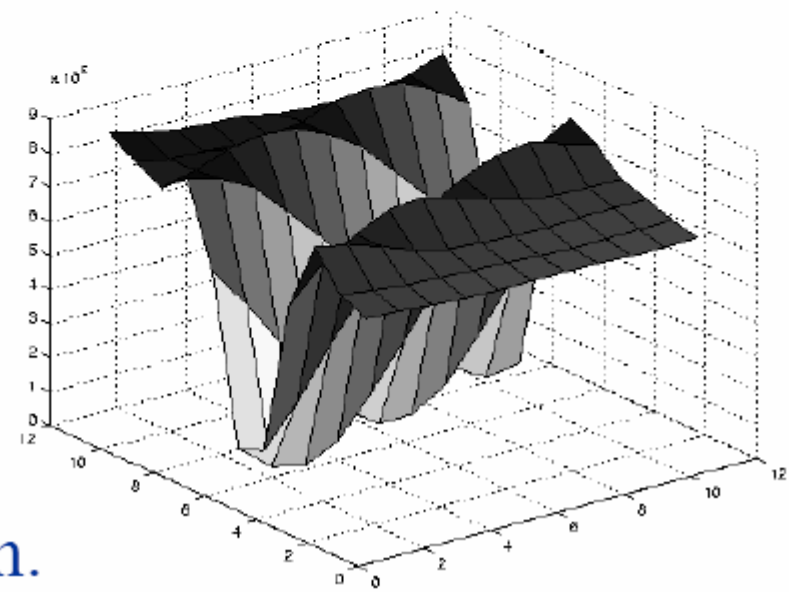
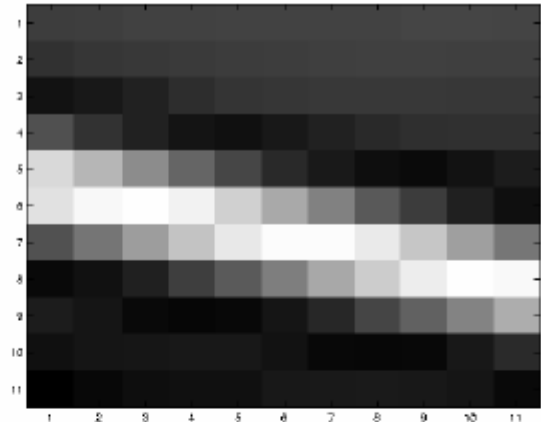
# Textured area



$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$$

Gradients in x and y.

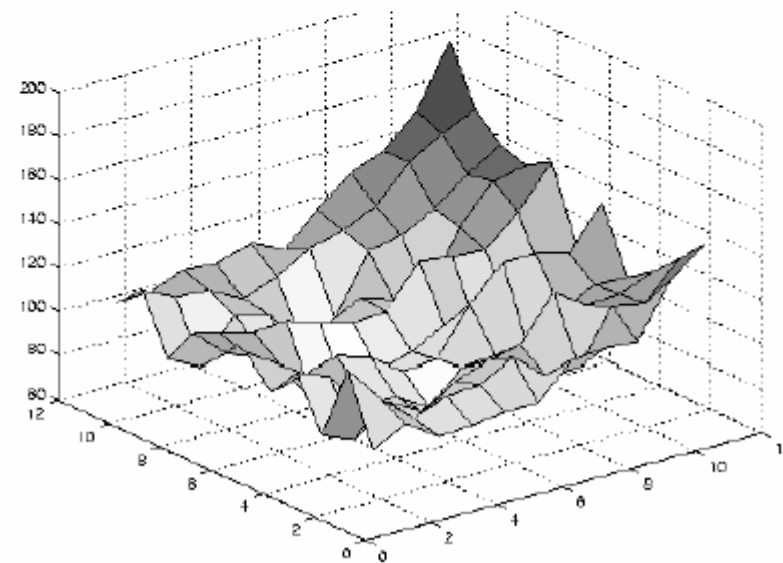
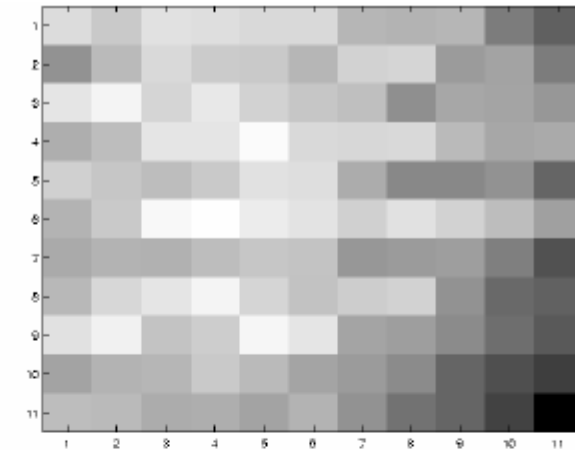
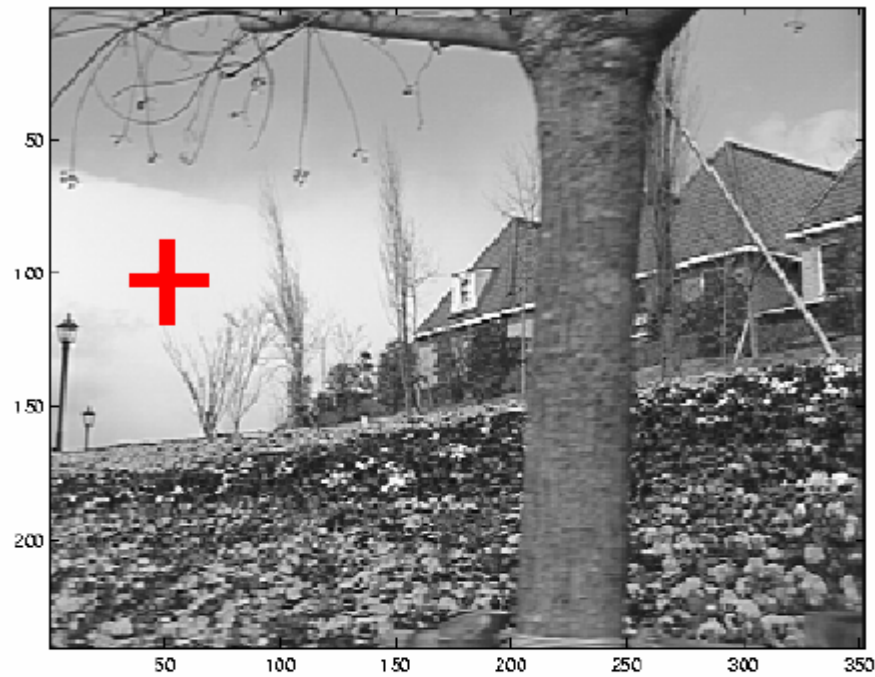
# Edge



$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$$

Gradients oriented in one direction.

# Homogenous area

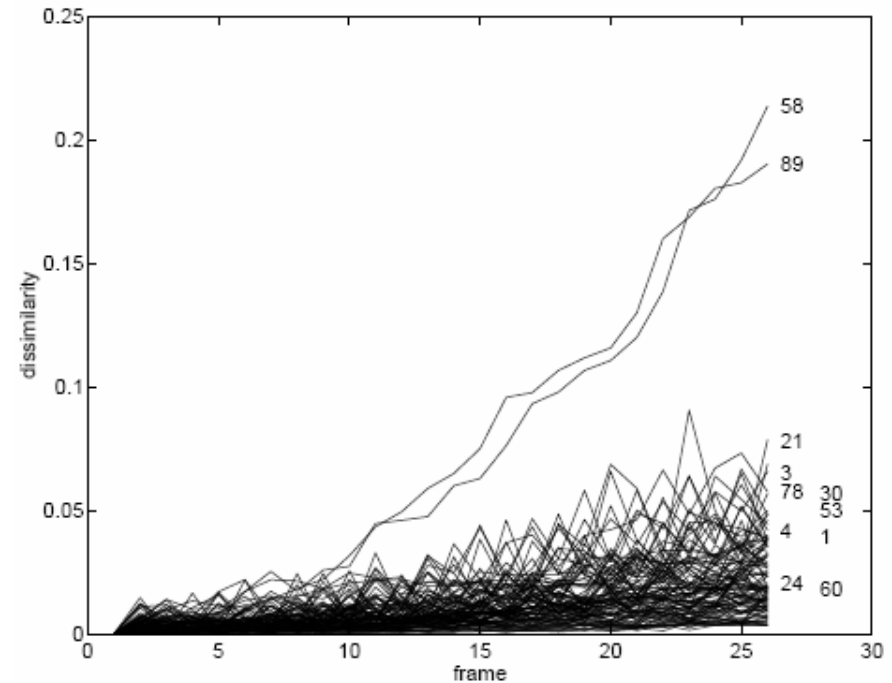
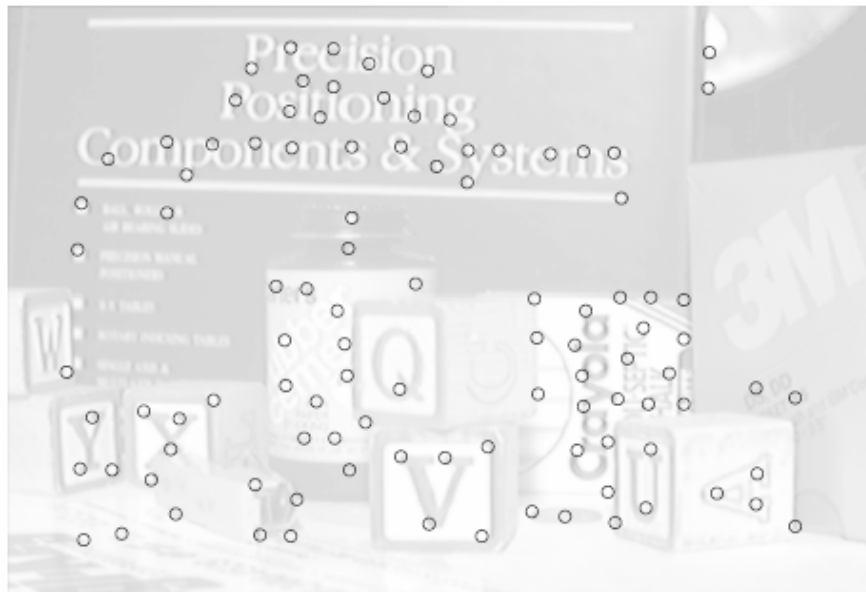


$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}$$

Weak gradients everywhere.

# KLT tracking

- Select feature by  $\min(\lambda_1, \lambda_2) > \lambda$
- Monitor features by measuring dissimilarity



# Translational Model



What's wrong with the translational assumption (ie constant motion within a region  $R$ )?

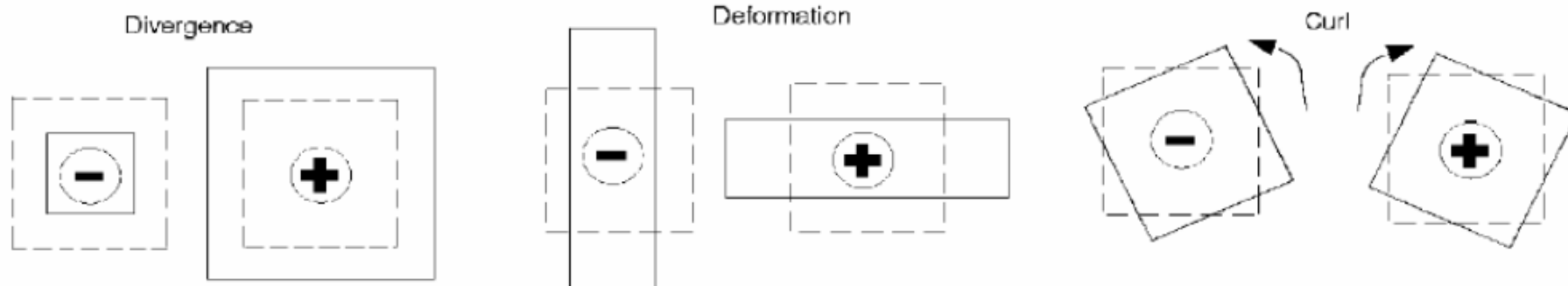
How can we generalize it?

# Affine Flow

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$$E(\mathbf{a}) = \sum_{x,y \in R} (\nabla I^T \mathbf{u}(\mathbf{x}; \mathbf{a}) + I_t)^2$$

$$\mathbf{u}(\mathbf{x}; \mathbf{a}) = \begin{bmatrix} u(\mathbf{x}; \mathbf{a}) \\ v(\mathbf{x}; \mathbf{a}) \end{bmatrix} = \begin{bmatrix} a_1 + a_2x + a_3y \\ a_4 + a_5x + a_6y \end{bmatrix}$$





# Optimization

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$$E(\mathbf{a}) = \sum_{x,y \in R} (I_x a_1 + I_x a_2 x + I_x a_3 y + I_y a_4 + I_y a_5 x + I_y a_6 y + I_t)^2$$

Differentiate wrt the  $a_i$  and set equal to zero.

$$\begin{bmatrix} \Sigma I_x^2 & \Sigma I_x^2 x & \Sigma I_x^2 y & \Sigma I_x I_y & \Sigma I_x I_y x & \Sigma I_x I_y y \\ \Sigma I_x^2 x & \Sigma I_x^2 x^2 & \Sigma I_x^2 xy & \Sigma I_x I_y x & \Sigma I_x I_y x^2 & \Sigma I_x I_y xy \\ & & & \vdots & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} -\Sigma I_x I_t \\ -\Sigma I_x I_t x \\ -\Sigma I_x I_t y \\ -\Sigma I_y I_t \\ -\Sigma I_y I_t x \\ -\Sigma I_y I_t y \end{bmatrix}$$

# KLT tracking

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<http://www.ces.clemson.edu/~stb/klt/>

# KLT tracking

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<http://www.ces.clemson.edu/~stb/klt/>

# SIFT tracking (matching actually)

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Frame 0



Frame 10

# SIFT tracking

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Frame 0



Frame 100

# SIFT tracking

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Frame 0



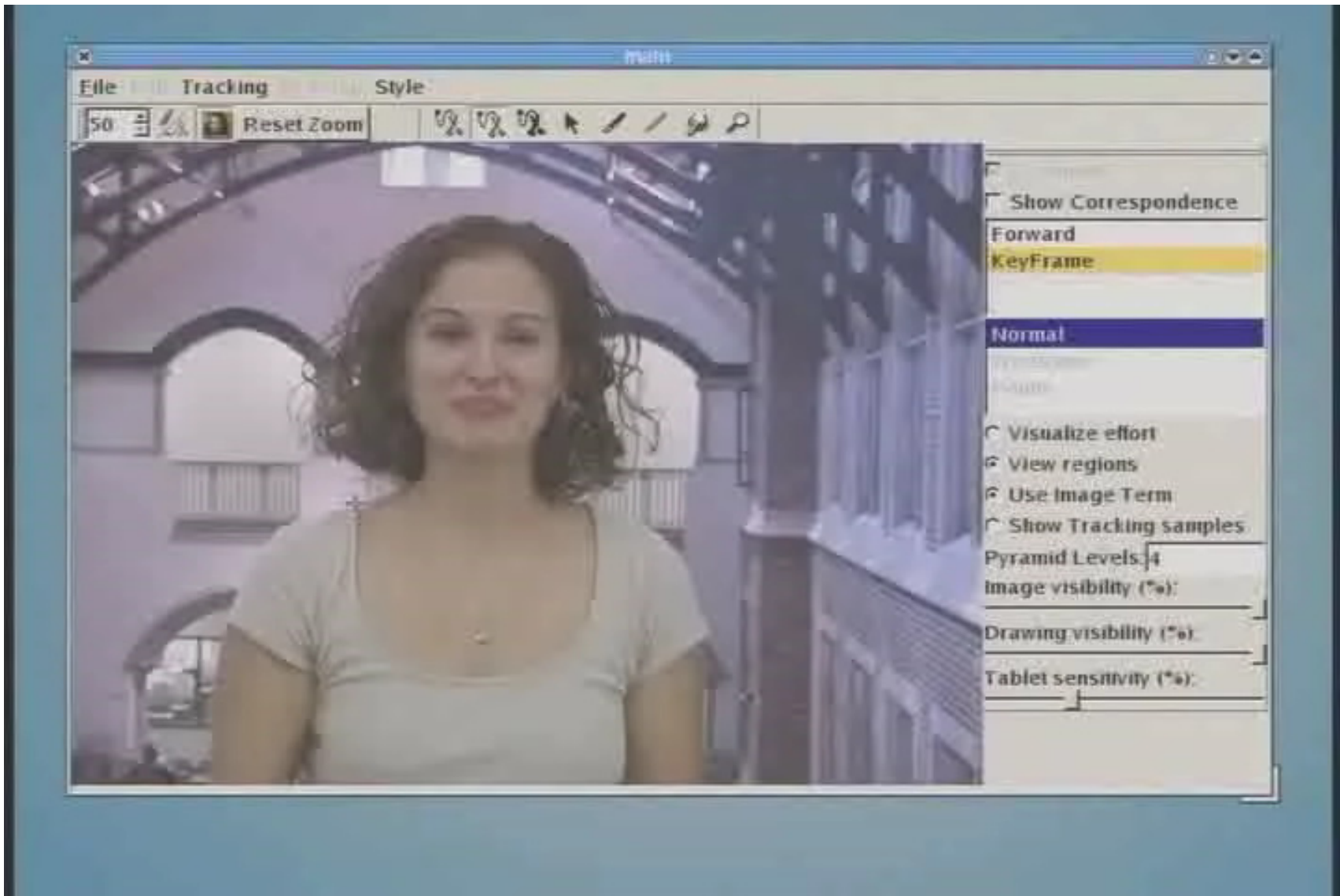
Frame 200

# KLT vs SIFT tracking

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- KLT has larger accumulating error; partly because our KLT implementation doesn't have affine transformation?
- SIFT is surprisingly robust

# Tracking for rotoscoping



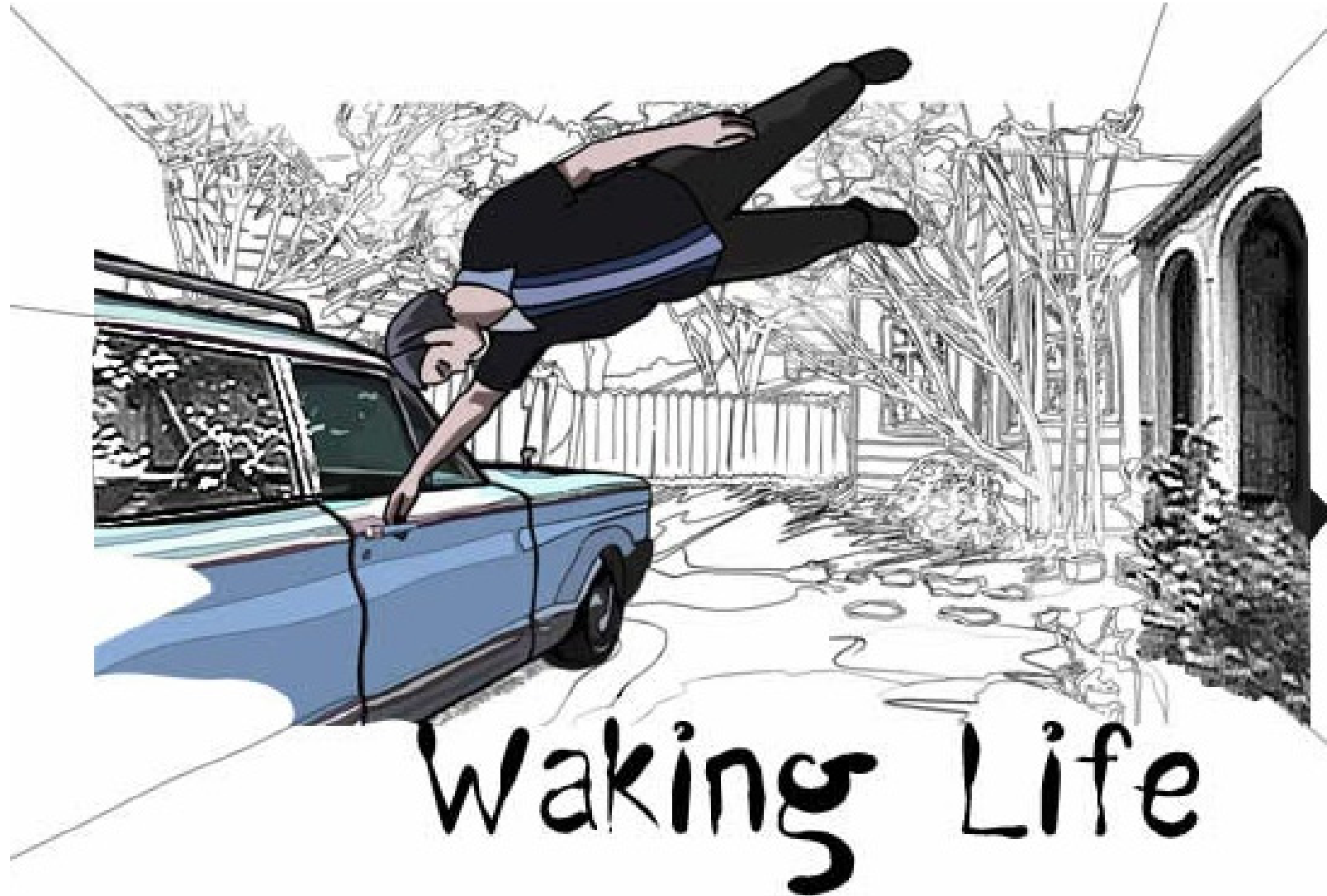


# Tracking for rotoscoping



# Waking life

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# Optical flow

# Single-motion assumption

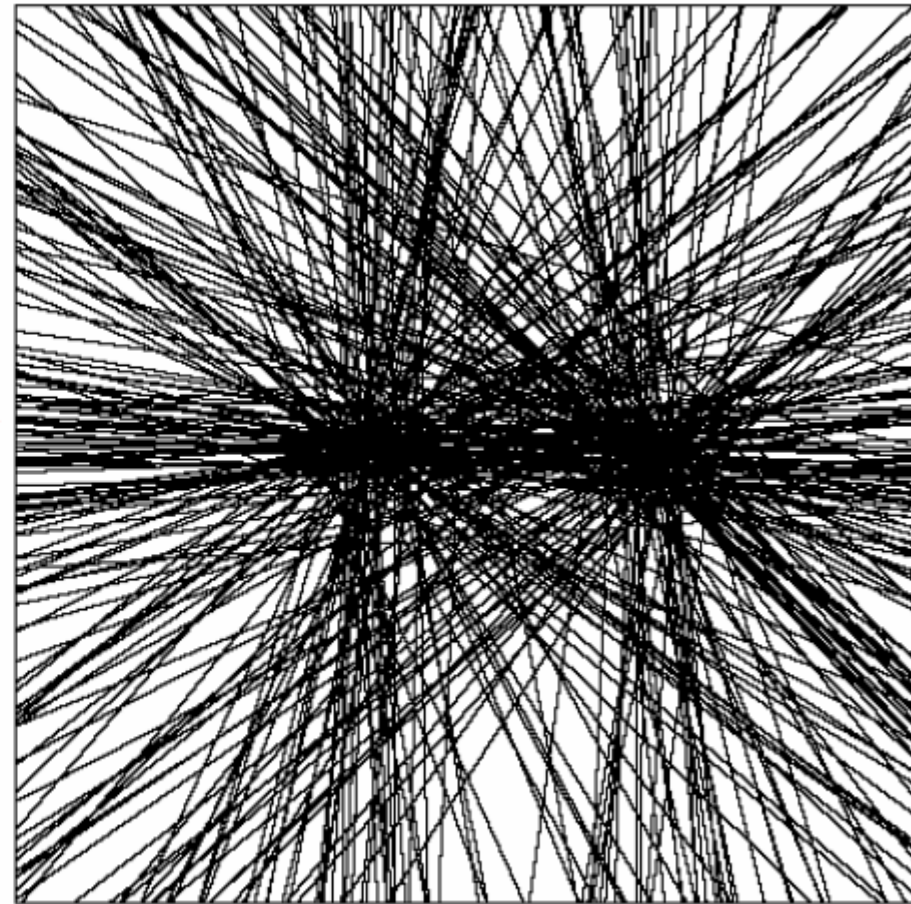
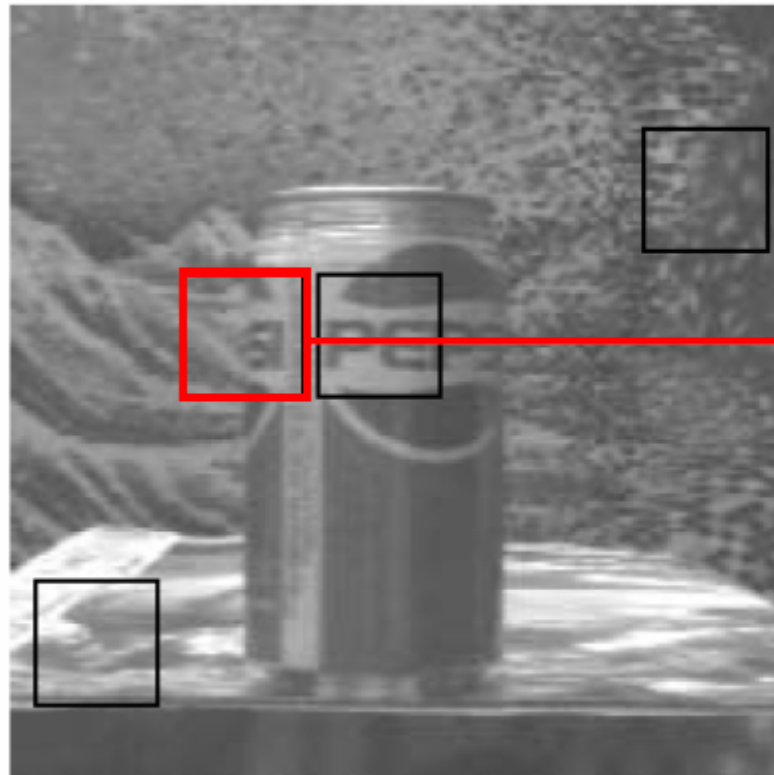
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Violated by

- Motion discontinuity
- Shadows
- Transparency
- Specular reflection
- ...

# Multiple motion

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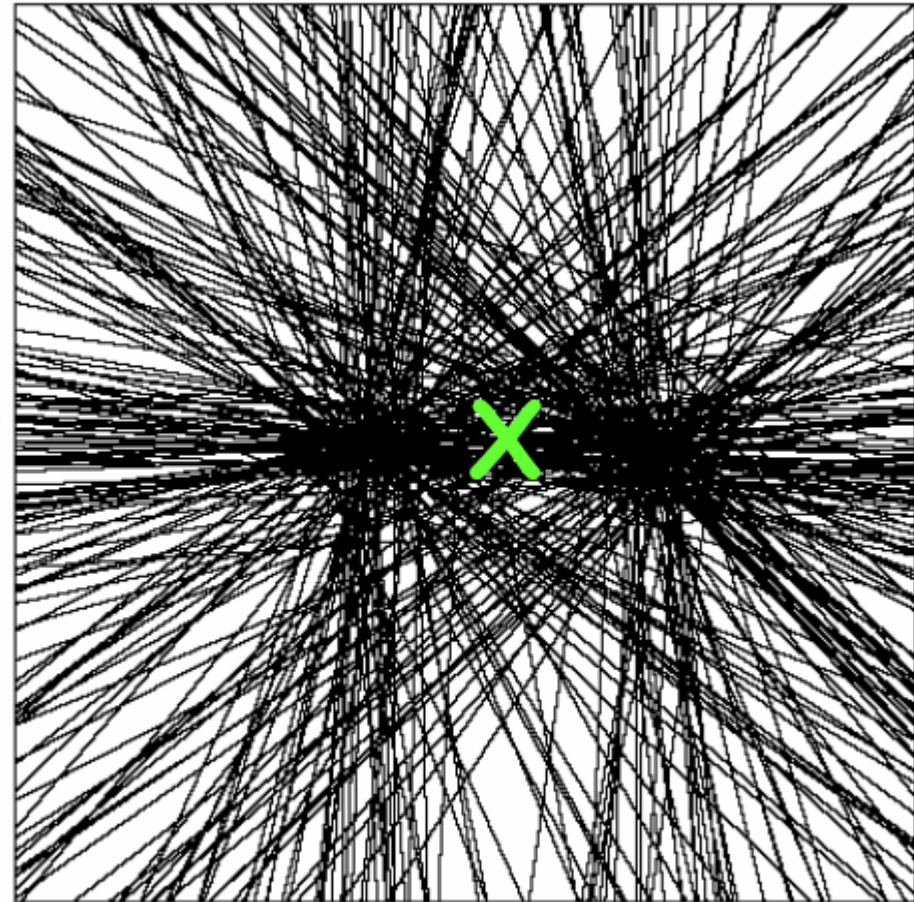
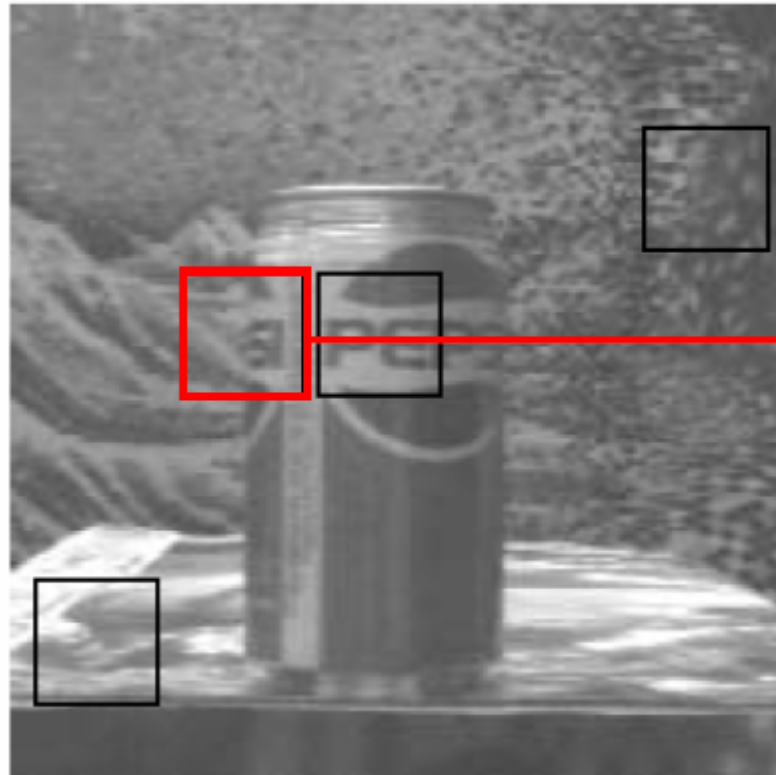


What is the “best” fitting translational motion?



# Multiple motion

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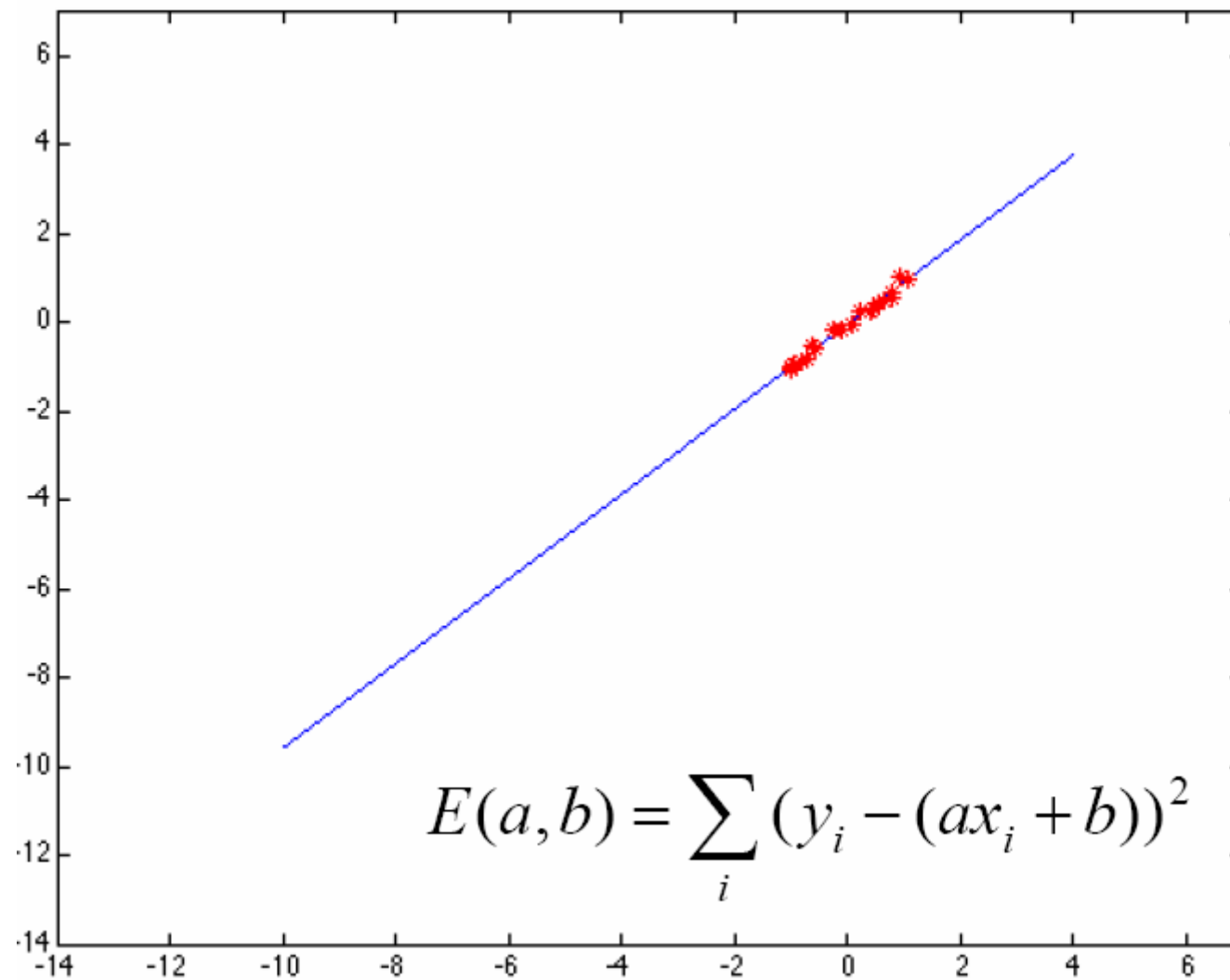


Least squares fit.

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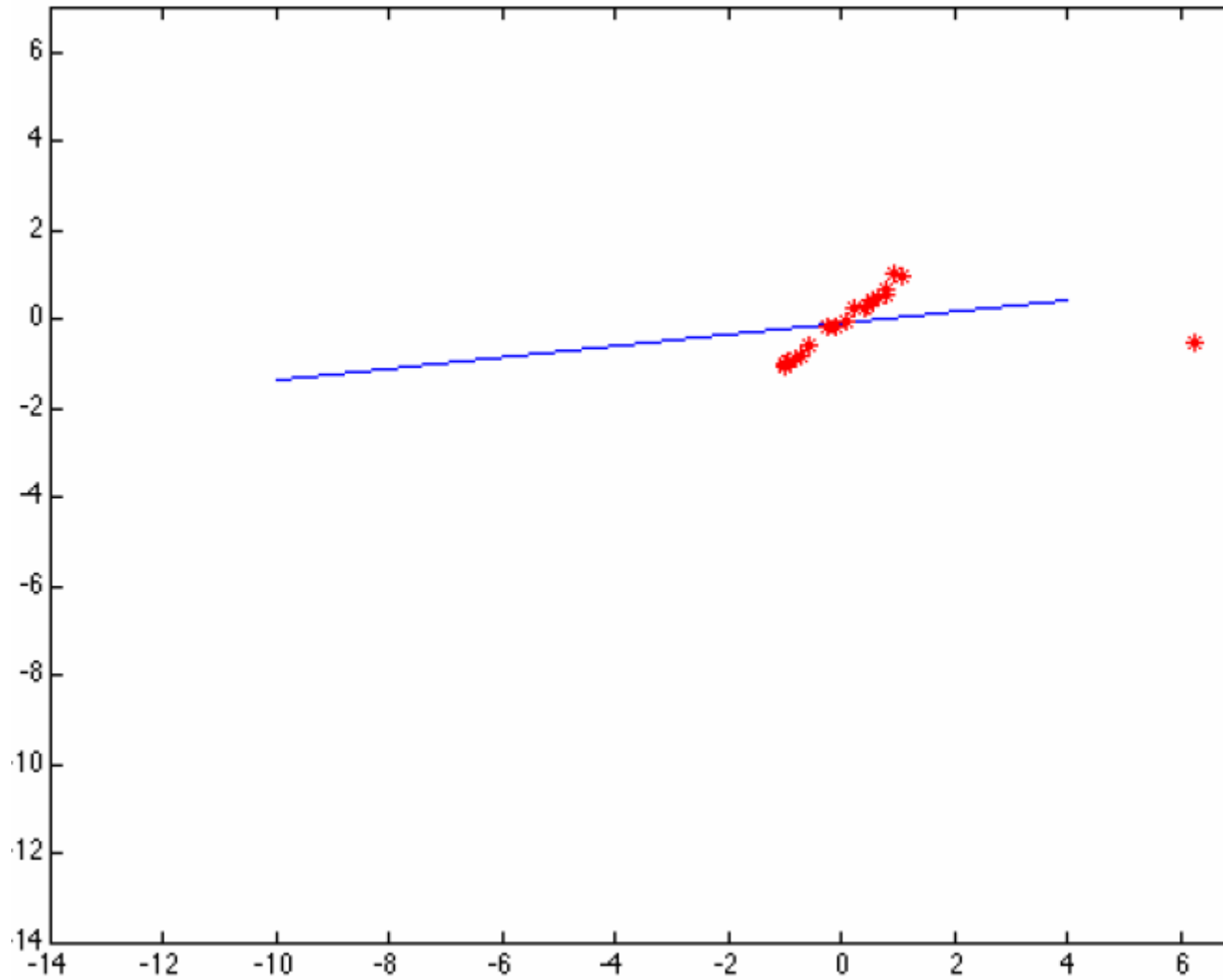
# Simple problem: fit a line

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# Least-square fit

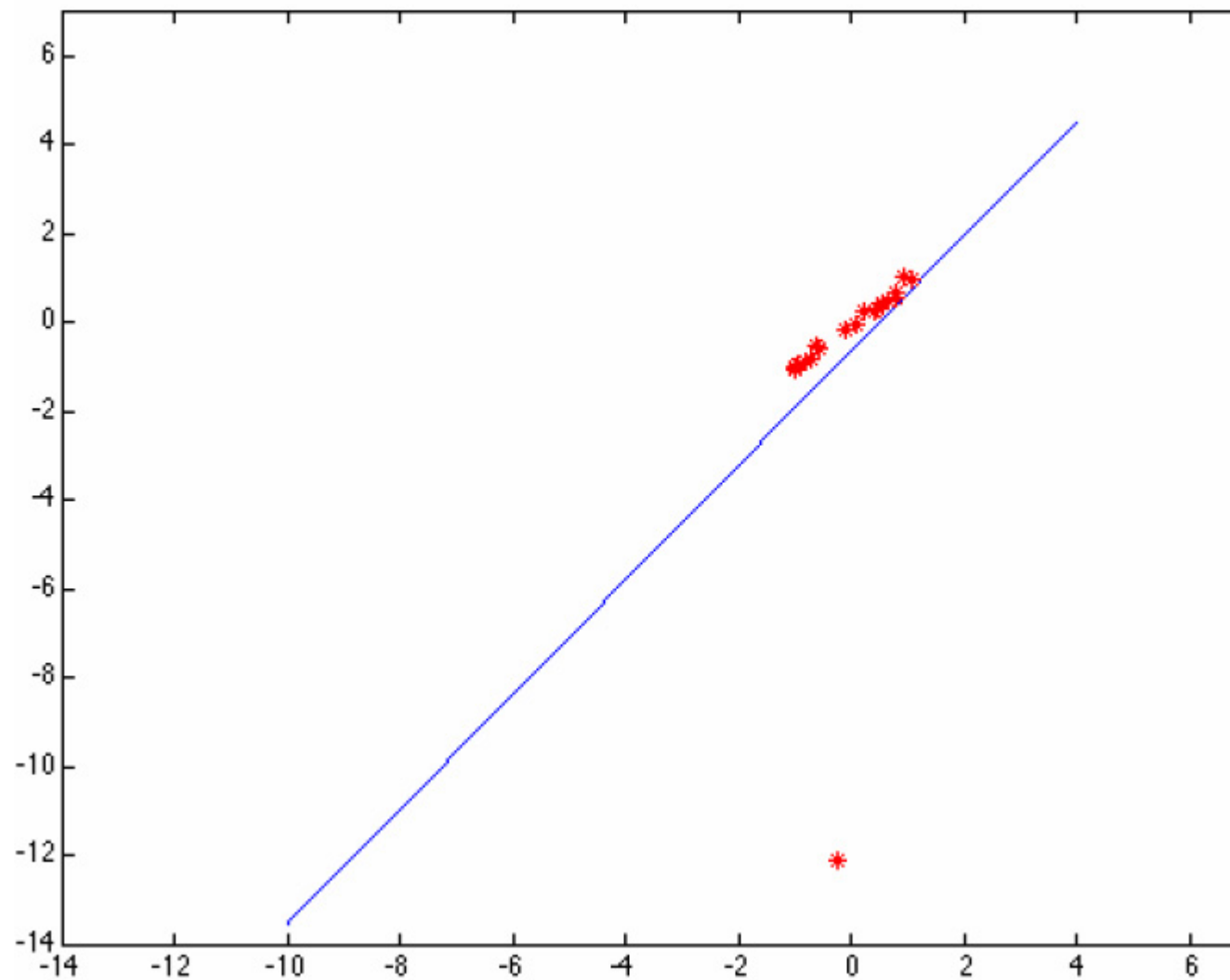
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# Least-square fit

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# Robust statistics

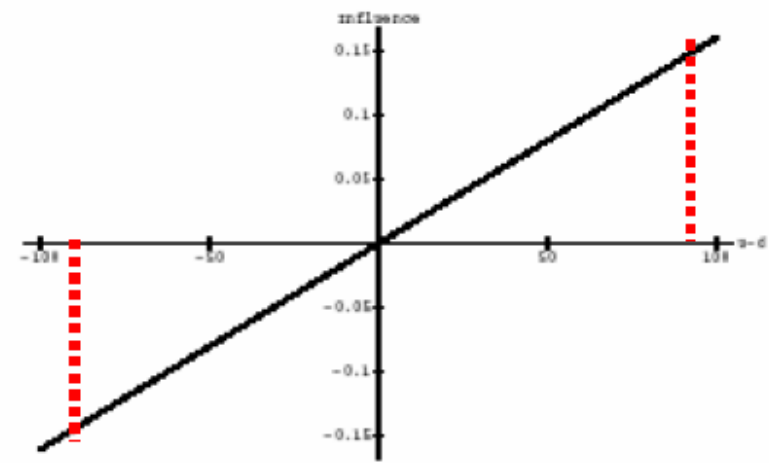
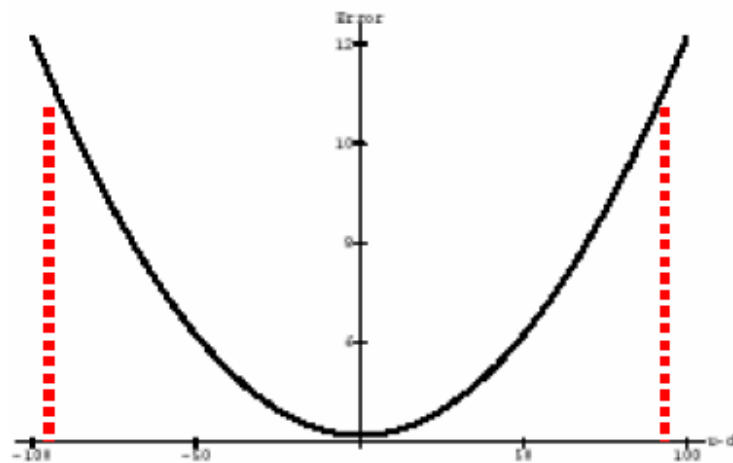
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- Recover the best fit for the **majority** of the data
- Detect and reject **outliers**

# Approach

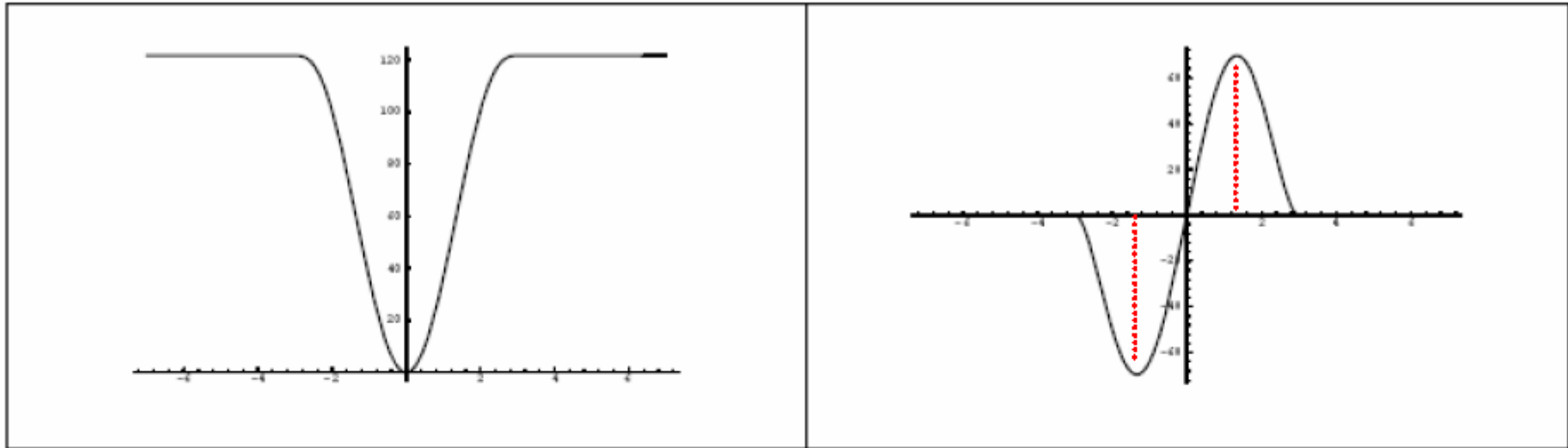
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Influence is proportional to the derivative of the  $\rho$  function.



Want to give less influence to points beyond some value.

# Robust weighting



Tukey's biweight.

Beyond a point, the influence begins to decrease.

Beyond where the second derivative is zero – outlier points

# Robust estimation

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$$E(\mathbf{a}) = \sum_{x,y \in R} \rho(I_x u + I_y v + I_t, \sigma)$$

Minimize: differentiate and set equal to zero:

$$\frac{\partial E}{\partial u} = \sum_{x,y \in R} \psi(I_x u + I_y v + I_t, \sigma) I_x = 0$$

$$\frac{\partial E}{\partial v} = \sum_{x,y \in R} \psi(I_x u + I_y v + I_t, \sigma) I_y = 0$$

*No closed form solution!*

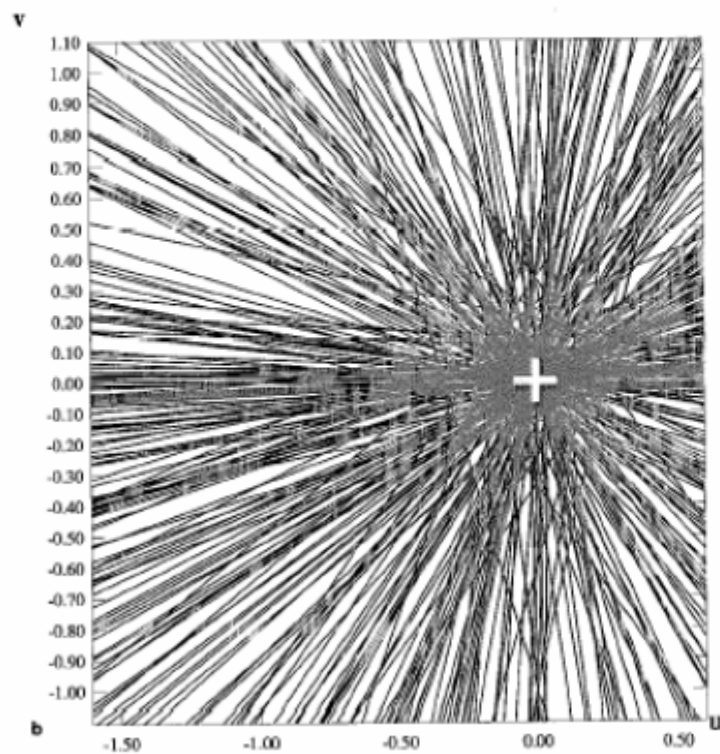
# — Fragmented Occlusion —



# Results

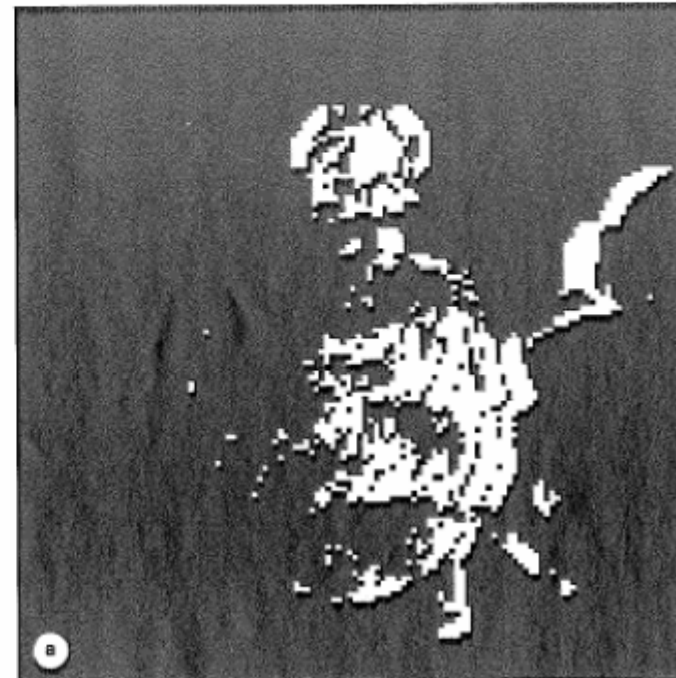
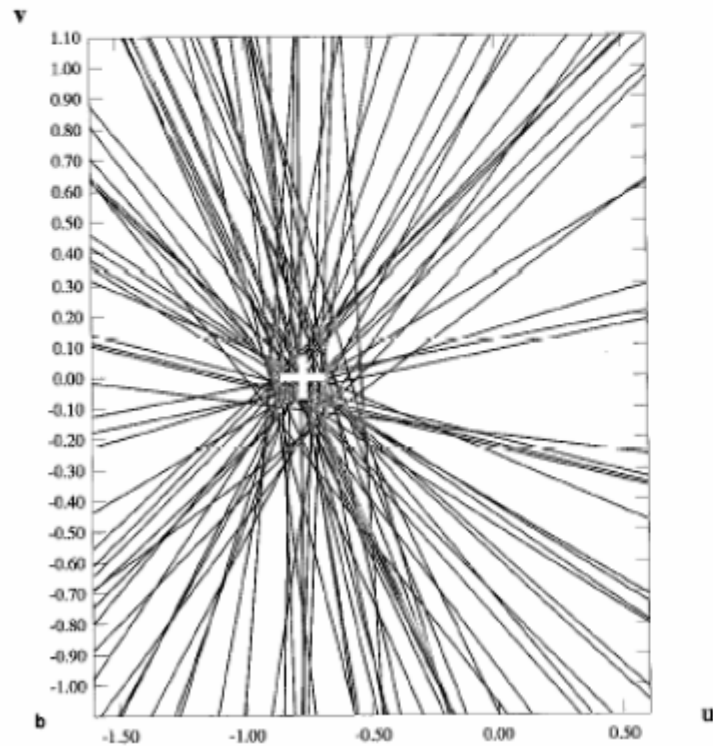
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Dominant Motion



# Results

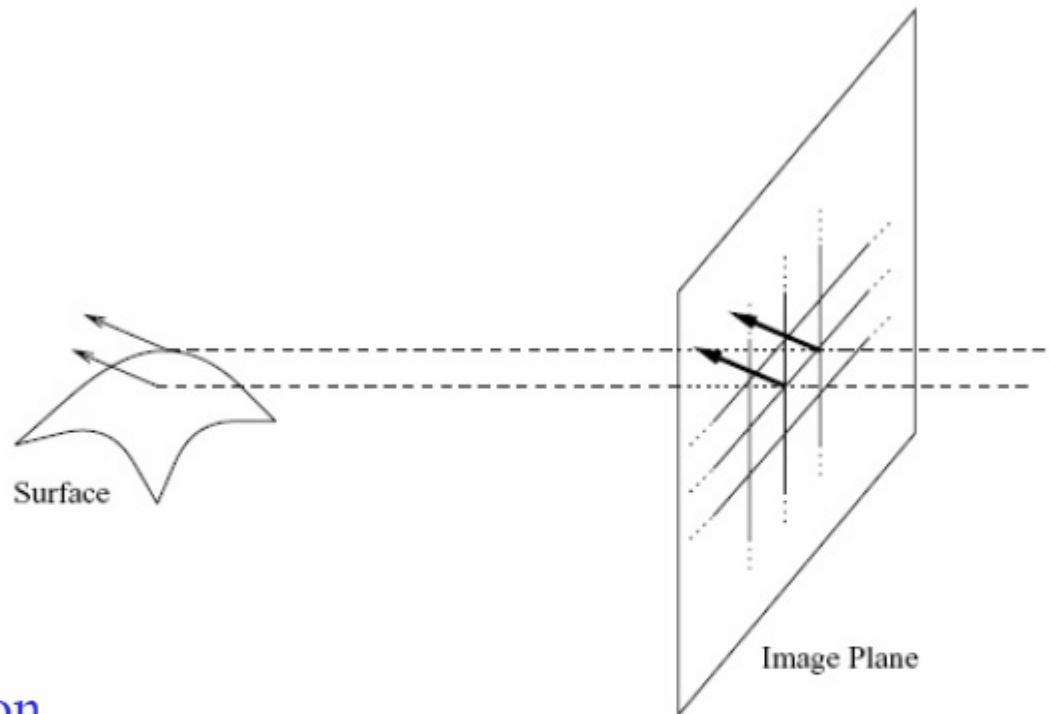
Secondary Motion





# Regularization and dense optical flow DigiVFX

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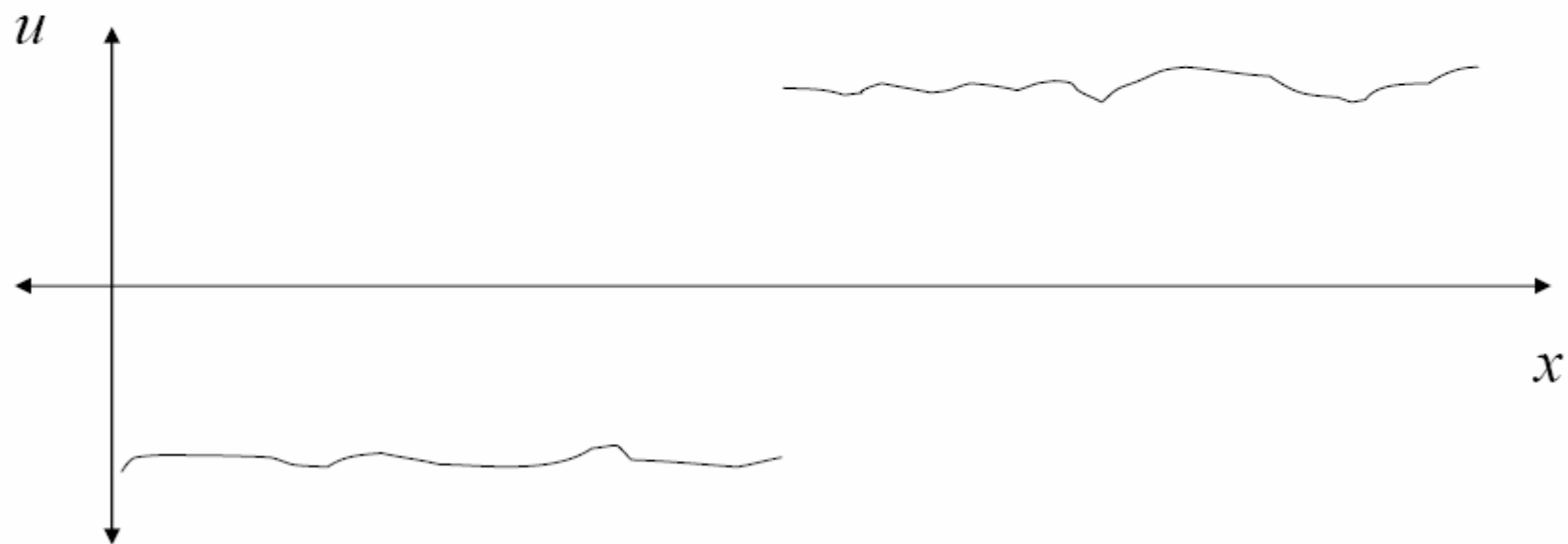


## Assumption

- \* Neighboring points in the scene typically belong to the same surface and hence typically have similar motions.
- \* Since they also project to nearby points in the image, we expect spatial coherence in image flow.

# Formalize this Idea

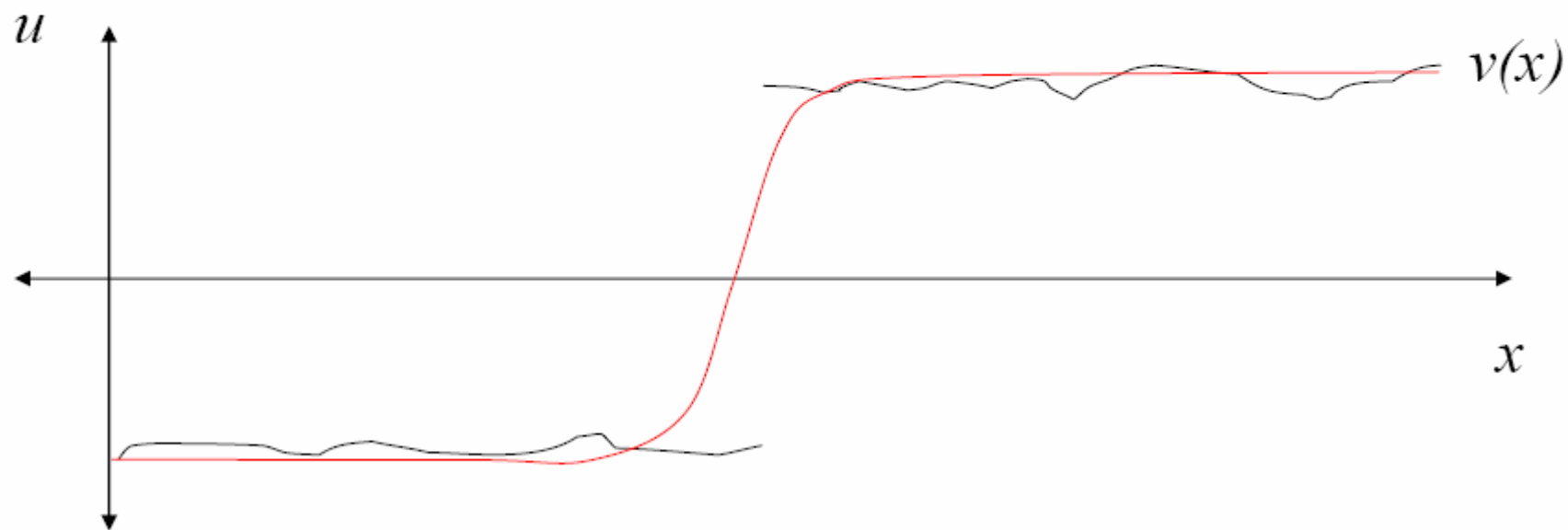
Noisy 1D signal:



Noisy measurements  $u(x)$

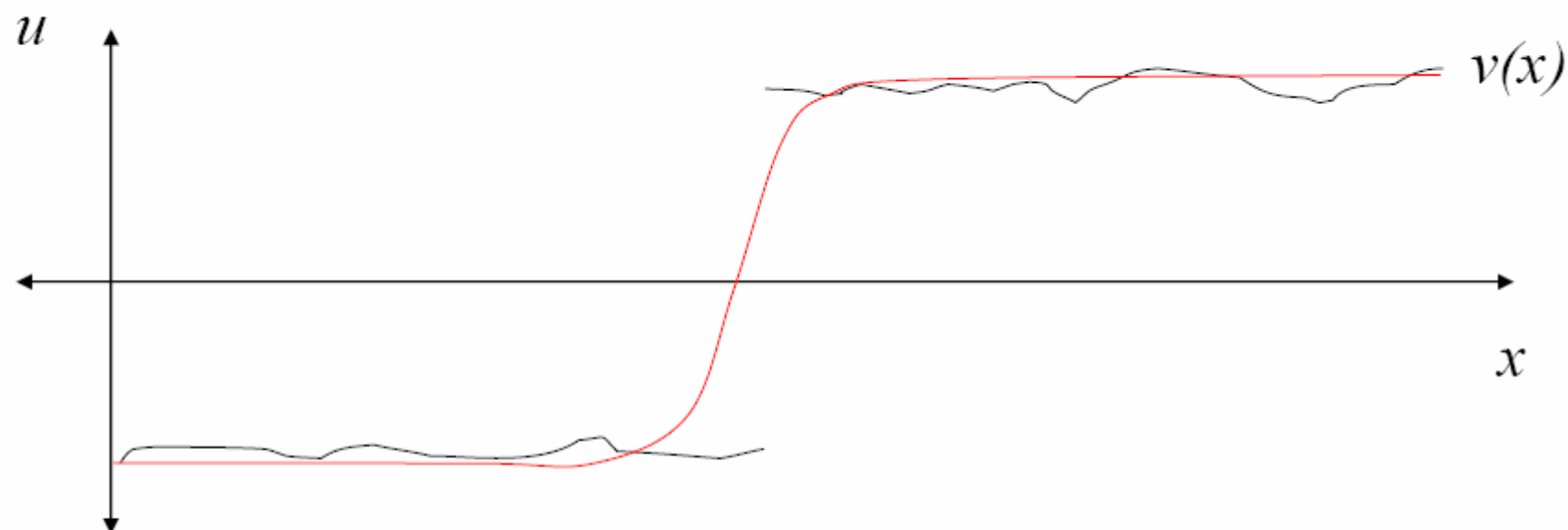
# Regularization

Find the “best fitting” smoothed function  $v(x)$



Noisy measurements  $u(x)$

# Regularization



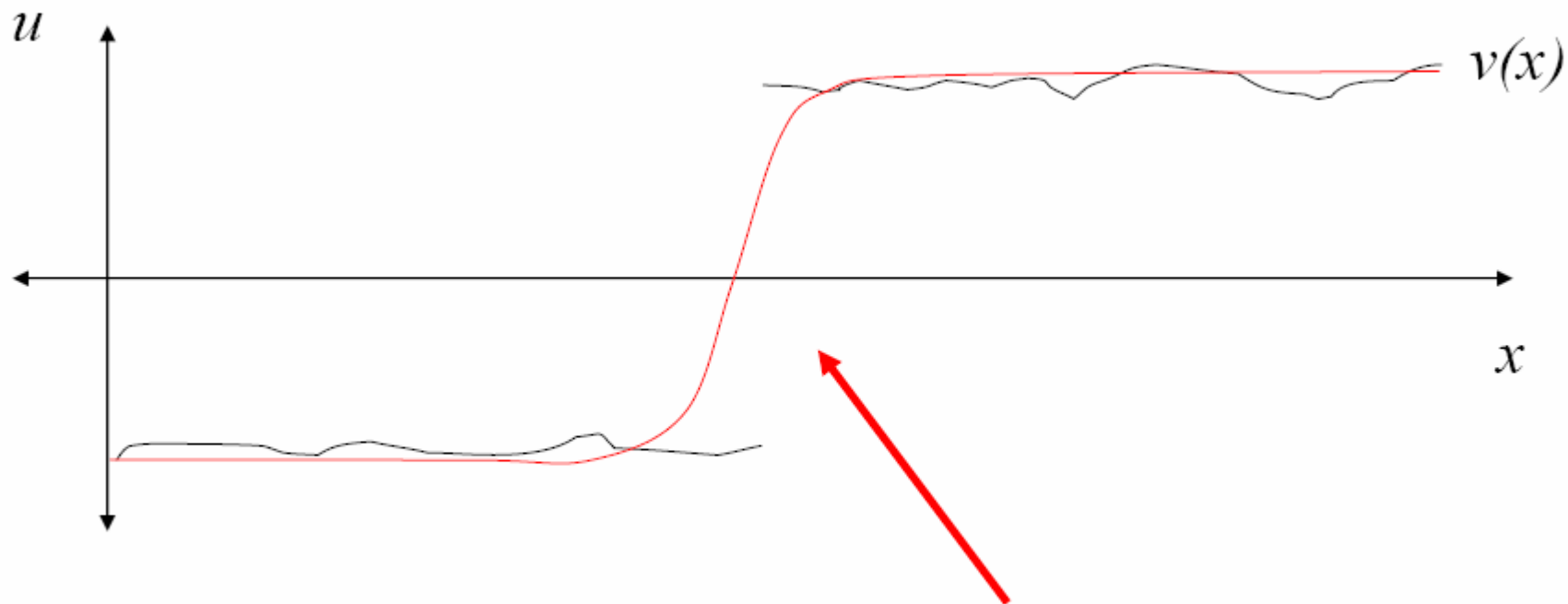
*Minimize:*

Faithful to the data

Spatial smoothness  
assumption

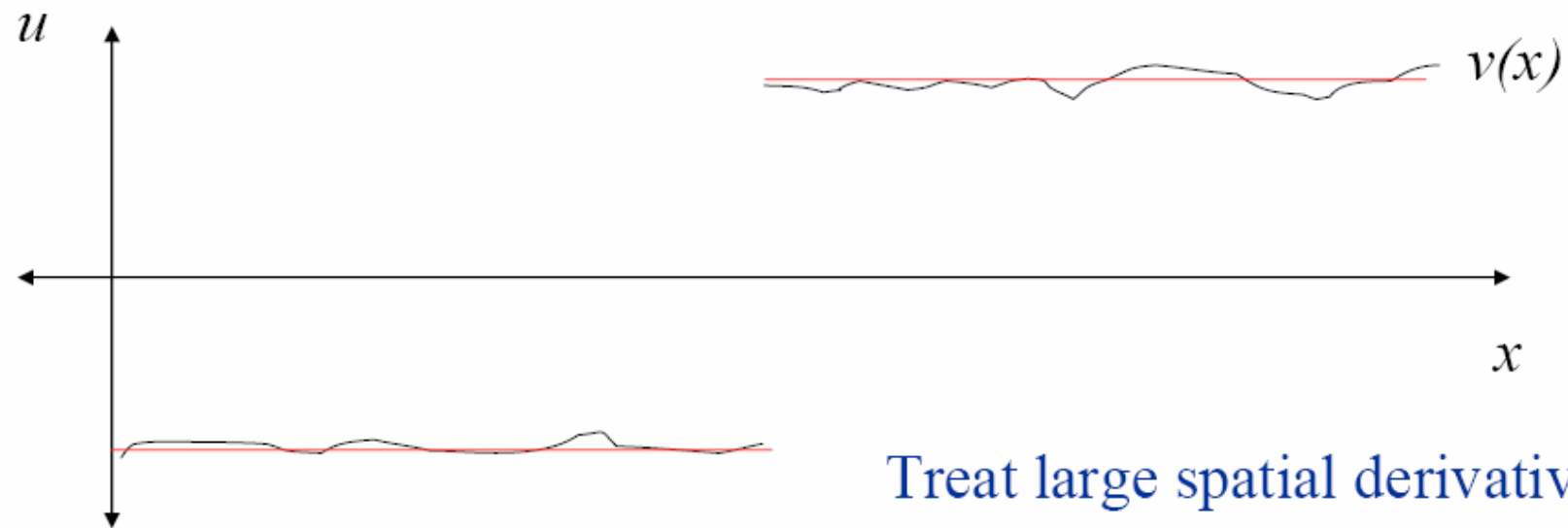
$$E(v) = \sum_{x=1}^N (v(x) - u(x))^2 + \lambda \sum_{x=1}^{N-1} (v(x+1) - v(x))^2$$

# Discontinuities



What about this discontinuity?  
What is happening here?  
What can we do?

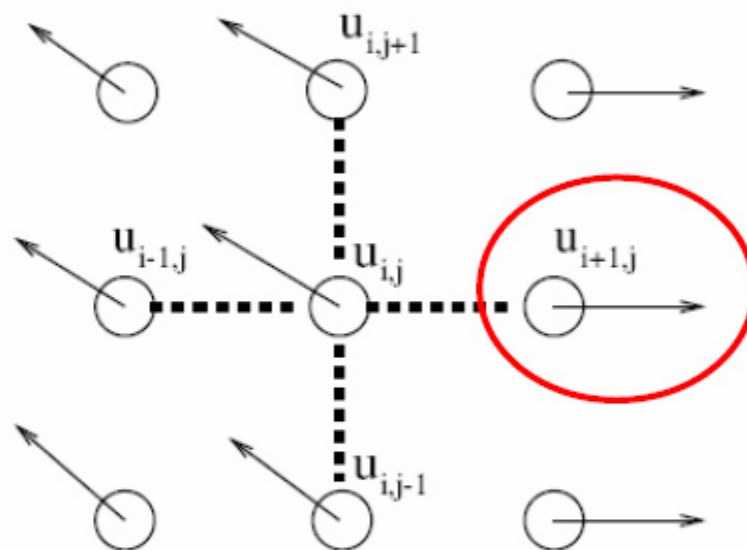
# Robust Regularization



*Minimize:*

$$E(v) = \sum_{x=1}^N \rho(v(x) - u(x), \sigma_1) + \lambda \sum_{x=1}^{N-1} \rho(v(x+1) - v(x), \sigma_2)$$

# Optical flow



Outlier with respect to neighbors.

Robust formulation of spatial coherence term

$$E_S(u, v) = \rho(u_x) + \rho(u_y) + \rho(v_x) + \rho(v_y)$$

# “Dense” Optical Flow

$$E_D(\mathbf{u}(\mathbf{x})) = \rho(I_x(\mathbf{x})u(\mathbf{x}) + I_y(\mathbf{x})v(\mathbf{x}) + I_t(\mathbf{x}), \sigma_D)$$

$$E_S(u, v) = \sum_{\mathbf{y} \in G(\mathbf{x})} [\rho(u(\mathbf{x}) - u(\mathbf{y}), \sigma_S) + \rho(v(\mathbf{x}) - v(\mathbf{y}), \sigma_S)]$$

Objective function:

$$E(\mathbf{u}) = \sum_{\mathbf{x}} E_D(\mathbf{u}(\mathbf{x})) + \lambda E_S(\mathbf{u}(\mathbf{x}))$$

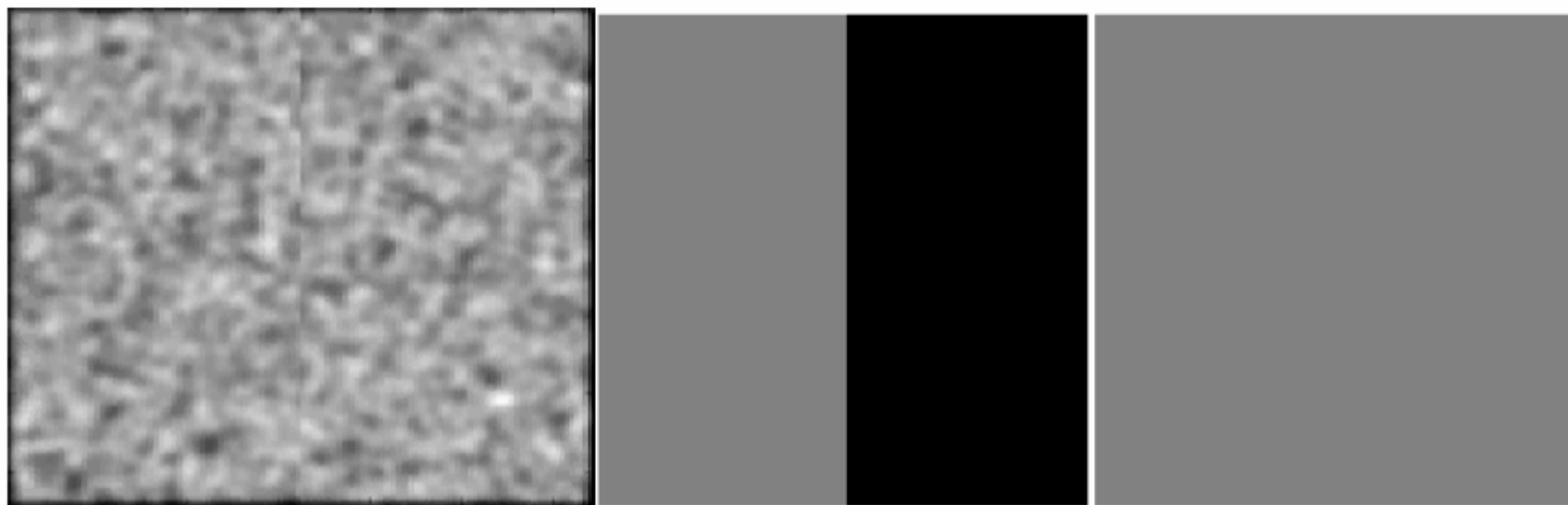
When  $\rho$  is quadratic = “Horn and Schunck”



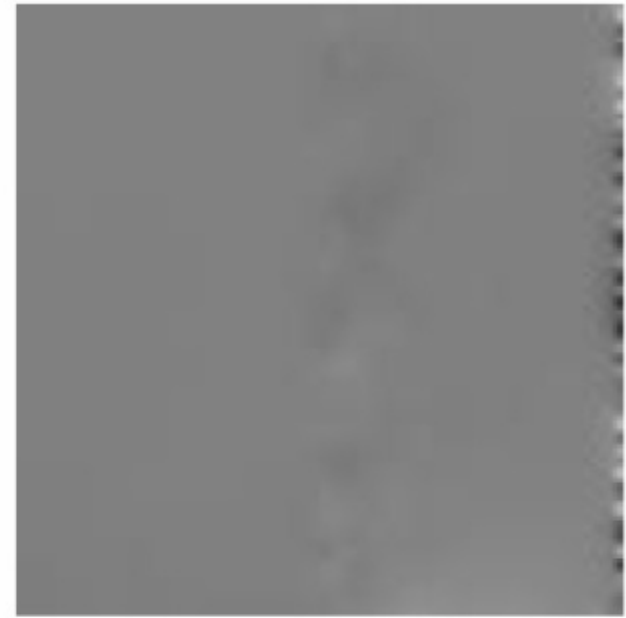
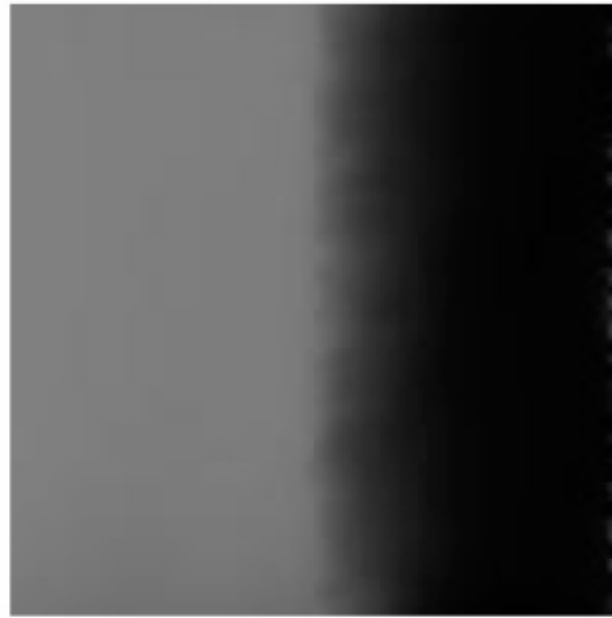
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# Example

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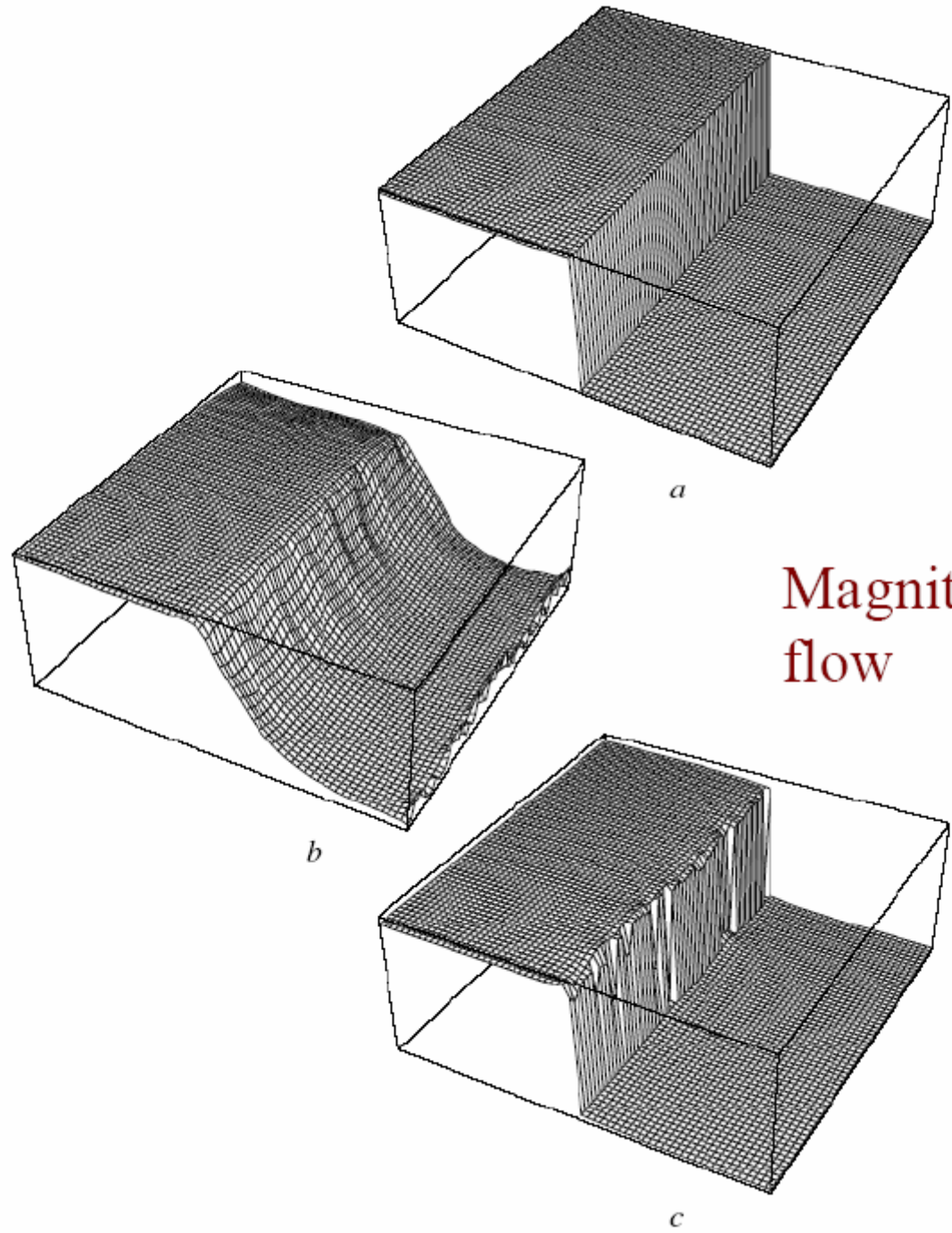


Quadratic:



Robust:





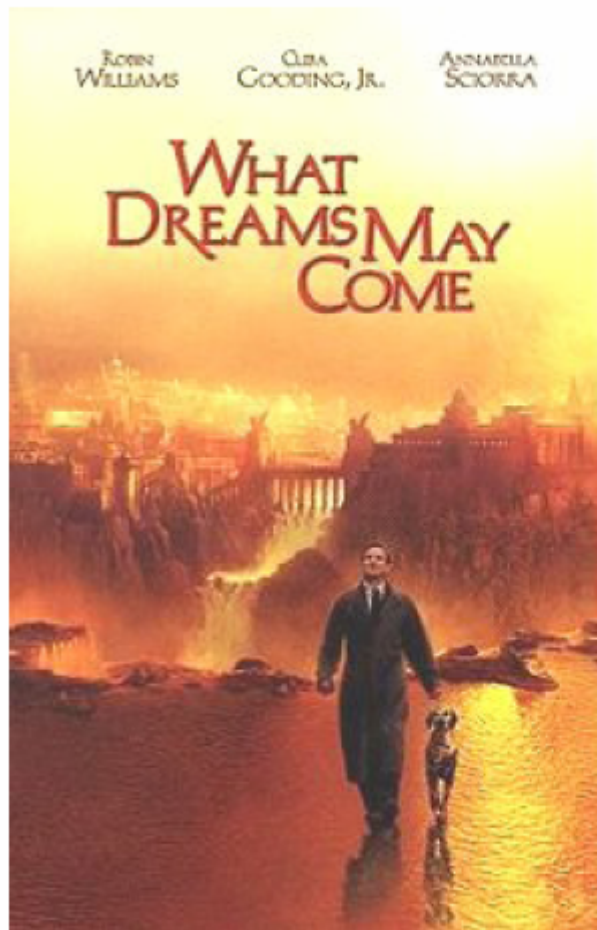
*a*

*b*

*c*

Magnitude of horizontal flow

# Applications of Optical Flow



Impressionist effect.  
Transfer motion of real world to a painting

# Input for the NPR algorithm

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# Brushes

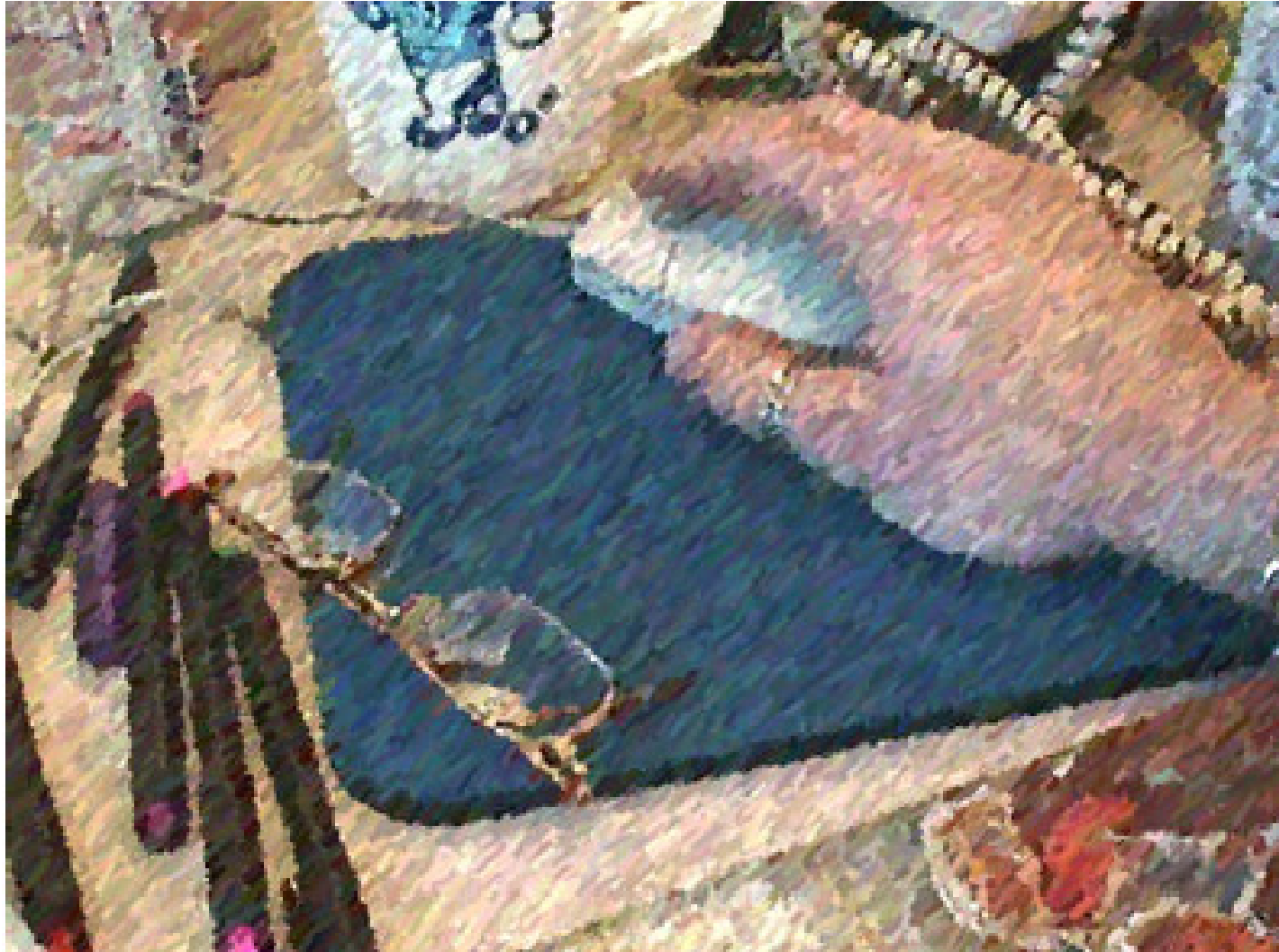
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# Edge clipping

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# Gradient

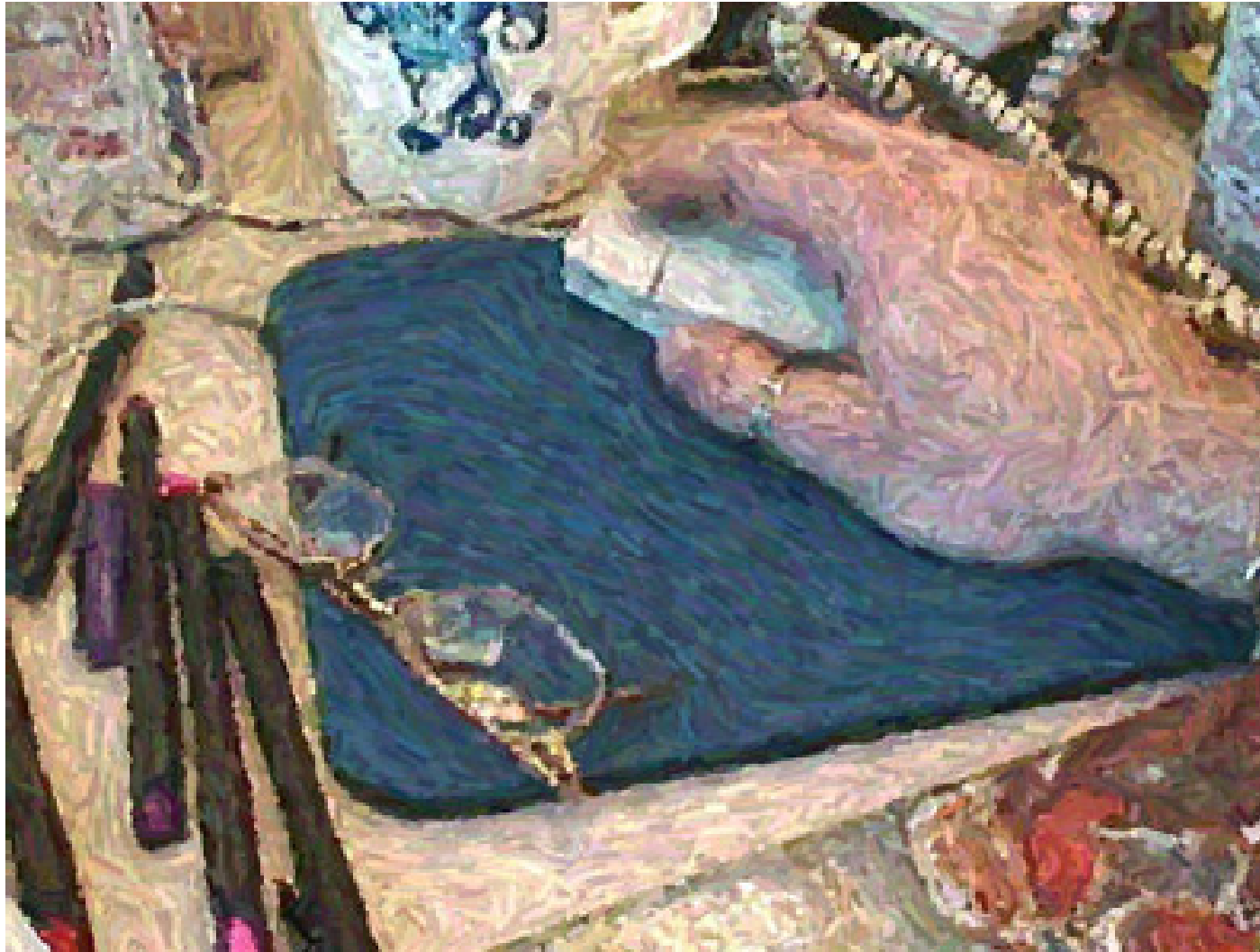
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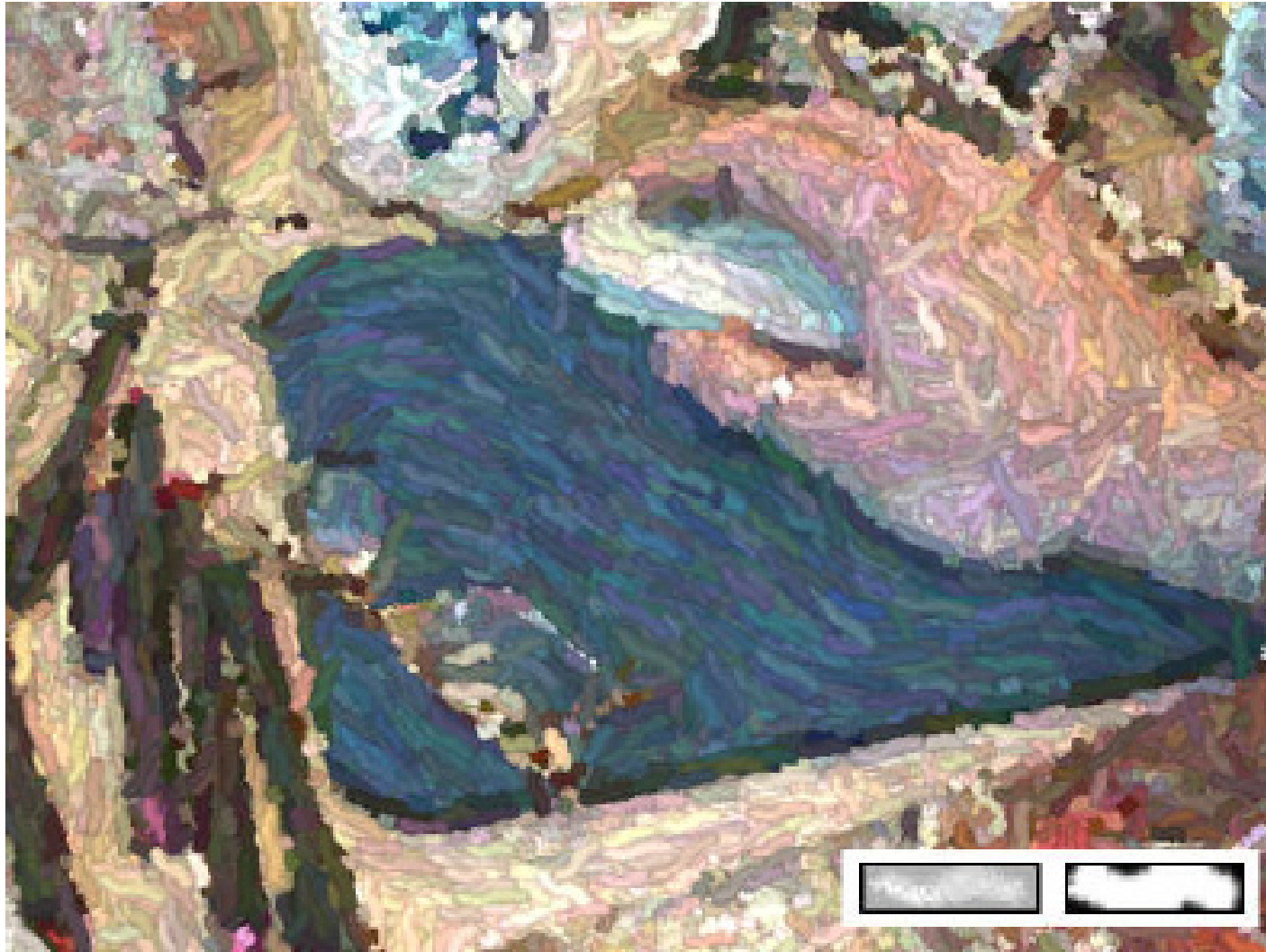
# Smooth gradient

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# Textured brush

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# Edge clipping

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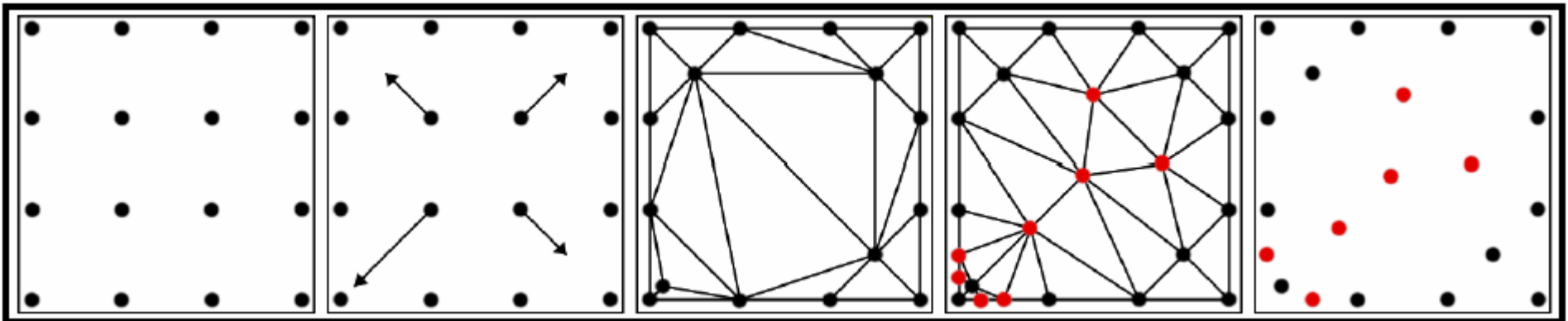
# Temporal artifacts

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Frame-by-frame application of the NPR algorithm

# Temporal coherence



# RE:Vision

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# What dreams may come

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# Reference

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