

# Image warping/morphing

Digital Visual Effects, Spring 2005

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2005/3/9

*with slides by Richard Szeliski, Steve Seitz and Alexei Efros*

## Announcements

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- Class time: 1:30-4:20 (with a 20-minute break)
- Last call: send [cyy@csie.ntu.edu.tw](mailto:cyy@csie.ntu.edu.tw) to subscribe vfx
- Course forum is set up (see course page)
- Scribe volunteers for today and next week
- A schedule for scribes will be posted in forum soon. Please fill in the schedule.

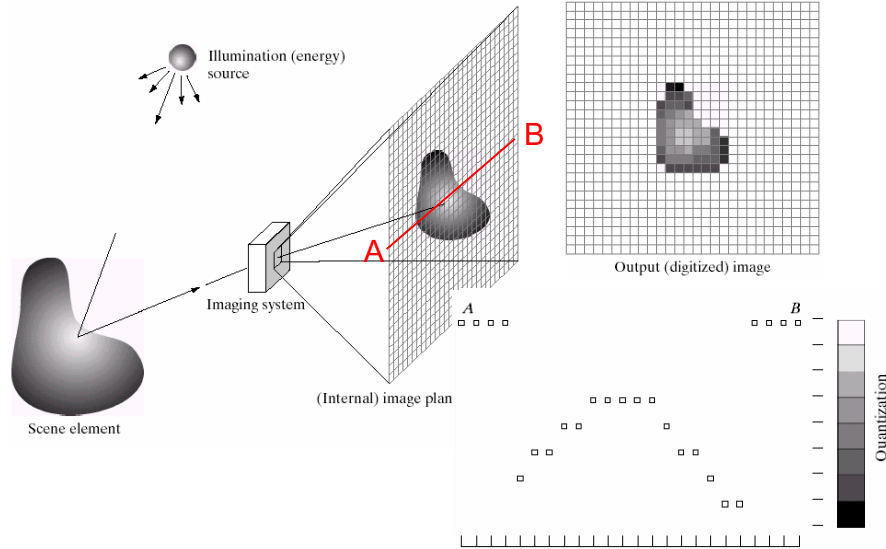
## Outline

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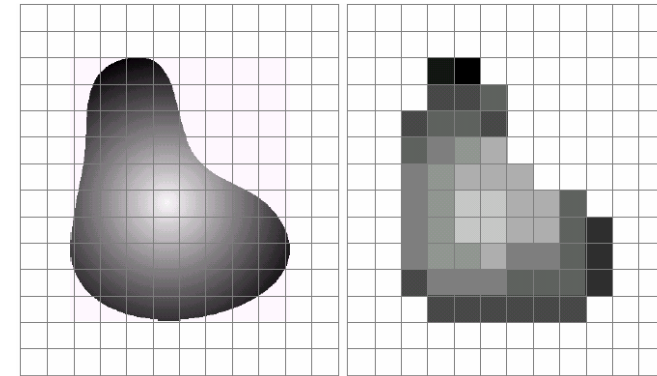
- Images
- Image warping
- Image morphing
- Project #1

## Image fundamentals

## Image formation

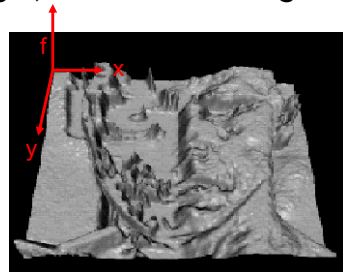


## Sampling and quantization



## What is an image

- We can think of an **image** as a function,  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ :
  - $f(x, y)$  gives the **intensity** at position  $(x, y)$
  - defined over a rectangle, with a finite range:
    - $f: [a,b] \times [c,d] \rightarrow [0,1]$



- A color image

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

## A digital image

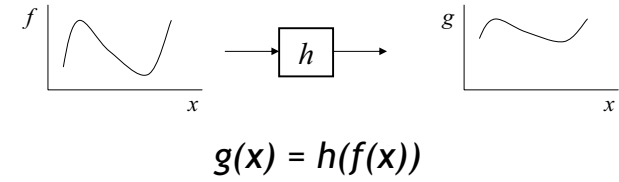
- We usually operate on **digital (discrete) images**:
  - **Sample** the 2D space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- If our samples are  $D$  apart, we can write this as:
 
$$f[i, j] = \text{Quantize}\{f(i D, j D)\}$$
- The image can now be represented as a matrix of integer values

	$j \rightarrow$							
$i \downarrow$	62	79	23	119	120	105	4	0
	10	10	9	62	12	78	34	0
	10	58	197	46	46	0	0	48
	176	135	5	188	191	68	0	49
	2	1	1	29	26	37	0	77
	0	89	144	147	187	102	62	208
	255	252	0	166	123	62	0	31
	166	63	127	17	1	0	99	30

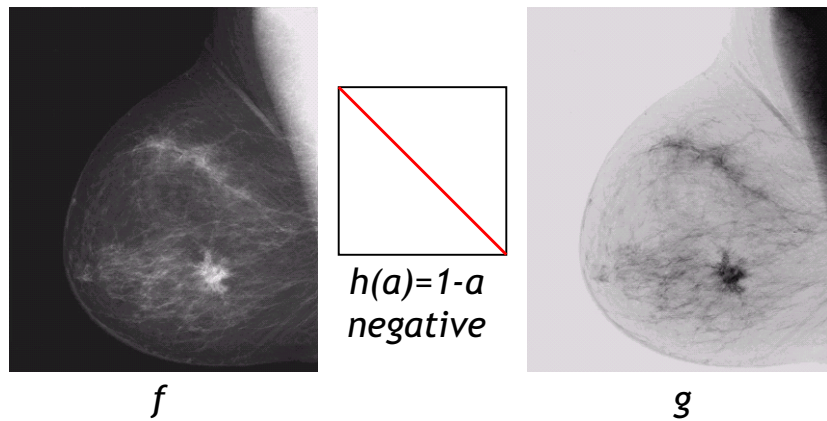
# Aliasing



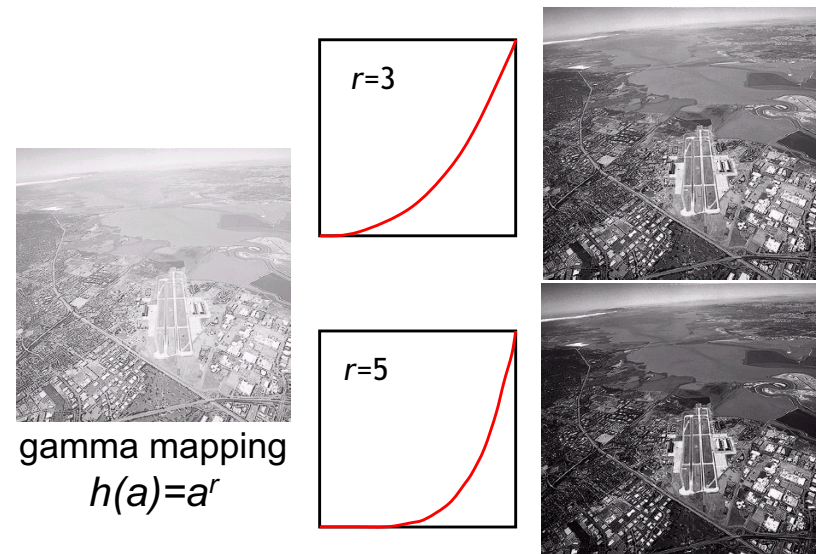
# Image processing



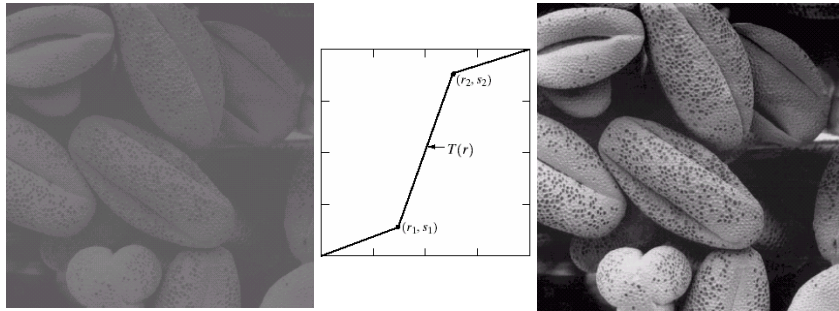
# Point processing



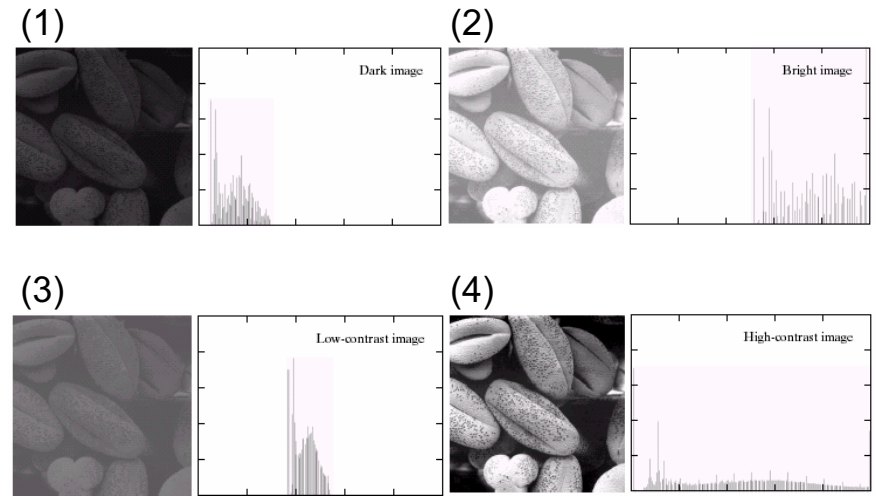
# Image enhancement



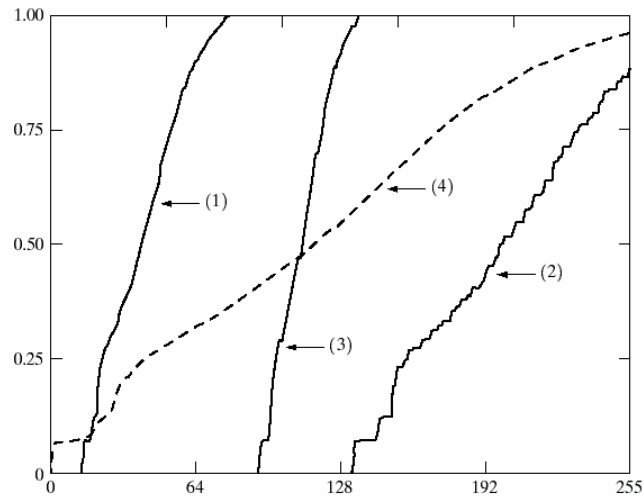
# Contrast stretching



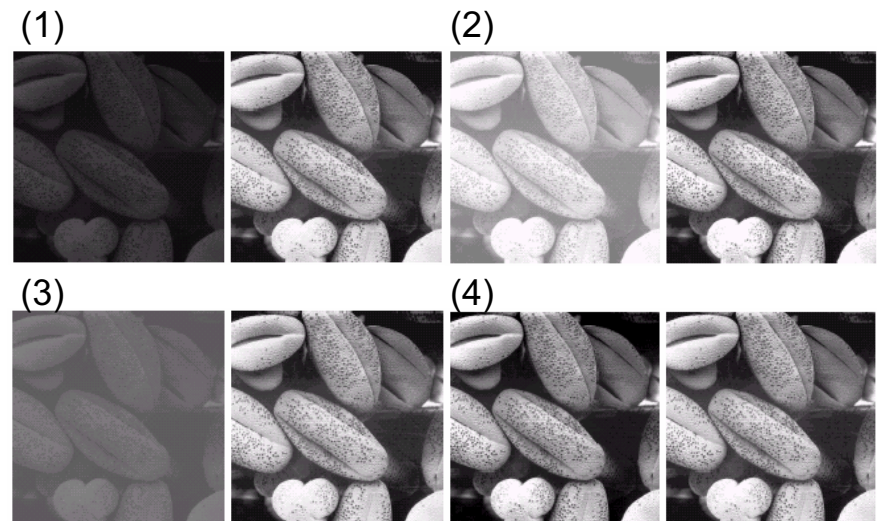
# Histogram



# Accumulated histogram



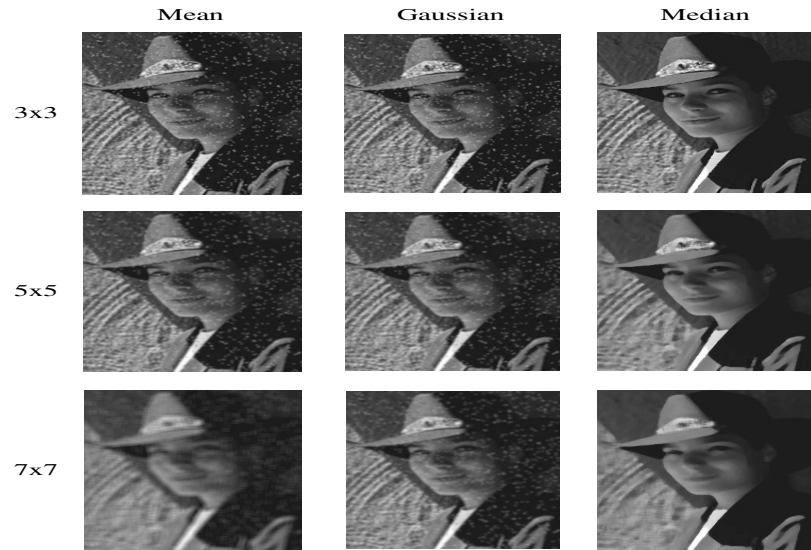
# Histogram equalization



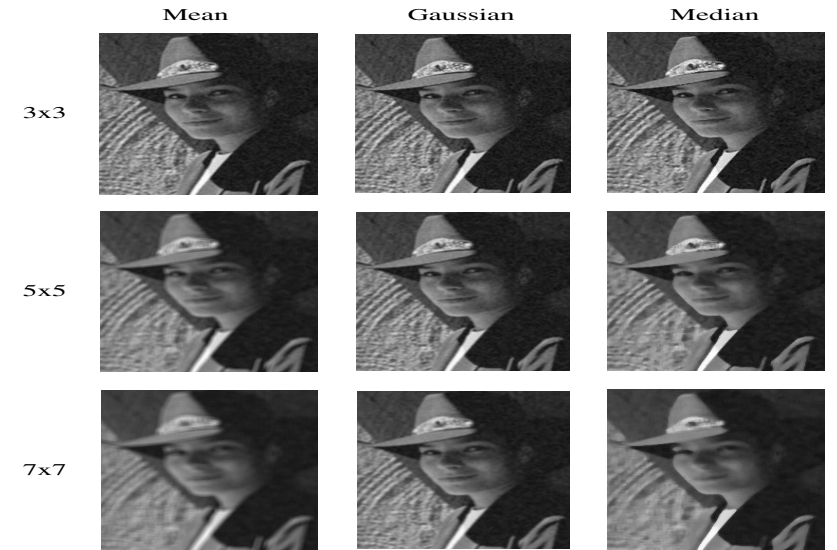




## Comparison: salt and pepper noise



## Comparison: Gaussian noise



## Image warping

## Image warping

image filtering: change **range** of image

$$g(x) = h(f(x))$$

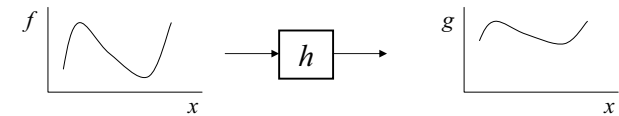


image warping: change **domain** of image

$$g(x) = f(h(x))$$

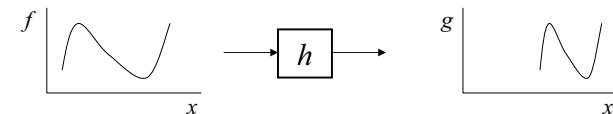


image filtering: change **range** of image

$$f(x) = h(g(x))$$

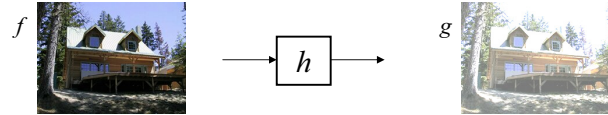
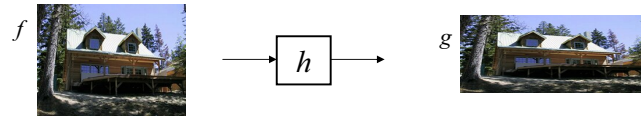


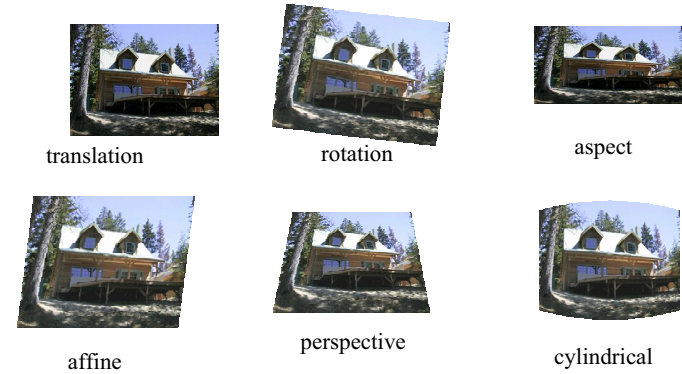
image warping: change **domain** of image

$$f(x) = g(h(x))$$



## Parametric (global) warping

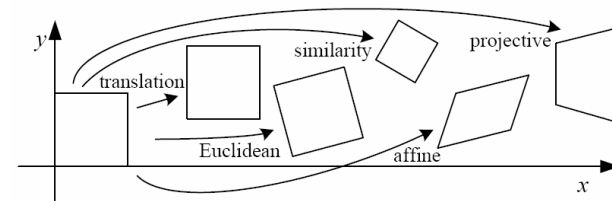
Examples of parametric warps:



## 2D coordinate transformations

- translation:  $\mathbf{x}' = \mathbf{x} + \mathbf{t}$        $\mathbf{x} = (x, y)$
- rotation:  $\mathbf{x}' = \mathbf{R} \mathbf{x} + \mathbf{t}$
- similarity:  $\mathbf{x}' = s \mathbf{R} \mathbf{x} + \mathbf{t}$
- affine:  $\mathbf{x}' = \mathbf{A} \mathbf{x} + \mathbf{t}$
- perspective:  $\mathbf{x}' \cong \mathbf{H} \underline{\mathbf{x}}$        $\underline{\mathbf{x}} = (x, y, 1)$   
 ( $\underline{\mathbf{x}}$  is a *homogeneous* coordinate)
- These all form a nested *group* (closed under composition w/ inv.)

## 2D image transformations

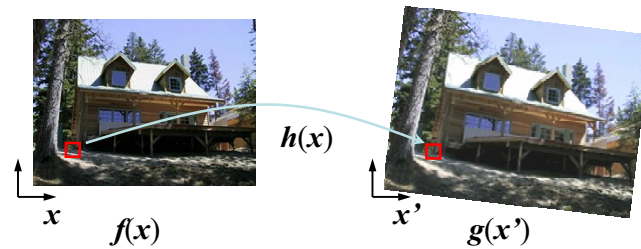


Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I & \mathbf{t} \end{bmatrix}_{2 \times 3}$	2	orientation + ...	
rigid (Euclidean)	$\begin{bmatrix} R & \mathbf{t} \end{bmatrix}_{2 \times 3}$	3	lengths + ...	
similarity	$\begin{bmatrix} sR & \mathbf{t} \end{bmatrix}_{2 \times 3}$	4	angles + ...	
affine	$\begin{bmatrix} A \\ \mathbf{t} \end{bmatrix}_{2 \times 3}$	6	parallelism + ...	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3 \times 3}$	8	straight lines	

## Image warping

DigiVFX

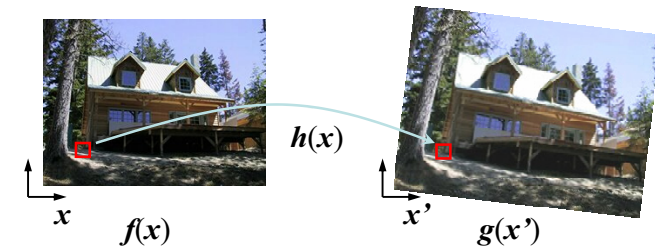
- Given a coordinate transform  $\mathbf{x}' = \mathbf{h}(\mathbf{x})$  and a source image  $f(\mathbf{x})$ , how do we compute a transformed image  $\mathbf{g}(\mathbf{x}') = f(\mathbf{h}(\mathbf{x}))$ ?



## Forward warping

DigiVFX

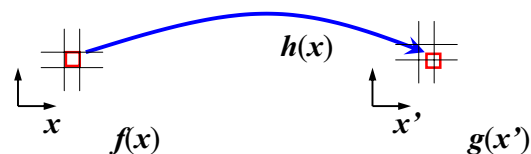
- Send each pixel  $f(\mathbf{x})$  to its corresponding location  $\mathbf{x}' = \mathbf{h}(\mathbf{x})$  in  $\mathbf{g}(\mathbf{x}')$
- What if pixel lands “between” two pixels?



## Forward warping

DigiVFX

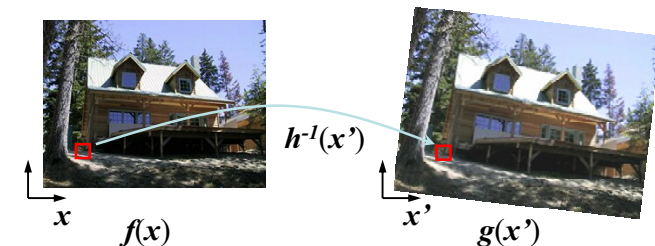
- Send each pixel  $f(\mathbf{x})$  to its corresponding location  $\mathbf{x}' = \mathbf{h}(\mathbf{x})$  in  $\mathbf{g}(\mathbf{x}')$
- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels, normalize later (*splatting*)



## Inverse warping

DigiVFX

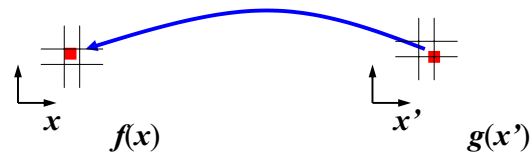
- Get each pixel  $\mathbf{g}(\mathbf{x}')$  from its corresponding location  $\mathbf{x} = \mathbf{h}^{-1}(\mathbf{x}')$  in  $f(\mathbf{x})$
- What if pixel comes from “between” two pixels?





## Inverse warping

- Get each pixel  $g(x')$  from its corresponding location  $x = h^{-1}(x')$  in  $f(x)$
- What if pixel comes from “between” two pixels?
- Answer: *resample* color value from *interpolated (prefiltered)* source image



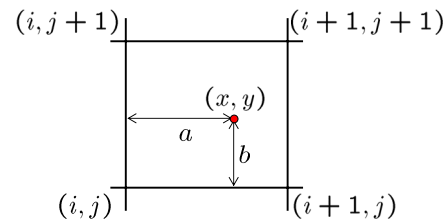
## Interpolation

- Possible interpolation filters:
  - nearest neighbor
  - bilinear
  - bicubic
  - sinc / FIR



## Bilinear interpolation

- A simple method for resampling images



$$f(x, y) = (1-a)(1-b) f[i, j] + a(1-b) f[i+1, j] + ab f[i+1, j+1] + (1-a)b f[i, j+1]$$

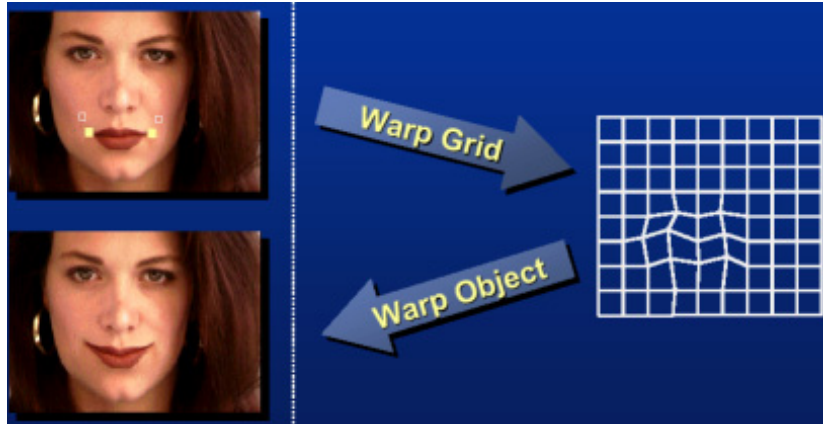
## Bicubic interpolation

<http://astronomy.swin.edu.au/~pbourke/colour/bicubic/>

## Non-parametric image warping

DigiVFX

- Specify a more detailed warp function
- Splines, meshes, optical flow (per-pixel motion)



## Image morphing

## Demo

DigiVFX

- <http://www.colonize.com/warp/>
- Warping is a useful operation for mosaics, video matching, view interpolation and so on.

## Image morphing

DigiVFX

- The goal is to synthesize a fluid transformation from one image to another.
- Cross dissolving is a common transition between cuts, but it is not good for morphing because of the ghosting effects.



image #1

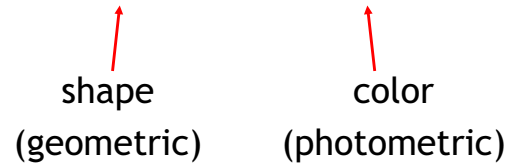
dissolving

image #2

## Image morphing

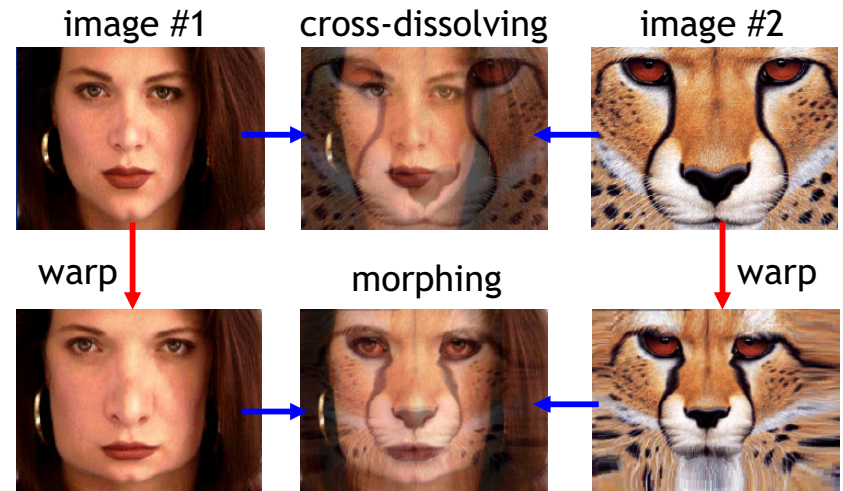
DigiVFX

- Why ghosting?
- Morphing = warping + cross-dissolving



## Image morphing

DigiVFX



## Morphing sequence

DigiVFX



## Artifacts of cross-dissolving

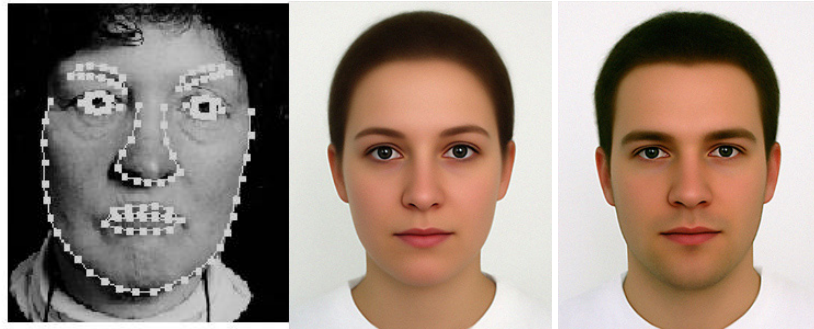
DigiVFX



<http://www.salavon.com/>

## Face averaging by morphing

DigiVFX



average faces

## Image morphing

DigiVFX

create a morphing sequence: for each time  $t$

1. Create an intermediate warping field (by interpolation)
2. Warp both images towards it
3. Cross-dissolve the colors in the newly warped images

## An ideal example

DigiVFX



$t=0$

morphing

$t=1$

## An ideal example

DigiVFX



$t=0$

middle face ( $t=0.5$ )

$t=1$



## Warp specification (mesh warping)

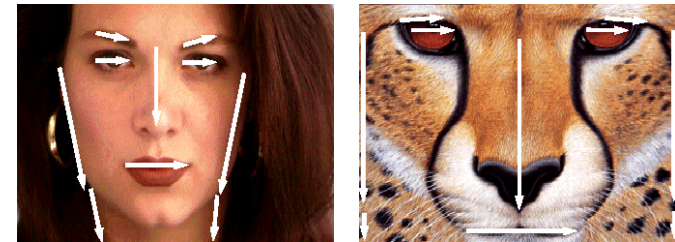
- How can we specify the warp?
  - Specify corresponding *spline control points*  
*interpolate* to a complete warping function



easy to implement, but less expressive

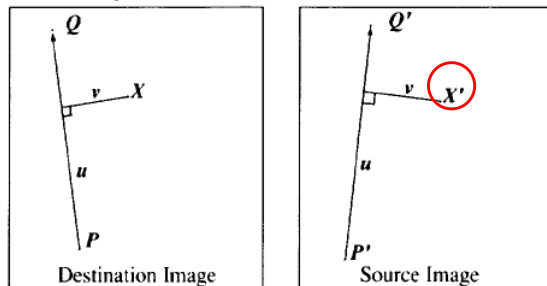
## Warp specification (field warping)

- How can we specify the warp?
  - Specify corresponding *vectors*
    - interpolate* to a complete warping function
    - The Beier & Neely Algorithm



## Beier&Neely (SIGGRAPH 1992)

- Single line-pair PQ to P'Q':



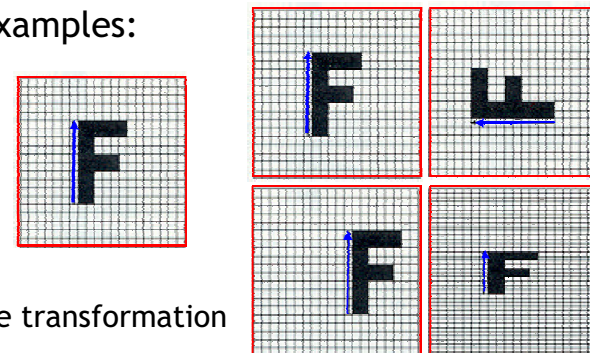
$$u = \frac{(X - P) \cdot (Q - P)}{\|Q - P\|^2} \quad (1)$$

$$v = \frac{(X - P) \cdot \text{Perpendicular}(Q - P)}{\|Q - P\|} \quad (2)$$

$$X' = P' + u \cdot (Q' - P') + \frac{v \cdot \text{Perpendicular}(Q' - P')}{\|Q' - P'\|} \quad (3)$$

## Algorithm (single line-pair)

- For each X in the destination image:
  - Find the corresponding u,v
  - Find X' in the source image for that u,v
  - destinationImage(X) = sourceImage(X')
- Examples:

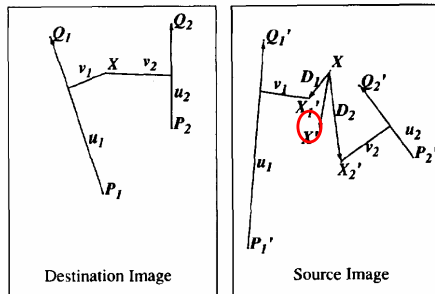


Affine transformation

## Multiple Lines

DigiVFX

$$D_i = X_i' - X_i$$



$$weight = \left( \frac{length^p}{(a + dist)} \right)^b$$

$length$  = length of the line segment,

$dist$  = distance to line segment

The influence of  $a, p, b$ . The same as the average of  $X_i'$

## Full Algorithm

DigiVFX

For each pixel  $X$  in the destination

$DSUM = (0,0)$

$weightsum = 0$

For each line  $P_i Q_i$

calculate  $u, v$  based on  $P_i Q_i$

calculate  $X_i'$  based on  $u, v$  and  $P_i' Q_i'$

calculate displacement  $D_i = X_i' - X_i$  for this line

$dist$  = shortest distance from  $X$  to  $P_i Q_i$

$weight = (length^p / (a + dist))^b$

$DSUM += D_i * weight$

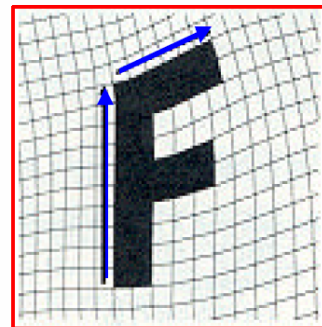
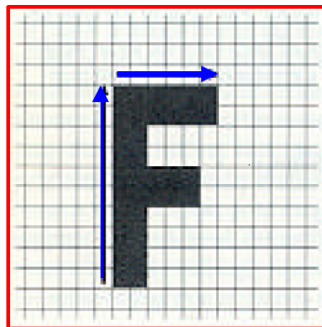
$weightsum += weight$

$X' = X + DSUM / weightsum$

$destinationImage(X) = sourceImage(X')$

## Resulting warp

DigiVFX



## Animated sequences

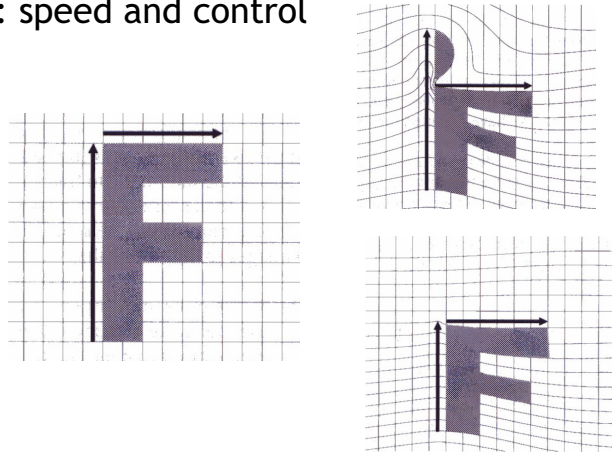
DigiVFX

- Specify keyframes and interpolate the lines for the inbetween frames
- Require a lot of tweaking

## Comparison to mesh morphing

DigiVFX

- Pros: more expressive
- Cons: speed and control



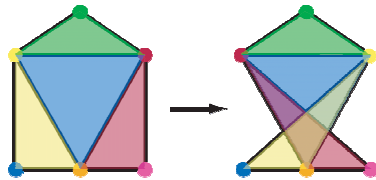
## Warp interpolation

DigiVFX

- How do we create an intermediate warp at time  $t$ ?
- For optical flow:
  - Easy. Interpolate each flow vector
- For feature point methods:
  - linear interpolation of each feature pair
- For Beier-Neely:
  - Can do the same for line end-points
  - But, a line rotating 180 degrees will become 0 length in the middle
  - One solution is to interpolate line mid-point and orientation angle
  - Not very intuitive

## Other Issues

DigiVFX



- Beware of folding
  - Can happen in any of the methods
  - You are probably trying to do something 3D-ish
- Extrapolation can sometimes produce interesting effects
  - Caricatures

## Results

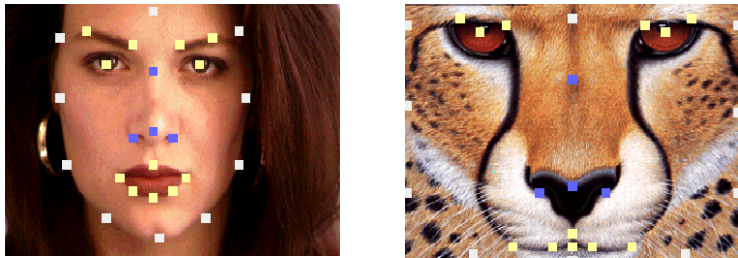
DigiVFX



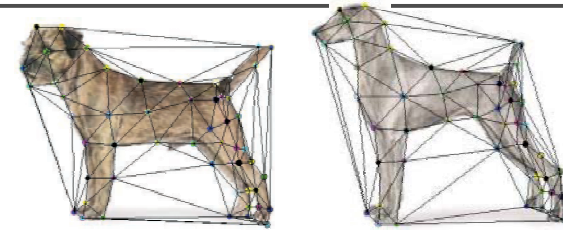
Michael Jackson's MTV "Black or White"

## Warp specification

- How can we specify the warp
- 3. Specify corresponding *points*
  - *interpolate* to a complete warping function



## Solution#1: convert to mesh warping



1. Define a triangular mesh over the points
  - Same mesh in both images!
  - Now we have triangle-to-triangle correspondences
2. Warp each triangle separately from source to destination
  - How do we warp a triangle?
  - 3 points = affine warp!
  - Just like texture mapping

## Solution#2: scattered point interpolation

- RBF
- Work minimization

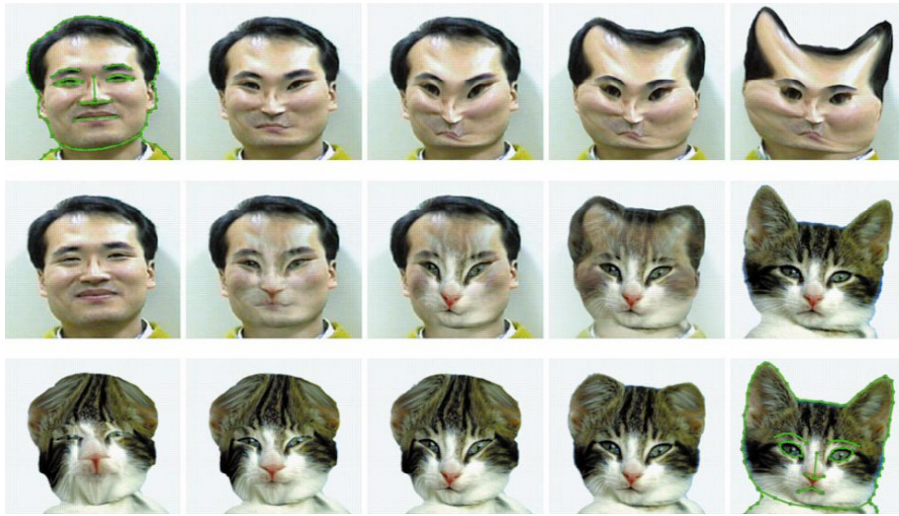
## Transition control





## Transition control

DigiVFX



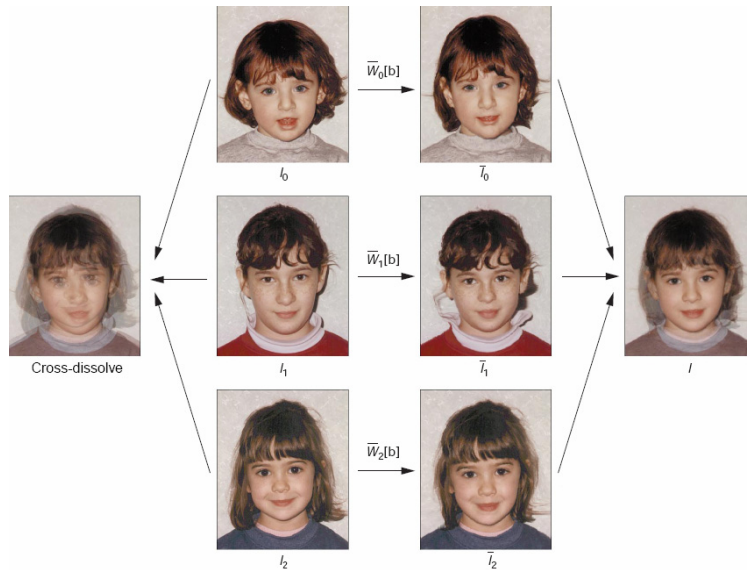
## Transition control

DigiVFX



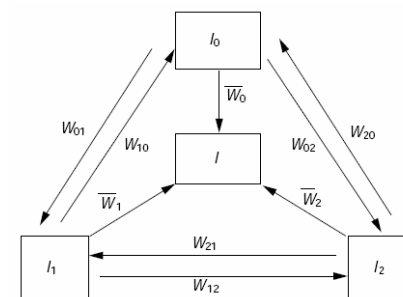
## Multi-source morphing

DigiVFX



## Multi-source morphing

DigiVFX

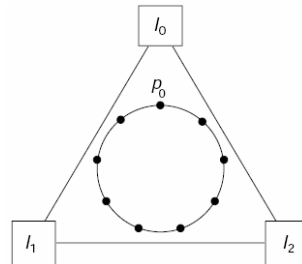


$$\bar{W}_i(p) = \sum_{j=1}^n b_j W_{ij}(p)$$

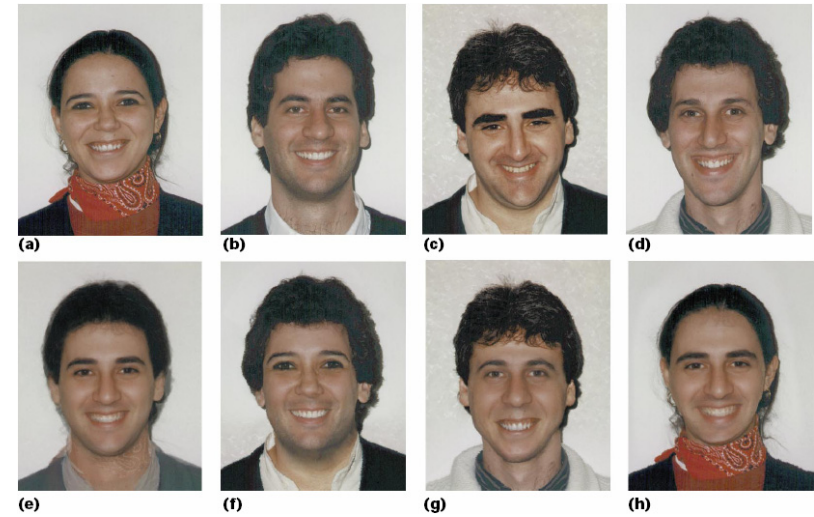
$$\bar{I}_i(r) = \bar{W}_i(p) \cdot b_i I_i(p)$$

$$I(r) = \sum_{i=1}^n \bar{I}_i(r)$$

## Multi-source morphing



## Multi-source morphing



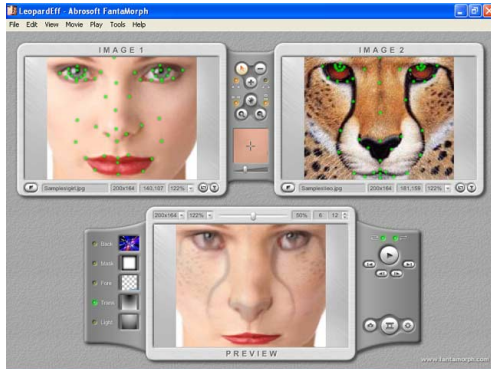
## Project #1: image morphing

## Project #1 image morphing

- Assigned: 3/9
- Due: 11:59pm 3/29
- Work in pairs
- Handout will be online by tomorrow noon. I will send a mail to vfx when it is available.
- We will provide a generic image library, gil.

## Reference software

- [Morphing software review](#)
- I used [FantaMorph](#) 30-day evaluation version. You can use any one you like.



## Morphing is not only for faces



## Morphing is not only for faces



## Bells and whistles

- Multi-source morphing
- Automatic morphing
- Morphing for animated sequences

## Submission

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- You have to turn in your complete source, the executable, a html report and an artifact.
- Report page contains:  
description of the project, what do you learn, algorithm, implementation details, results, bells and whistles...
- Artifacts must be made using your own program.  
artifacts voting on forum
- Submission mechanism will be announced later.