Cameras

Digital Image Synthesis

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with slides by Pat Hanrahan and Matt Pharr

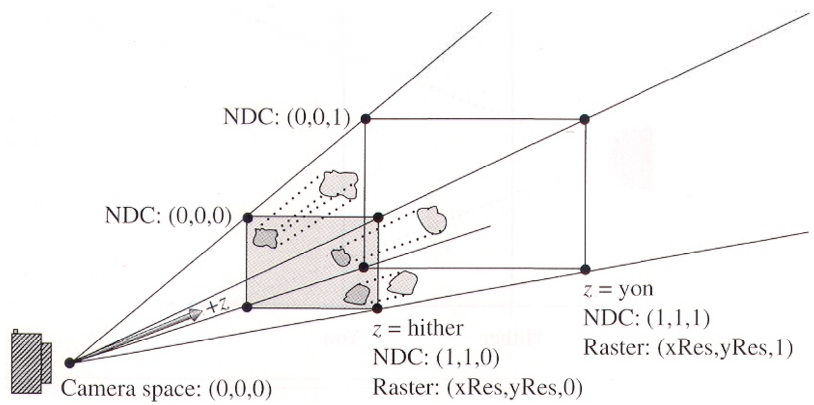
Camera

```
class Camera {
public:     return a weight, useful for simulating real lens
  virtual float GenerateRay(const CameraSample
                     &sample, Ray *ray) const = 0;
  ...                sample position       corresponding
                     at the image plane   normalized ray in
                                          the world space
  virtual float GenerateRayDifferential(
        const CameraSample &sample,
        RayDifferential *rd) const;
data members
  AnimatedTransform CameraToWorld;
  float ShutterOpen, ShutterClose;
  Film *film;
};
```

GenerateRayDifferential

```
float Camera::GenerateRayDifferential(…) {
  float wt = GenerateRay(sample, rd); default
  CameraSample sshift = sample;       implementation
  ++(sshift.imageX);
  Ray rx;
  float wtx = GenerateRay(sshift, &rx);
  rd->rxOrigin = rx.o;    rd->rxDirection = rx.d;

  --(sshift.imageX);    ++(sshift.imageY);
  Ray ry;
  float wty = GenerateRay(sshift, &ry);
  rd->ryOrigin = ry.o;    rd->ryDirection = ry.d;
  if (wtx == 0.f || wty == 0.f) return 0.f;
  rd->hasDifferentials = true;
  return wt;
}
```

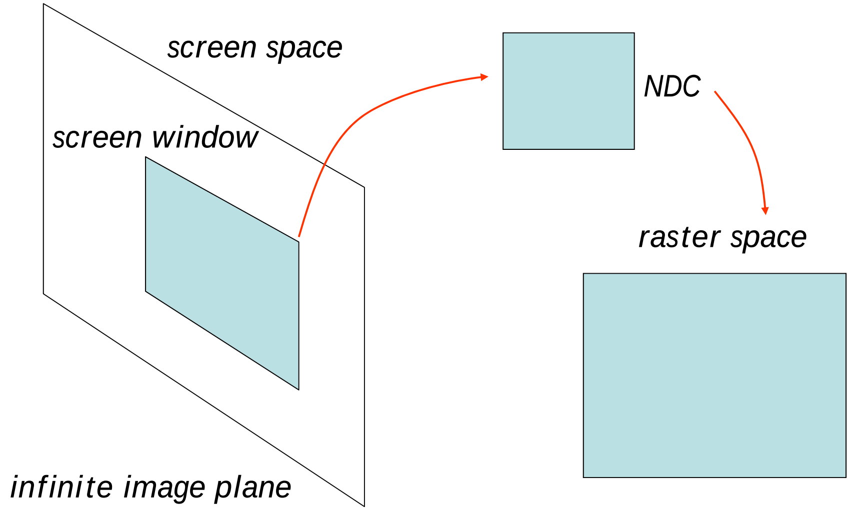
Camera space



Coordinate spaces

- world space
- object space
- camera space (origin: camera position, z: viewing direction, y: up direction)
- screen space: a 3D space defined on the image plane, z ranges from 0(near) to 1(far); it defines the visible window
- normalized device space (NDC): (x, y) ranges from (0,0) to (1,1) for the rendered image, z is the same as the screen space
- raster space: similar to NDC, but the range of (x,y) is from (0,0) to (xRes, yRes)

Screen space



Projective camera models

- Transform a 3D scene coordinate to a 2D image coordinate by a 4x4 projective matrix

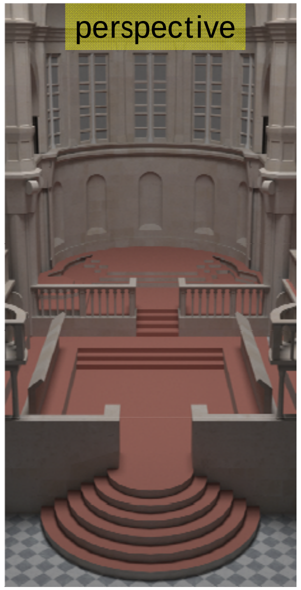
```
class ProjectiveCamera : public Camera {
public:
  ProjectiveCamera(AnimatedTransform
    &cam2world, Transform &proj,
    float Screen[4],float sopen,float sclose,
    float lensr, float focald, Film *film);
protected:
  Transform CameraToScreen, RasterToCamera;
  Transform ScreenToRaster, RasterToScreen;
  float lensRadius, focalDistance;
};
```

camera to screen projection (3D to 2D)

Projective camera models

```
ProjectiveCamera::ProjectiveCamera(...)
  :Camera(cam2world, sopen, sclose, f) {
  ...
  CameraToScreen=proj;
  WorldToScreen=CameraToScreen*WorldToCamera;
  ScreenToRaster
   = Scale(float(film->xResolution),
           float(film->yResolution), 1.f)*
     Scale(1.f / (Screen[1] - Screen[0]),
           1.f / (Screen[2] - Screen[3]), 1.f)*
     Translate(Vector(-Screen[0],-Screen[3],0.f));
  RasterToScreen = Inverse(ScreenToRaster);
  RasterToCamera =
      Inverse(CameraToScreen) * RasterToScreen;
}
```

Projective camera models



Orthographic camera

```
Transform Orthographic(float znear, float zfar)
{
  return Scale(1.f, 1.f, 1.f/(zfar-znear))
    *Translate(Vector(0.f, 0.f, -znear));
}

OrthoCamera::OrthoCamera( ... )
  : ProjectiveCamera(cam2world,
     Orthographic(0., 1.),
   Screen, sopen, sclose, lensr, focald, f)
{ All differential rays have the same dir and origin shift
   dxCamera = RasterToCamera(Vector(1, 0, 0));
   dyCamera = RasterToCamera(Vector(0, 1, 0));
}
```

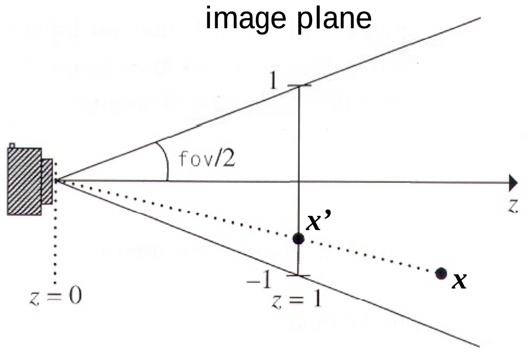
OrthoCamera::GenerateRay

```
float OrthoCamera::GenerateRay(const
  CameraSample &sample, Ray *ray) const {
  Point Pras(sample.imageX,sample.imageY,0);
  Point Pcamera;
  RasterToCamera(Pras, &Pcamera);
  *ray = Ray(Pcamera, Vector(0,0,1),
             0.f, INFINITY);
  <Modify ray for depth of field>
  ray->time = Lerp(sample.time,
              shutterOpen, shutterClose);
  CameraToWorld(*ray, ray);
  return 1.f;
}
```

OrthoCamera::GenerateRayDifferential

```
float OrthoCamera::GenerateRay(const
  CameraSample &sample,RayDifferential *ray) {
  Point Pras(sample.imageX,sample.imageY,0);
  Point Pcamera;
  RasterToCamera(Pras, &Pcamera);
  *ray = RayDifferential(Pcamera,
         Vector(0,0,1), 0., INFINITY);
  <Modify ray for depth of field>
  ray->time = Lerp(sample.time,
              shutterOpen, shutterClose);
  ray->rxOrigin = ray->o + dxCamera;
  ray->ryOrigin = ray->o + dyCamera;
  ray->rxDirection = ray->ryDirection = ray->d;
  ray->hasDifferentials = true;
  CameraToWorld(*ray, ray);
  return 1.f;
}
```

Perspective camera



$x' = x / z$

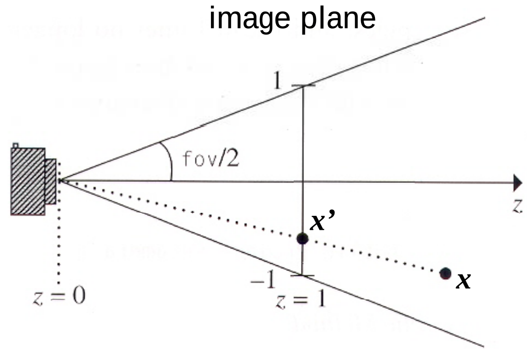
$y' = y / z$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$z' = \frac{z-n}{f-n}$?

But, you must divide by z because of x' and y'

Perspective camera



$x' = x / z$

$y' = y / z$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$z' = \frac{f(z-n)}{z(f-n)}$$

Perspective camera

```
Transform Perspective(float fov,float n,float f)
{                                         near_z     far_z
  Matrix4x4 *persp =
  new Matrix4x4(1, 0,       0,           0,
                0, 1,       0,           0,
                0, 0,   f/(f-n),    -f*n/(f-n),
                0, 0,       1,           0);

  float invTanAng= 1.f / tanf(Radians(fov)/2.f);
  return Scale(invTanAng, invTanAng, 1) *
         Transform(persp);
}
```

PerspectiveCamera::GenerateRay

```
float PerspectiveCamera::GenerateRay
      (const CameraSample &sample, Ray *ray) const
{
  // Generate raster and camera samples
  Point Pras(sample.imageX, sample.imageY, 0);
  Point Pcamera;
  RasterToCamera(Pras, &Pcamera);
  *ray = Ray(Point(0,0,0), Vector(Pcamera),
             0.f, INFINITY);
  <Modify ray for depth of field>
  ray->time = Lerp(sample.time,
                   shutterOpen, shutterClose);
  CameraToWorld(*ray, ray);
  return 1.f;
}
```

GenerateRayDifferential

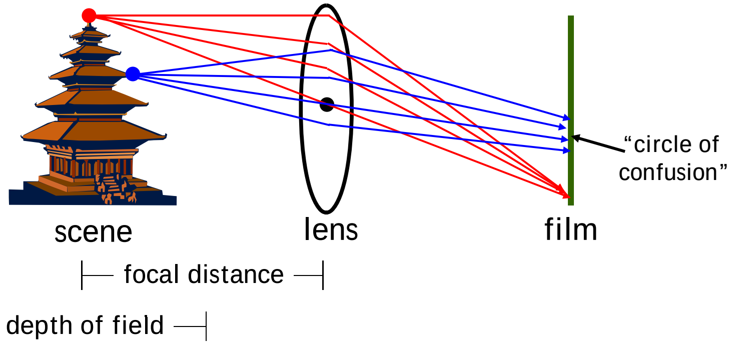
the same as GenerateRay

precomputed in the constructor

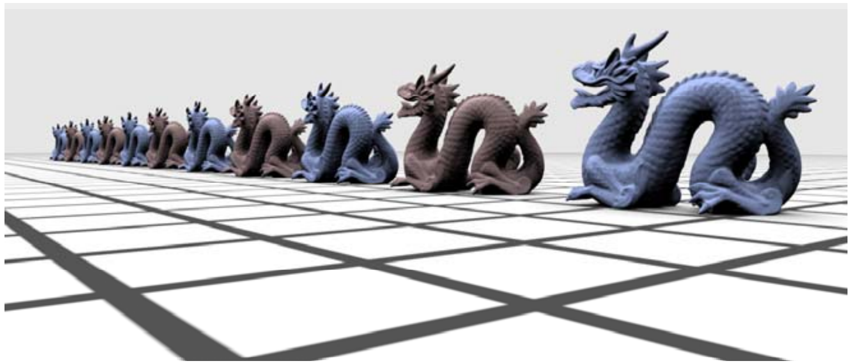
```
dxCamera = RasterToCamera(Point(1,0,0)) -
           RasterToCamera(Point(0,0,0));
dyCamera = RasterToCamera(Point(0,1,0)) –
           RasterToCamera(Point(0,0,0));
ray->rxOrigin = ray->ryOrigin = ray->o;
ray->rxDirection = Normalize(Vector(Pcamera)
                             + dxCamera);
ray->ryDirection = Normalize(Vector(Pcamera)
                             + dyCamera);

ray->time = Lerp(sample.time,
                 shutterOpen, shutterClose);
CameraToWorld(*ray, ray);
ray->hasDifferentials = true;
return 1.f;
```

Depth of field

- Circle of confusion $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
- Depth of field: the range of distances from the lens at which objects appear in focus (circle of confusion roughly smaller than a pixel)

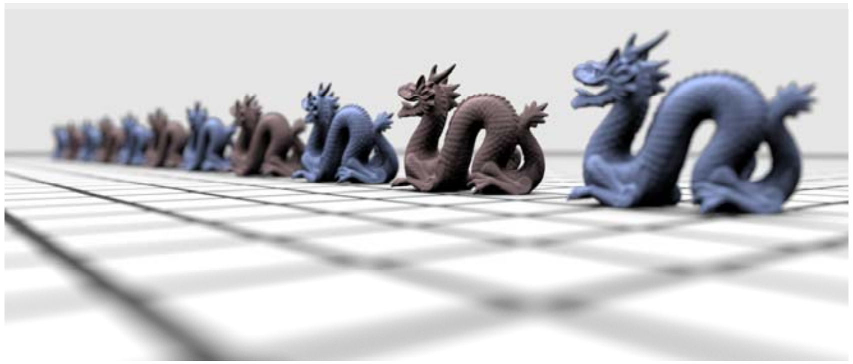


Depth of field



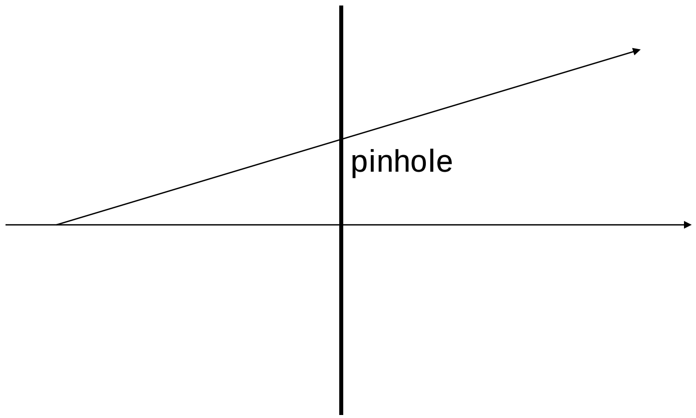
without depth of field

Depth of field

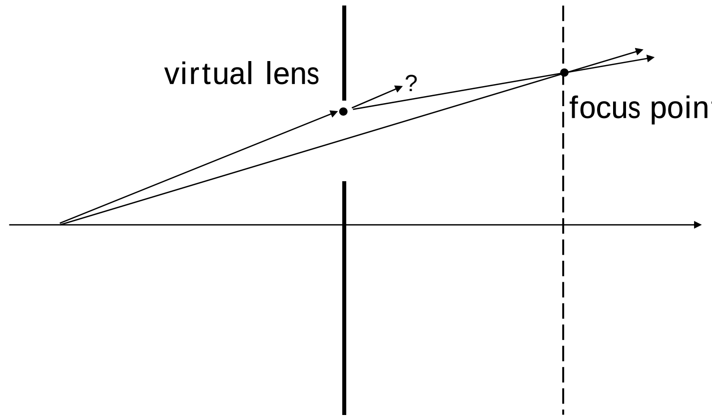


with depth of field

Sample the lens



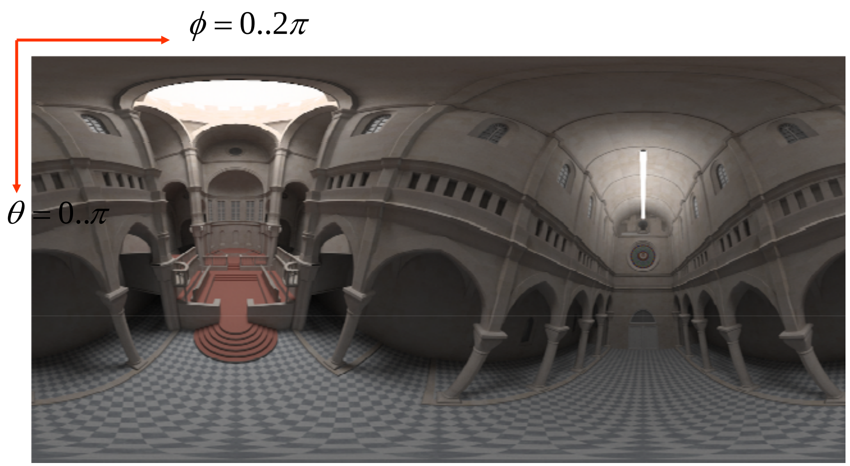
Sample the lens



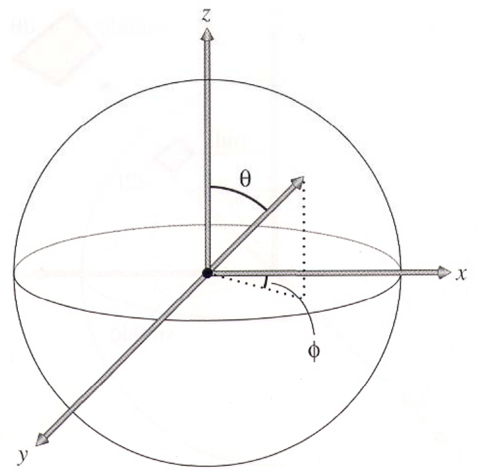
In GenerateRay(…)

```
if (LensRadius > 0.) {
  // Sample point on lens
  float lensU, lensV;
  ConcentricSampleDisk(sample.lensU,
  sample.lensV,
                       &lensU, &lensV);
  lensU *= lensRadius;
  lensV *= lensRadius;
  // Compute point on plane of focus
  float ft = focalDistance / ray->d.z;
  Point Pfocus = (*ray)(ft);
  // Update ray for effect of lens
  ray->o = Point(lensU, lensV, 0.f);
  ray->d = Normalize(Pfocus - ray->o);
}
```

Environment camera



Environment camera



$x=\sin\theta\cos\phi$
$y=\sin\theta\sin\phi$
$z=\cos\theta$

EnvironmentCamera

```
EnvironmentCamera::
  EnvironmentCamera(const Transform &world2cam,
                    float hither, float yon,
                    float sopen, float sclose,
                    Film *film)
  : Camera(world2cam, hither, yon,
           sopen, sclose, film)
{
  rayOrigin = CameraToWorld(Point(0,0,0));
}
```

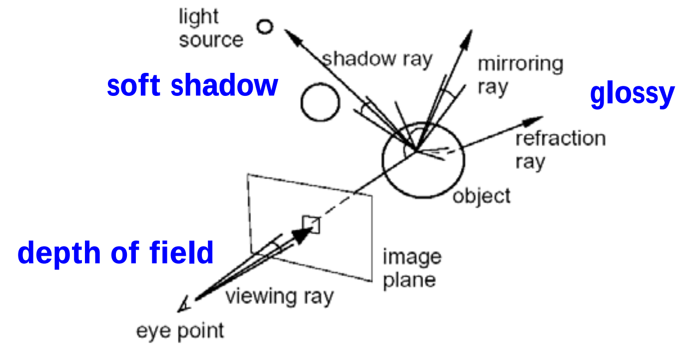
in world space

EnvironmentCamera::GenerateRay

```
float EnvironmentCamera::GenerateRay
      (CameraSample &sample, Ray *ray) const
{
  float time = Lerp(sample.time,
                    shutterOpen, shutterClose);
  float theta=M_PI*sample.imageY/film->yResolution;
  float phi=2*M_PI*sample.imageX/film->xResolution;
  Vector dir(sinf(theta)*cosf(phi), cosf(theta),
             sinf(theta)*sinf(phi));
  *ray = Ray(Point(0,0,0), dir, 0.f, INFINITY,time);
  CameraToWorld(*ray, ray);
  return 1.f;
}
```

Distributed ray tracing

- *SIGGRAPH 1984, by Robert L. Cook, Thomas Porter and Loren Carpenter from LucasFilm.*
- Apply distribution-based sampling to many parts of the ray-tracing algorithm.



Distributed ray tracing

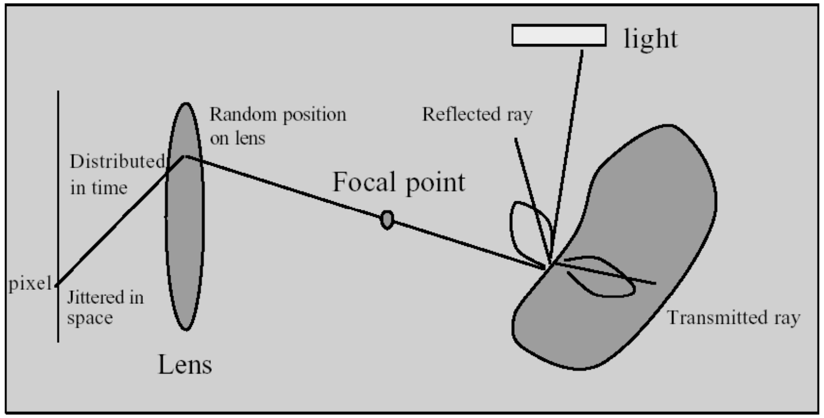
Gloss/Translucency
- Perturb directions reflection/transmission, with distribution based on angle from ideal ray

Depth of field
- Perturb eye position on lens

Soft shadow
- Perturb illumination rays across area light

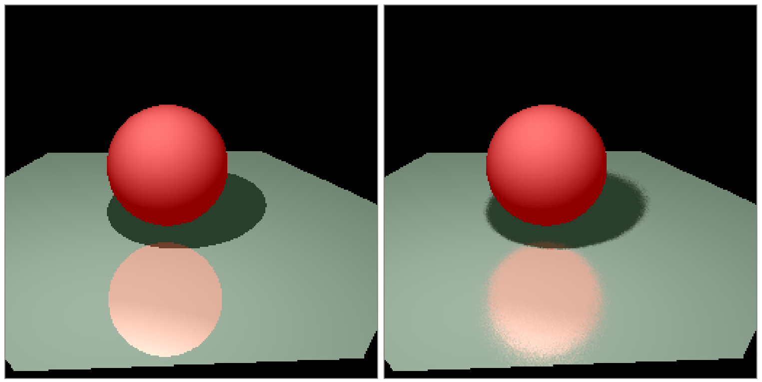
Motion blur
- Perturb eye ray samples in time

Distributed ray tracing



DRT: Gloss/Translucency

- Blurry reflections and refractions are produced by randomly perturbing the reflection and refraction rays from their "true" directions.

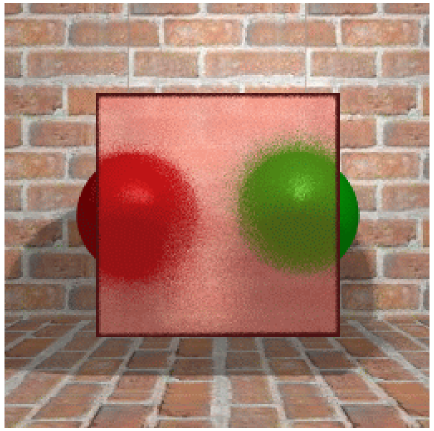


Glossy reflection

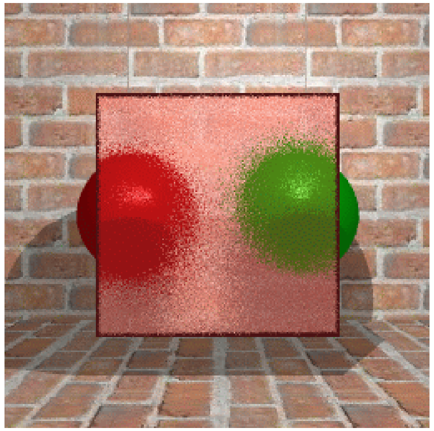
4 rays

64 rays

Translucency

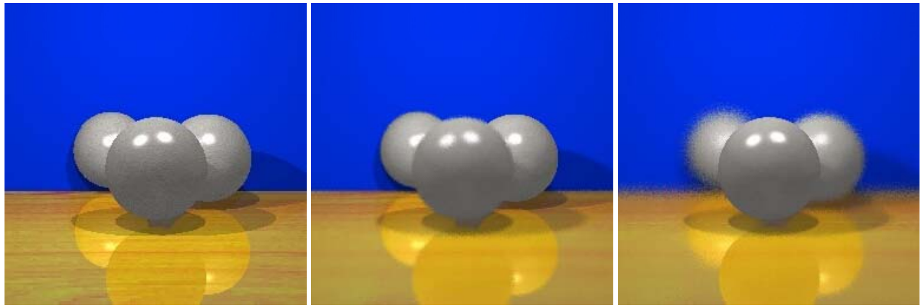


4 rays

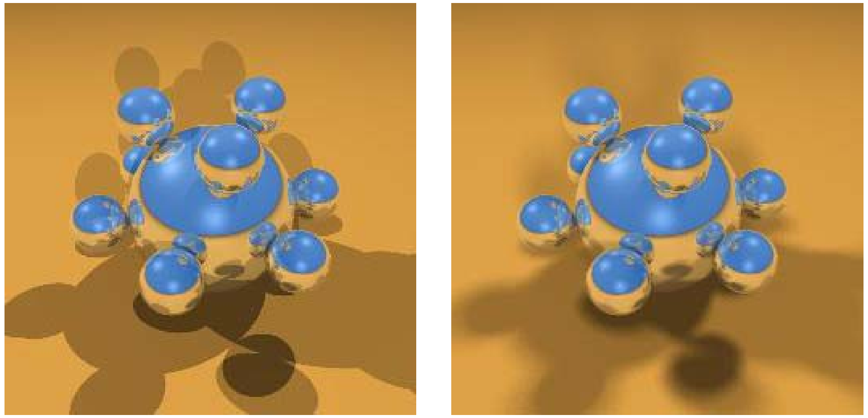


16 rays

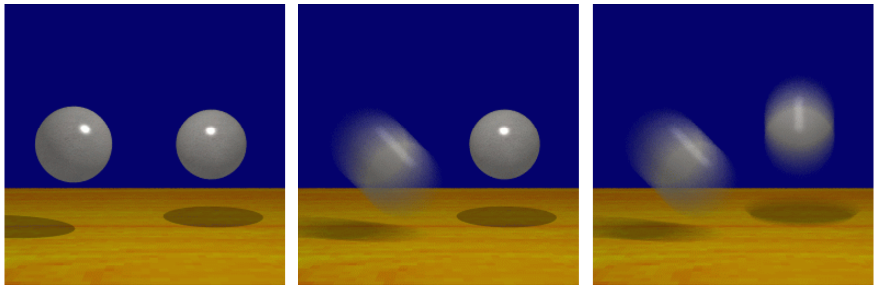
Depth of field



Soft shadows

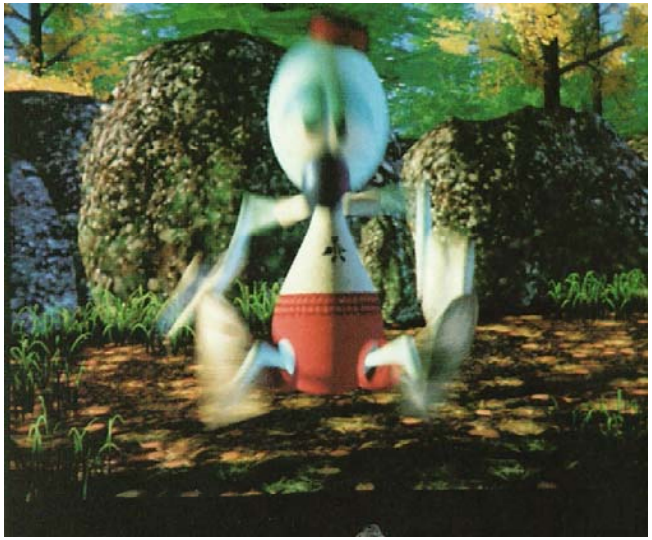


Motion blur



Results

Adventures of Andre & Wally B (1986)



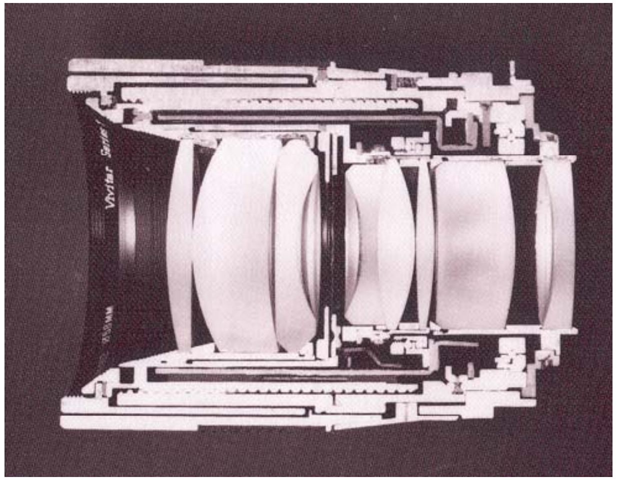
Realistic camera model

- Most camera models in graphics are not geometrically or radiometrically correct.
- Model a camera with a lens system and a film backplane. A lens system consists of a sequence of simple lens elements, stops and apertures.

Why a realistic camera model?

- Physically-based rendering. For more accurate comparison to empirical data.
- Seamlessly merge CGI and real scene, for example, VFX.
- For vision and scientific applications.
- The camera metaphor is familiar to most 3d graphics system users.

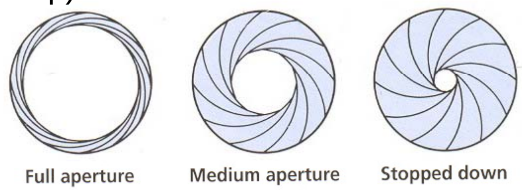
Real Lens



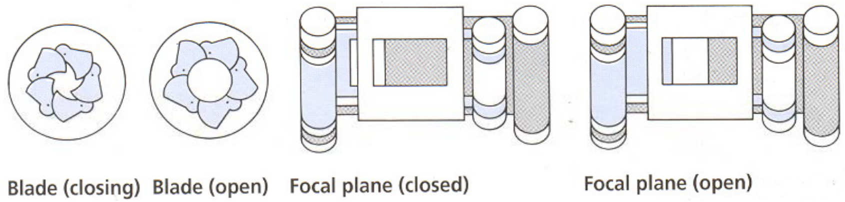
Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*

Exposure

- Two main parameters:
  - Aperture (in f stop)

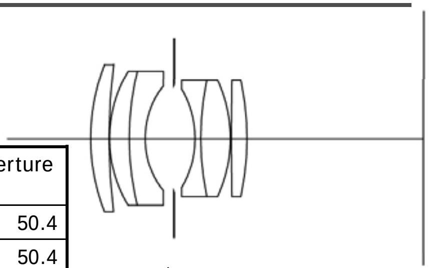


  - Shutter speed (in fraction of a second)



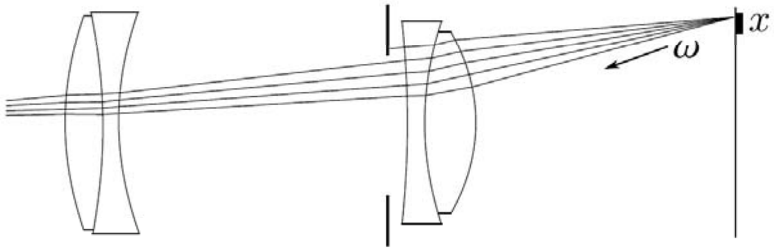
Double Gauss

Radius (mm)	Thick (mm)	n_d	V-no	aperture
58.950	7.520	1.670	47.1	50.4
169.660	0.240			50.4
38.550	8.050	1.670	47.1	46.0
81.540	6.550	1.699	30.1	46.0
25.500	11.410			36.0
	9.000			34.2
-28.990	2.360	1.603	38.0	34.0
81.540	12.130	1.658	57.3	40.0
-40.770	0.380			40.0
874.130	6.440	1.717	48.0	40.0
-79.460	72.228			40.0



Data from W. Smith, Modern Lens Design, p 312

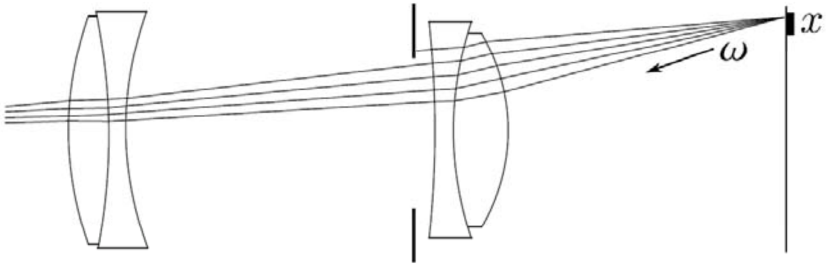
Measurement equation



$$R = \int\int\int\int L(T(x,\omega,\lambda);\lambda)S(x,t)P(x,\lambda)\cos\theta\; dx\, d\omega\, dt\, d\lambda$$

L: radiance
T: image to object space transformation
S: shutter function
P: sensor response characteristics

Measurement equation



$$R = \Delta t \cdot \int \int L(T(x,\omega)) \cos\theta \; dx \; d\omega$$

L: radiance T: image to object space transformation

Solving the integral

Problem: given a function f and domain Ω, how to calculate

$$\int_\Omega f(x)dx$$

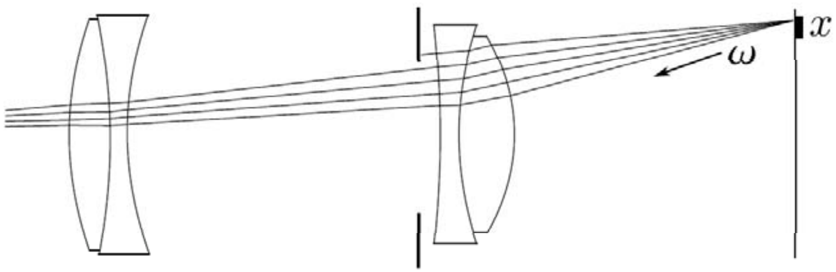
Solution: Monte Carlo method:

$$\int_\Omega f(x)dx \approx [\frac{1}{N}\sum_{i=1}^{N} f(x_i)] \cdot \int_\Omega dx$$

where $x_1, x_2, \ldots, x_N$ are uniform distributed random samples in Ω.

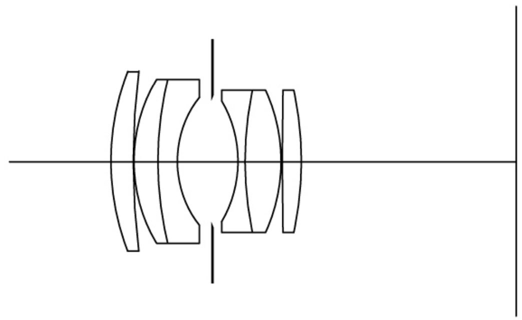
Algorithm

1. For each pixel on the image, generate some random samples x_i and ω_i uniformly.
2. For each x_i and ω_i, calculate $T(x_i, \omega_i)$.
3. Shoot the ray according to the result of $T(x_i, \omega_i)$ into the scene, and calculate the radiance.
4. Set the pixel value to the average of radiance.



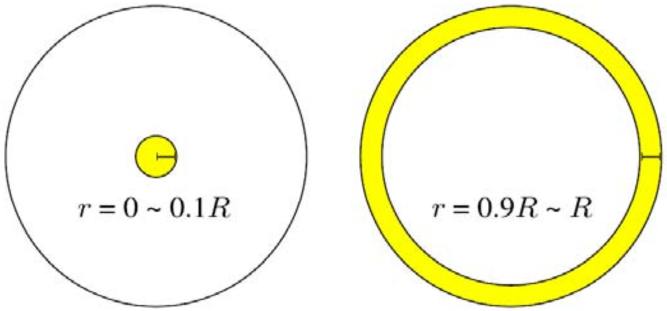
Tracing rays through lens system

1. $R = Ray(x_i, \omega_i)$
2. Calculate the intersection point p for each lens element E_i from rear to front.
    1. Return zero if p is outside the aperture of E_i.
    2. Compute the new direction by Snell's law if the medium is different.



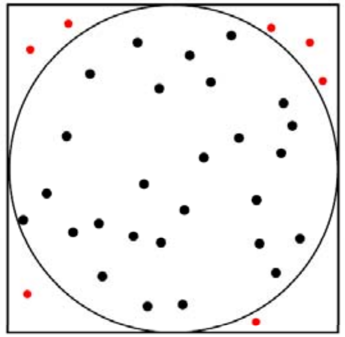
Sampling a disk uniformly

- Now we need to obtain random samples on a disk uniformly.
- How about uniformly sample r in $[0, R]$ and θ in $[0, 2\pi]$ and let $x = r\cos\theta, y = r\sin\theta$?
  - The result is not uniform due to coordinate transformation.



Rejection

1. Uniformly sample a point in the bounding square of the disk.
2. If the sample lies outside the disk, reject it and sample another one.



Another method

- Sample r and θ in a specific way so that the result is uniform after coordinate transformation.
- Let

$$r = \sqrt{\xi_1},\ \theta = 2\pi\xi_2$$

  where ξ_1 and ξ_2 are random samples distributed in $[0, 1]$ uniforml uniformly.
- This produce uniform samples on a disk after coordinate transformation. We will prove it later in chapter 14 "Monte Carlo integration".

Ray Tracing Through Lenses

200 mm telephoto

35 mm wide-angle

50 mm double-gauss

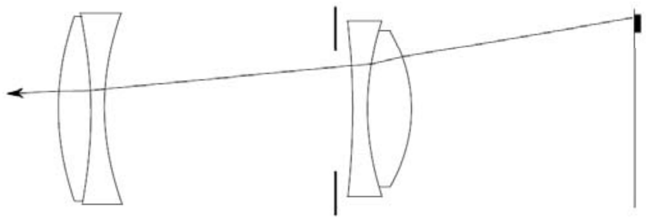
16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)

Assignment #2

- Write the "realistic" camera plugin for PBRT which implements the realistic camera model.
- The description of lens system will be provided.
- `GenerateRay(const Sample &sample, Ray *ray)`
  - PBRT generate rays by calling `GenerateRay()`, which is a virtual function of `Camera`.
  - PBRT will give you pixel location in `sample`.
  - You need to fill the content of `ray` and return a value for its weight.

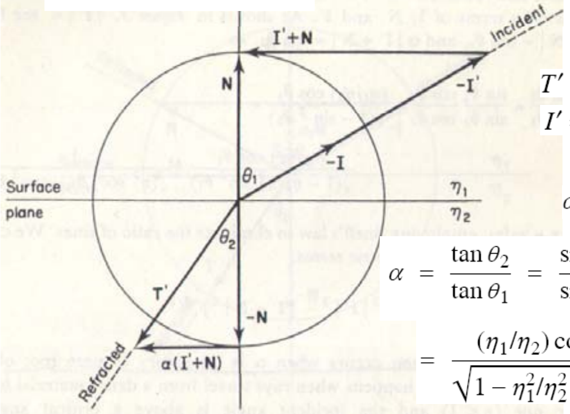
Assignment #2



1. Sample a point on the exit pupil uniformly.
   - Hint: `sample.lensU` and `sample.lensV` are two random samples distributed in $[0, 1]$ uniformly.
2. Trace this ray through the lens system. You can return zero if this ray is blocked by an aperture stop.
3. Fill `ray` with the result and return $\frac{\cos^4\theta'}{Z^2}$ as its weight.

Whitted's method

$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$



$$T' = \alpha(I' + N) - N \text{ for some } \alpha$$

$$I' = I/(-I \cdot N)$$

$$|I' + N| = \tan\theta_1$$

$$\alpha|I' + N| = \tan\theta_2$$

$$\alpha = \frac{\tan\theta_2}{\tan\theta_1} = \frac{\sin\theta_2}{\sin\theta_1}\frac{\cos\theta_1}{\cos\theta_2} = \frac{(\eta_1/\eta_2)\cos\theta_1}{\sqrt{1-\sin^2\theta_2}}$$

$$= \frac{(\eta_1/\eta_2)\cos\theta_1}{\sqrt{1-\eta_1^2/\eta_2^2\sin^2\theta_1}} = \frac{1}{\sqrt{n^2\sec^2\theta_1 - \tan^2\theta_1}}$$

$$\uparrow \; |I'| = \sec\theta_1$$

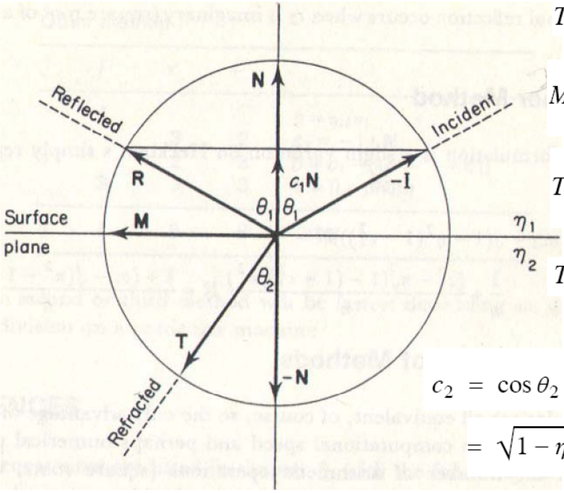
$$\alpha = (n^2|I'|^2 - |I' + N|^2)^{-1/2}$$

Whitted's method

Whitted's Method

$\sqrt{\ }$	/	×	+	
	1			$n = \eta_2/\eta_1$
	3	3	2	$I' = I/(-I \cdot N)$
			3	$J = I' + N$
1	1	8	5	$\alpha = 1/\sqrt{n^2(I' \cdot I') - (J \cdot J)}$
		3	3	$T' = \alpha J - N$
1	3	3	2	$T = T'/\|T'\|$
2	8	17	15	TOTAL

Heckber's method



$$T = \sin\theta_2 M - \cos\theta_2 N$$

$$M = \frac{I_{perp}}{|I_{perp}|} = \frac{I + c_1 N}{\sin\theta_1}$$

$$T = \frac{\sin\theta_2}{\sin\theta_1}(I + c_1 N) - \cos\theta_2 N$$

$$T = \eta I + (\eta c_1 - c_2)N$$

$$c_2 = \cos\theta_2 = \sqrt{1 - \sin^2\theta_2}$$

$$= \sqrt{1 - \eta^2 \sin^2\theta_1} = \sqrt{1 - \eta^2(1 - c_1^2)}$$

Heckbert's method

Heckbert's Method

$\sqrt{\ }$	/	×	+	
	1			$\eta = \eta_1/\eta_2$
		3	2	$c_1 = -I \cdot N$
1		3	2	$c_2 = \sqrt{1 - \eta^2(1 - c_1^2)}$
		7	4	$T = \eta I + (\eta c_1 - c_2)N$
1	1	13	8	TOTAL

Other method

$$T = \eta I + (\eta c_1 - \sqrt{1 - \eta^2(1 - c_1^2)})N$$

$$= \frac{I}{n} + \frac{c_1 - n\sqrt{1 - (1 - c_1^2)/n^2}}{n} N$$

$$= \frac{I + (c_1 - \sqrt{n^2 - 1 + c_1^2})N}{n}$$

Other Method

$\sqrt{\ }$	/	×	+	
	1			$n = \eta_2/\eta_1$
		3	2	$c_1 = -I \cdot N$
1		2	3	$\beta = c_1 - \sqrt{n^2 - 1 + c_1^2}$
	3	3	3	$T = (I + \beta N)/n$
1	4	8	8	TOTAL

Comparisons

Whitted's Method

$\sqrt{\ }$	/	×	+	
	1			$n = \eta_2/\eta_1$
	3	3	2	$I' = I/(-I \cdot N)$
			3	$J = I' + N$
1	1	8	5	$\alpha = 1/\sqrt{n^2(I' \cdot I') - (J \cdot J)}$
		3	3	$T' = \alpha J - N$
1	3	3	2	$T = T'/\lvert T'\rvert$
2	8	17	15	TOTAL

Heckbert's Method

$\sqrt{\ }$	/	×	+	
	1			$\eta = \eta_1/\eta_2$
		3	2	$c_1 = -I \cdot N$
1		3	2	$c_2 = \sqrt{1 - \eta^2(1 - c_1^2)}$
		7	4	$T = \eta I + (\eta c_1 - c_2)N$
1	1	13	8	TOTAL

Other Method

$\sqrt{\ }$	/	×	+	
	1			$n = \eta_2/\eta_1$
		3	2	$c_1 = -I \cdot N$
1		2	3	$\beta = c_1 - \sqrt{n^2 - 1 + c_1^2}$
	3	3	3	$T = (I + \beta N)/n$
1	4	8	8	TOTAL