Color and Radiometry

Digital Image Synthesis

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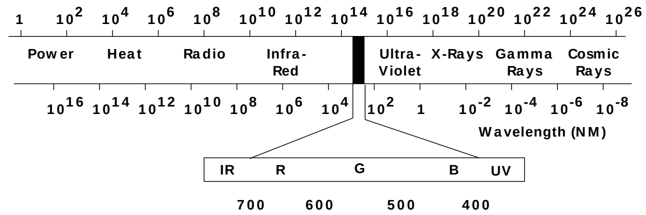
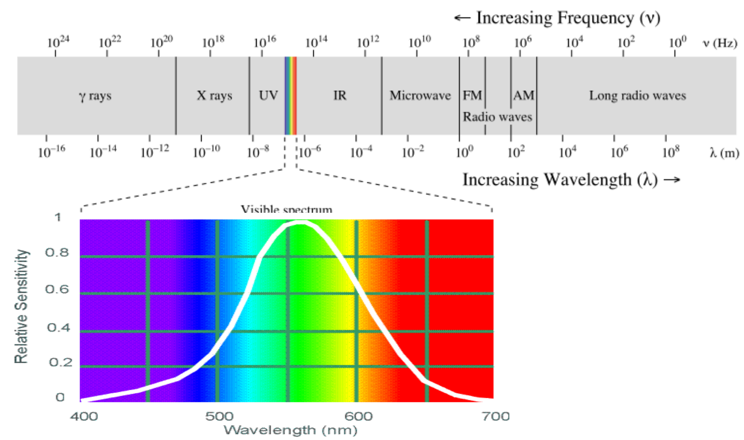
with slides by Svetlana Lazebnik, Pat Hanrahan and Matt Pharr

Radiometry

- Radiometry: study of the propagation of electromagnetic radiation in an environment
- Four key quantities: flux, intensity, irradiance and radiance
- These radiometric quantities are described by their spectral power distribution (SPD)
- Human visible light ranges from 370nm to 730nm



Electromagnetic spectrum



Human Luminance Sensitivity Function

Why do we see light at these wavelengths?

Because that's where the sun radiates electromagnetic energy

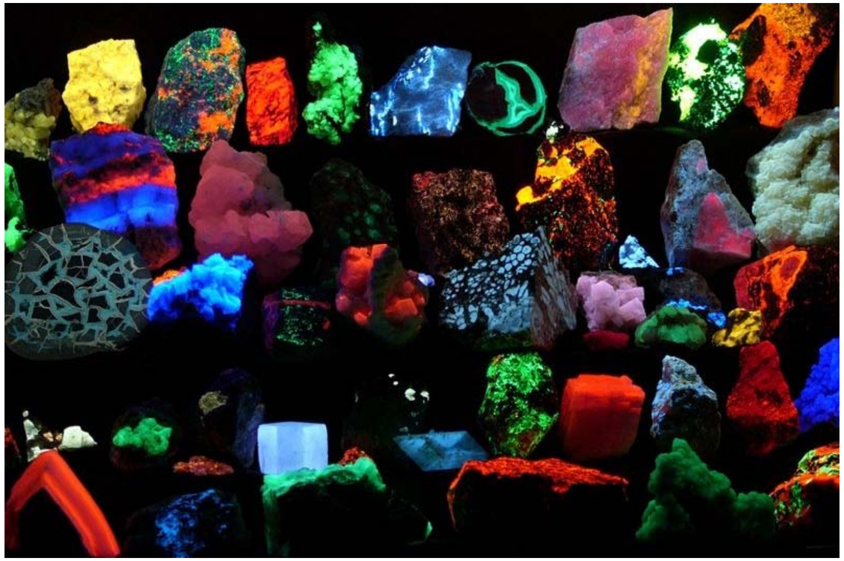
Basic radiometry

- pbrt is based on radiative transfer: study of the transfer of radiant energy based on radiometric principles and operates at the geometric optics level (light interacts with objects much larger than the light's wavelength)
- It is based on the particle model. Hence, **diffraction** and **interference** can't be easily accounted for.

Basic assumptions about light behavior

- **Linearity**: the combined effect of two inputs is equal to the sum of effects
- **Energy conservation**: scattering event can't produce more energy than they started with
- **Steady state**: light is assumed to have reached equilibrium, so its radiance distribution isn't changing over time.
- **No polarization**: we only care the frequency of light but not other properties (such as phases)
- **No fluorescence or phosphorescence**: behavior of light at a wavelength or time doesn't affect the behavior of light at other wavelengths or time

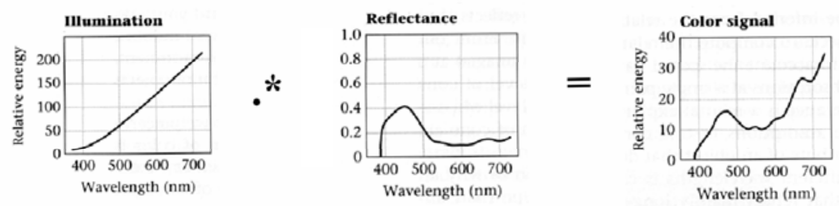
Fluorescent materials



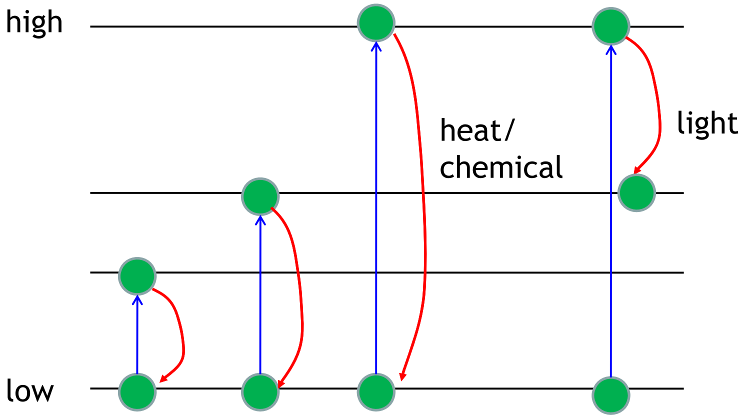
Color

Interaction of light and surfaces

- Reflected color is the result of interaction of light source spectrum with surface reflectance
- Spectral radiometry
  - All definitions and units are now "per unit wavelength"
  - All terms are now "spectral"



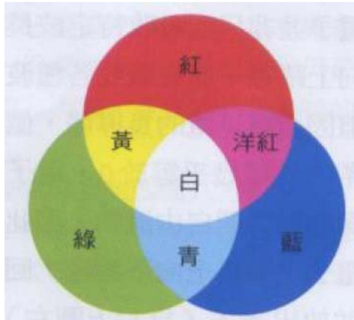
Why reflecting different colors



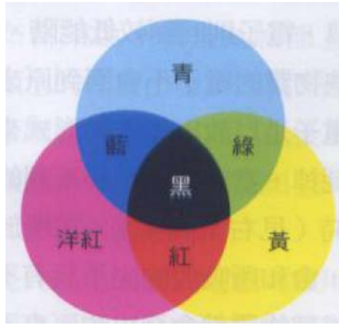
Light with specific wavelengths are absorbed.

Fluorescent

Primary colors



Primary colors for addition (light sources)

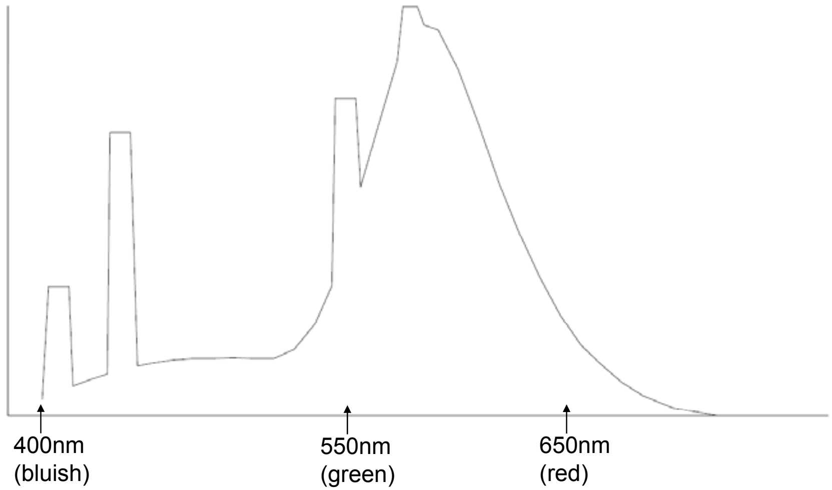


Primary colors for subtraction (reflection)

Heat generates light

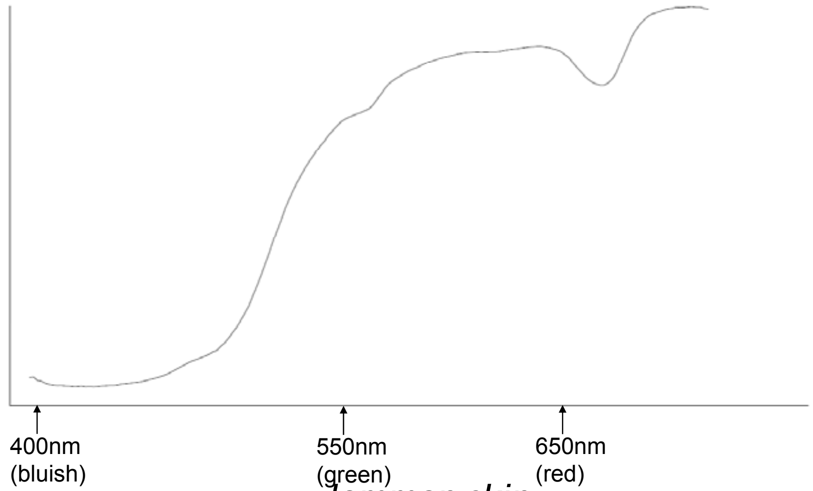
- Vibration of atoms or electrons due to heat generates electromagnetic radiation as well. If its wavelength is within visible light (>1000K), it generates color as well.
- Color only depends on temperature, but not property of the object.
- Human body radiates IR light under room temperature.
- 2400-2900K: color temperature of incandescent light bulb

Spectral power distribution



fluorescent light（日光燈）

Spectral power distribution

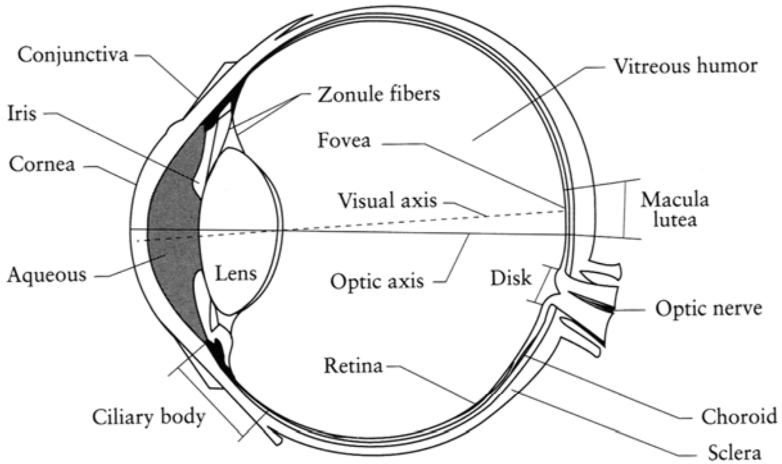


lemmon skin

Color

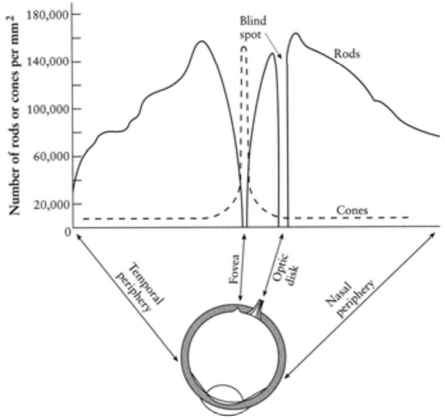
- Need a compact, efficient and accurate way to represent functions like these
- Find proper basis functions to map the infinite-dimensional space of all possible SPD functions to a low-dimensional space of coefficients
- For example, B(λ)=1 is a trivial but bad approximation
- Fortunately, according to *tristimulus theory*, all visible SPDs can be accurately represented with three values.

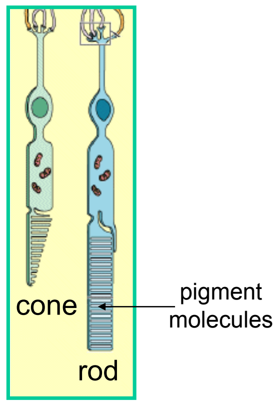
The Eye



Slide by Steve Seitz

Density of rods and cones





Rods and cones are *non-uniformly* distributed on the retina

- Rods responsible for intensity, cones responsible for color
- **Fovea** - Small region (1 or 2°) at the center of the visual field containing the highest density of cones (and no rods).
- Less visual acuity in the periphery—many rods wired to the same neuron

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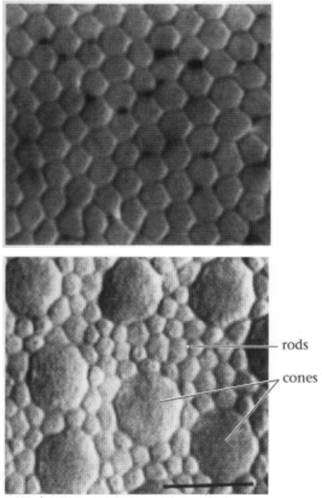
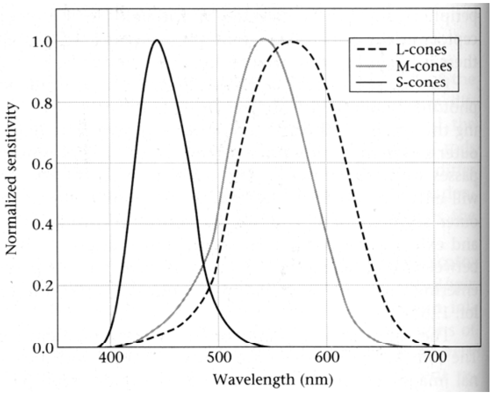
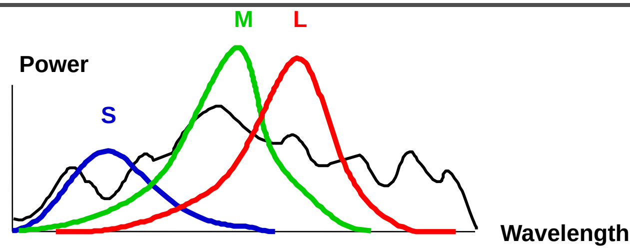
Human Photoreceptors





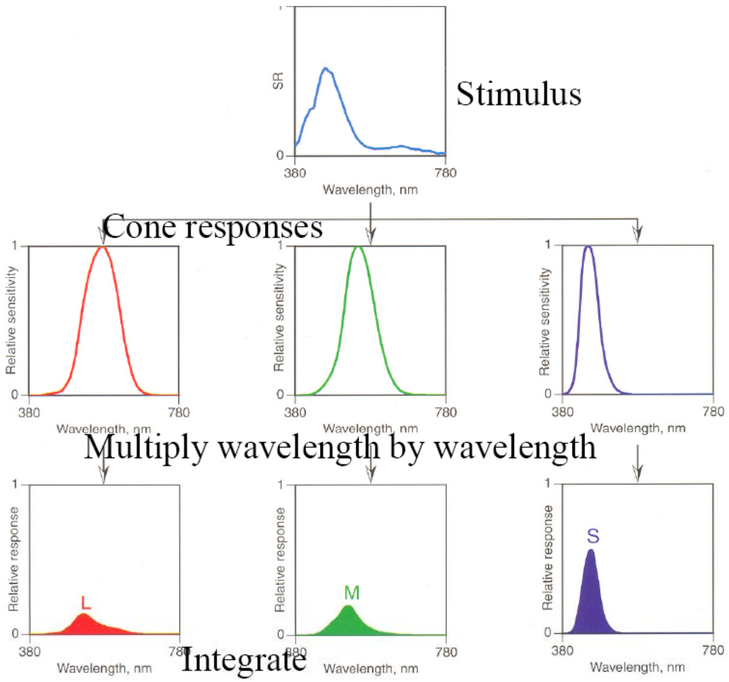
Color perception



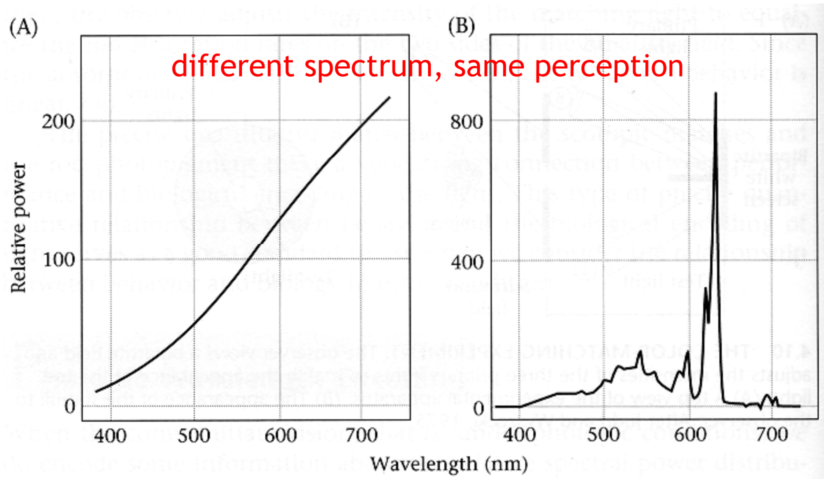
Rods and cones act as filters on the spectrum

- To get the output of a filter, multiply its response curve by the spectrum, integrate over all wavelengths
  - Each cone yields one number
  - Q: How can we represent an entire spectrum with 3 numbers?
  - A: We can't! Most of the information is lost.
    - As a result, two different spectra may appear indistinguishable
      - » such spectra are known as **metamers**

Slide by Steve Seitz

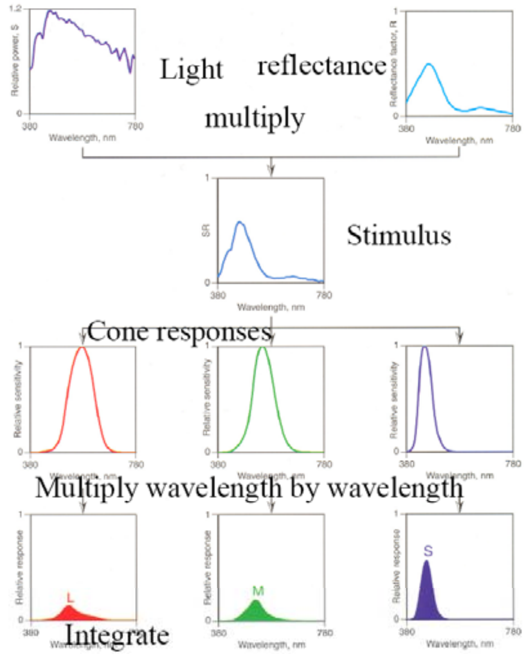


Metamers

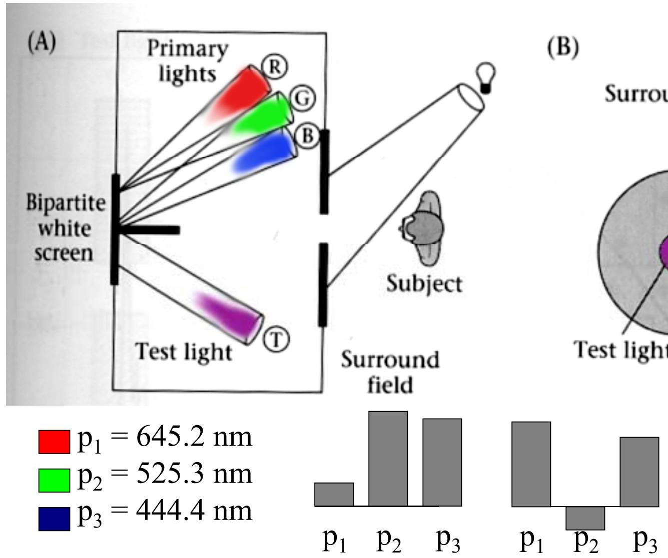


tungsten (鎢絲) bulb

television monitor

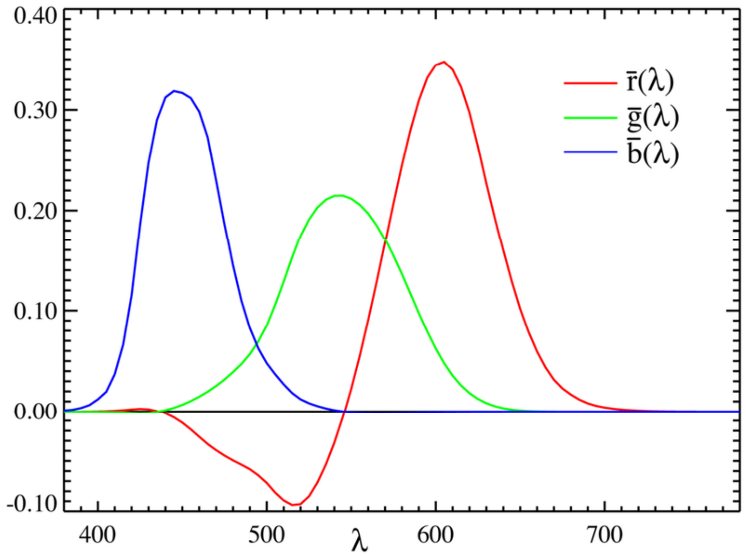


Color matching experiment



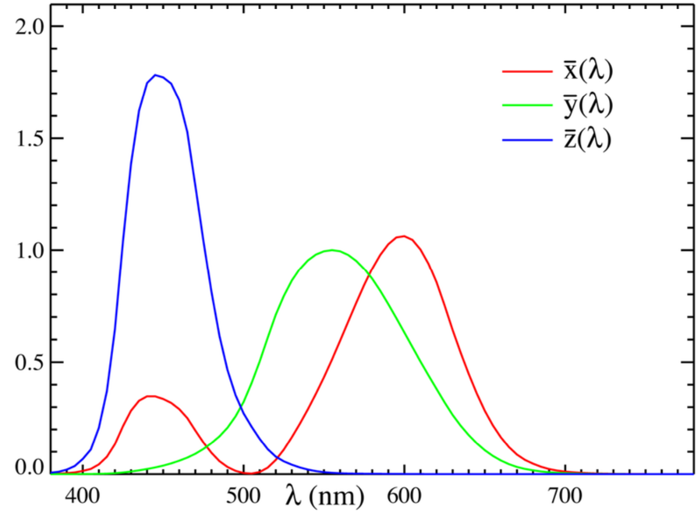
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Color matching experiment



Color matching experiment

- To avoid negative parameters



Spectrum

- In **core/spectrum.***
- Two representations: **RGBSpectrum** (default) and **SampledSpectrum**
- The selection is done at compile time with a **typedef** in **core/pbrt.h**

```
typedef RGBSpectrum Spectrum;
```

- Both stores a fixed number of samples at a fixed set of wavelengths.

CoefficientSpectrum

```
template <int nSamples>
class CoefficientSpectrum {
  +=, +, -, /, *, *= (CoefficientSpectrum)
  ==, != (CoefficientSpectrum)
  IsBlack, Clamp
  *, *=, /, /= (float)
protected:
  float c[nSamples];
}

Sqrt, Pow, Exp
```

SampledSpectrum

- Represents a SPD with uniformly spaced samples between a starting and an ending wavelength (400 to 700 nm for HVS). The number of samples, 30, is generally more than enough.

```
static const int sampledLambdaStart = 400;
static const int sampledLambdaEnd = 700;
static const int nSpectralSamples = 30;
```

SampledSpectrum

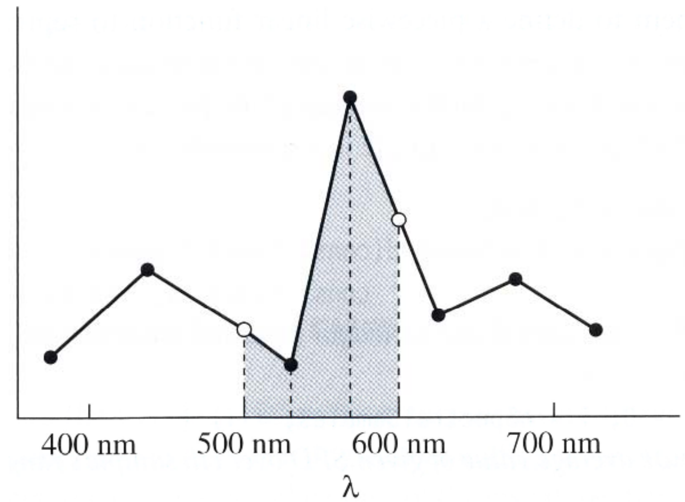
```
class SampledSpectrum : public
  CoefficientSpectrum<nSpectralSamples> {
  …
}
```

It is possible to convert SPD with irregular spaced samples and more or fewer samples into a **SampledSpectrum**. For example, sampled BRDF.

SampledSpectrum

```
static SampledSpectrum FromSampled(
        float *lambda, float *v, int n) {
  <Sort samples if unordered>
  SampledSpectrum r;
  for (int i = 0; i<nSpectralSamples; ++i) {
    lambda0=Lerp(i/float(nSpectralSamples),
      sampledLambdaStart, sampledLambdaEnd);
    lambda1=Lerp((i+1)/float(nSpectralSamples),
      sampledLambdaStart, sampledLambdaEnd);
    r.c[i]=AverageSpectrumSamples(lambda,
            v, n, lambda0, lambda1);
  }
  return r;
}
```

AverageSpectrumSamples



Human visual system

- Tristimulus theory: all visible SPDs S can be accurately represented for human observers with three values, x_λ, y_λ and z_λ.
- The basis are the *spectral matching curves*, X(λ), Y(λ) and Z(λ) determined by CIE (國際照明委員會).

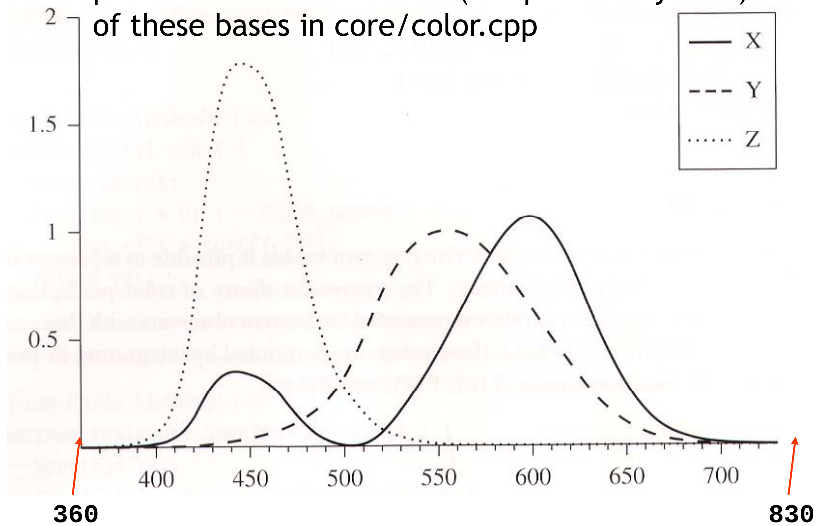
$$x_\lambda = \int_\lambda S(\lambda)X(\lambda)d\lambda$$

$$y_\lambda = \int_\lambda S(\lambda)Y(\lambda)d\lambda$$

$$z_\lambda = \int_\lambda S(\lambda)Z(\lambda)d\lambda$$

XYZ basis

pbrt has discrete versions (sampled every 1nm) of these bases in core/color.cpp



XYZ color

- Good for representing visible SPD to human observer, but not good for spectral computation.
- A product of two SPD's XYZ values is likely different from the XYZ values of the SPD which is the product of the two original SPDs.
- It is frequent to convert our samples into XYZ
- In **`Init()`**, we initialize the following

```
static SampledSpectrum X, Y, Z;
static float yint;
```

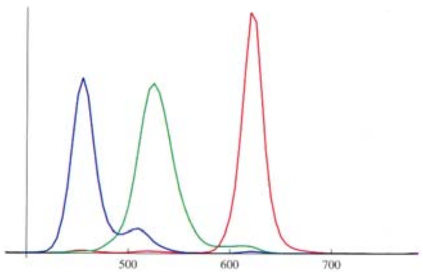
`X.c[i]` stores the sum of X function within the ith wavelength interval using **`AverageSpectrumSamples`**

`yint` stores the sum of **`Y.c[i]`**

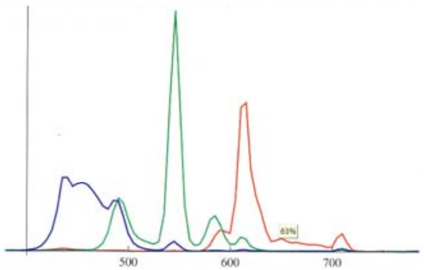
XYZ color

```
void ToXYZ(float xyz[3]) const {
  xyz[0] = xyz[1] = xyz[2] = 0.;
  for (int i = 0; i < nSpectralSamples; ++i)
  {
    xyz[0] += X.c[i] * c[i];
    xyz[1] += Y.c[i] * c[i];
    xyz[2] += Z.c[i] * c[i];
  }
  xyz[0] /= yint;
  xyz[0] /= yint;
  xyz[0] /= yint;
}
```

RGB color

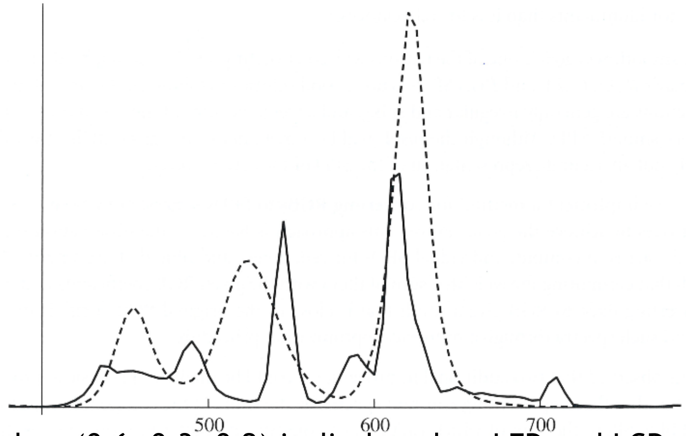


SPD for LCD displays



SPD for LED displays

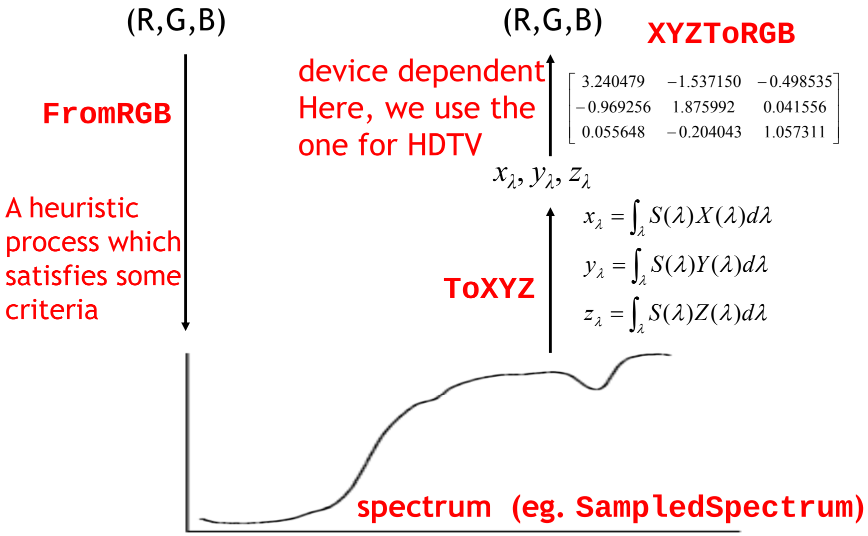
RGB color



SPDs when (0.6, 0.3, 0.2) is displayed on LED and LCD displays

We need to know display characteristics to display the color described by RGB values correctly.

Conversions



$$\begin{bmatrix} 3.240479 & -1.537150 & -0.498535 \\ -0.969256 & 1.875992 & 0.041556 \\ 0.055648 & -0.204043 & 1.057311 \end{bmatrix}$$

$x_\lambda, y_\lambda, z_\lambda$

$$x_\lambda = \int_\lambda S(\lambda)X(\lambda)d\lambda$$

$$y_\lambda = \int_\lambda S(\lambda)Y(\lambda)d\lambda$$

$$z_\lambda = \int_\lambda S(\lambda)Z(\lambda)d\lambda$$

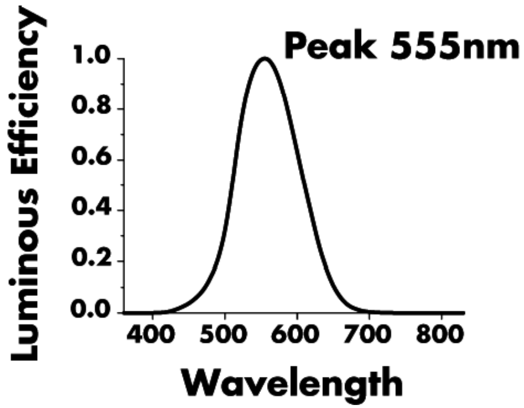
RGBSpectrum

- Note that RGB representation is ill-defined. Same RGB values display different SPDs on different displays. To use RGB to display a specific SPD, we need to know display characteristics first. But, it is convenient, computation and storage efficient.

```
class RGBSpectrum : public
  CoefficientSpectrum<3> {
  using CoefficientSpectrum<3>::c;
  …
}
```

Radiometry

Photometry



Luminance

$$Y = \int V(\lambda)L(\lambda)d\lambda$$

Basic quantities

non-directional

Flux: power, (W)

Irradiance: flux density per area, (W/m^2)

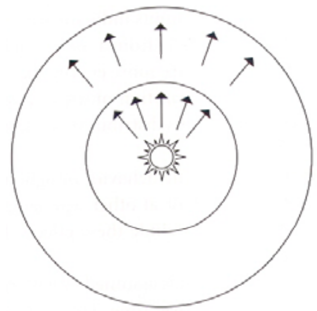
directional

Intensity: flux density per solid angle

Radiance: flux density per solid angle per area

Flux (Φ)

- Radiant flux, power
- Total amount of energy passing through a surface per unit of time (J/s,W)



Irradiance (E)

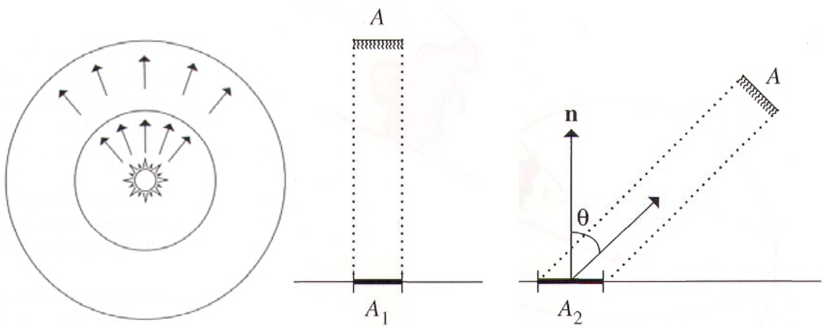
- Area density of flux (W/m^2) $E = \frac{d\Phi}{dA}$

Inverse square law

$$E = \frac{\Phi}{4\pi r^2}$$

Lambert's law

$$E = \frac{\Phi}{A} \qquad E = \frac{\Phi \cos\theta}{A}$$

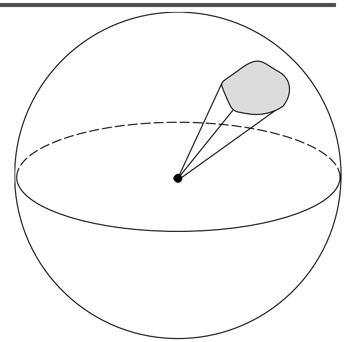


Angles and solid angles

- Angle $\theta = \frac{l}{r}$

  ⇒ **circle has 2π radians**

- Solid angle $\Omega = \frac{A}{R^2}$



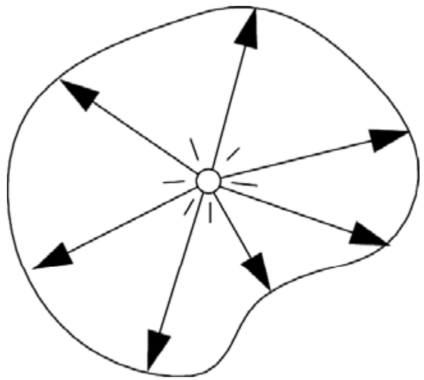
The solid angle subtended by a surface is defined as the surface area of a unit sphere covered by the surface's projection onto the sphere.

⇒ **sphere has 4π steradians**

Intensity (I)

- Flux density per solid angle $I = \frac{d\Phi}{d\omega}$
- Intensity describes the directional distribution of light

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

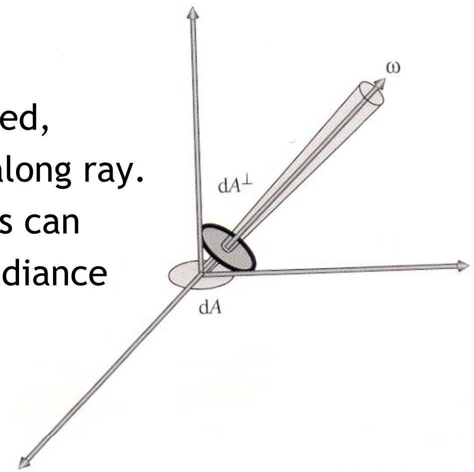


Radiance (L)

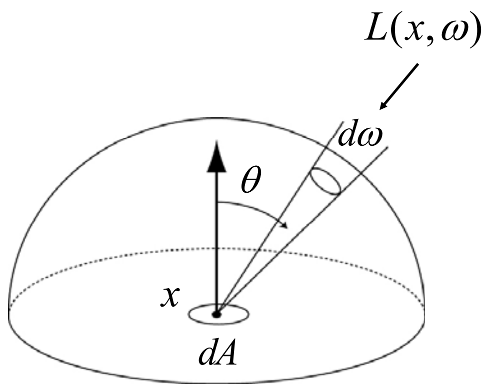
- Flux density per unit area per solid angle

$$L = \frac{d\Phi}{d\omega dA^{\perp}}$$

- Most frequently used, remains constant along ray.
- All other quantities can be derived from radiance



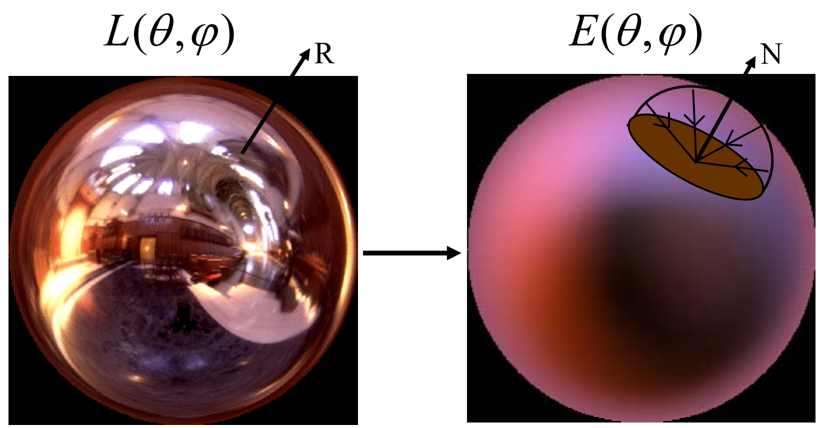
Calculate irradiance from radiance



Light meter

$$E(x) = \frac{d\Phi}{dA} = \int_{\Omega} L(x,\omega)\cos\theta d\omega$$

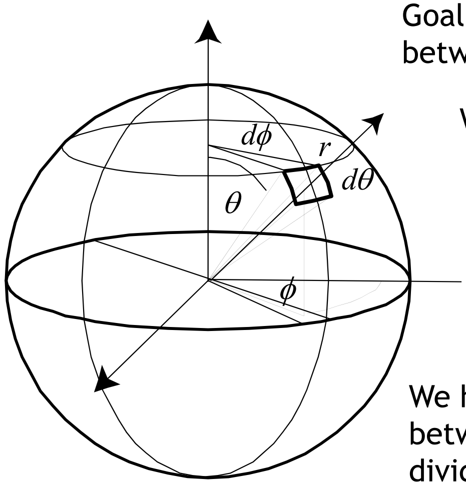
Irradiance Environment Maps



Radiance Environment Map

Irradiance Environment Map

Differential solid angles



Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

Why? In the integral,

$$\int_{S^2} f(\omega) d\omega$$

$d\omega$ is uniformly divided. To convert the integral to

$$\int\int f(\theta,\phi) d\theta d\phi$$

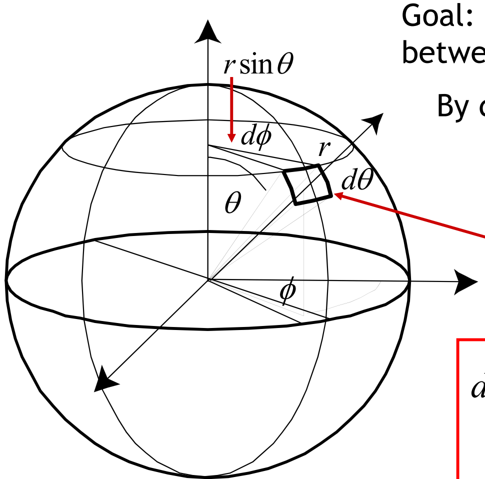
We have to find the relationship between $d\omega$ and uniformly divided $d\theta$ and $d\phi$.

Differential solid angles

- Can we find the surface area of a unit sphere by $\int_0^{2\pi}\int_0^{\pi} d\theta d\phi$?

$$\int_0^{2\pi}\int_0^{\pi} d\theta d\phi =$$

Differential solid angles



Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

By definition, we know that

$$d\omega = \frac{dA}{r^2}$$

$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$

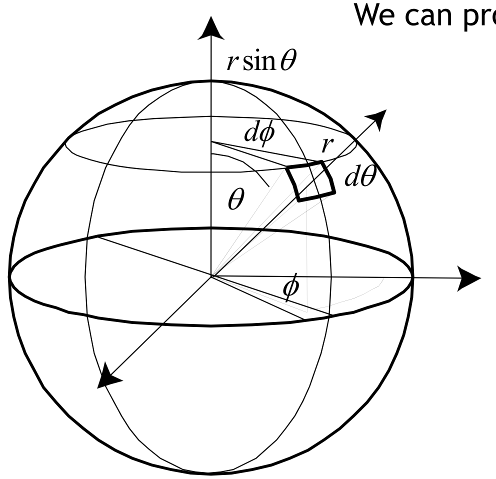
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

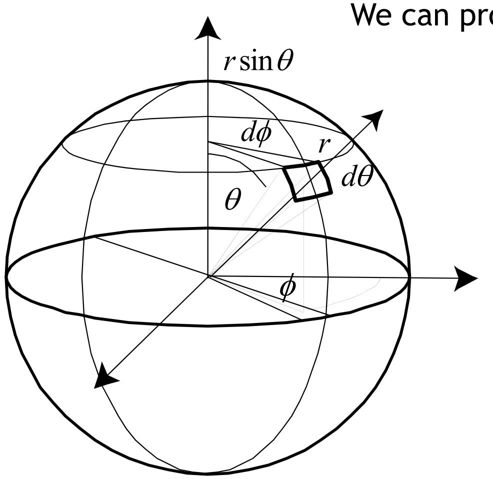
$$= -d\cos\theta d\phi$$

Differential solid angles

We can prove that $\Omega = \int_{S^2} d\omega = 4\pi$



Differential solid angles



We can prove that $\Omega = \int_{S^2} d\omega = 4\pi$

$$\Omega = \int_{S^2} d\omega$$

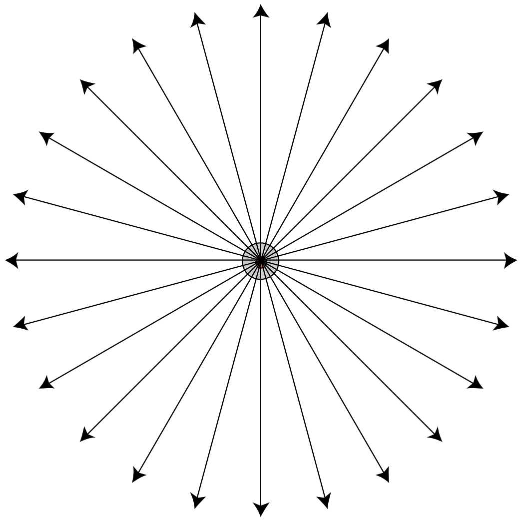
$$= \int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta$$

$$= 2\pi \int_1^{-1} -d\cos\theta$$

$$= 4\pi$$

Isotropic point source



If the total flux of the light source is Φ, what is the intensity?

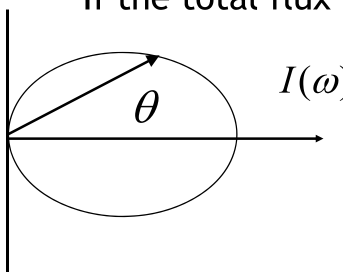
$$\Phi = \int_{S^2} I\, d\omega$$

$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Warn's spotlight

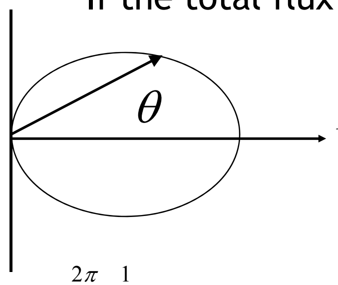
If the total flux is Φ, what is the intensity?



$$I(\omega) \propto \cos^S \theta$$

Warn's spotlight

If the total flux is Φ, what is the intensity?



$$I(\omega) = \begin{cases} c\cos^S\theta & \theta < \pi/2 \\ 0 & otherwise \end{cases}$$

$$\Phi = c\int_0^{2\pi}\int_0^{1} \cos^S\theta d\cos\theta d\phi = 2\pi c \int_0^1 \cos^S\theta d\cos\theta$$

$$= 2\pi c \left.\frac{y^{S+1}}{S+1}\right|_{y=0}^{y=1} = \frac{2\pi c}{S+1} \longrightarrow c = \frac{S+1}{2\pi}\Phi$$