

Color and Radiometry

Digital Image Synthesis

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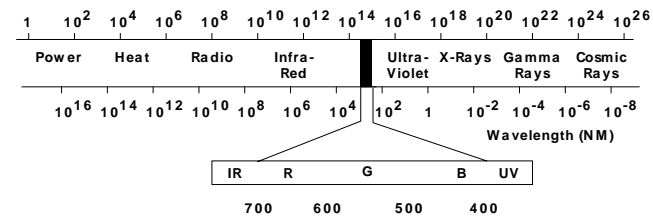
10/19/2006

with slides by Pat Hanrahan and Matt Pharr

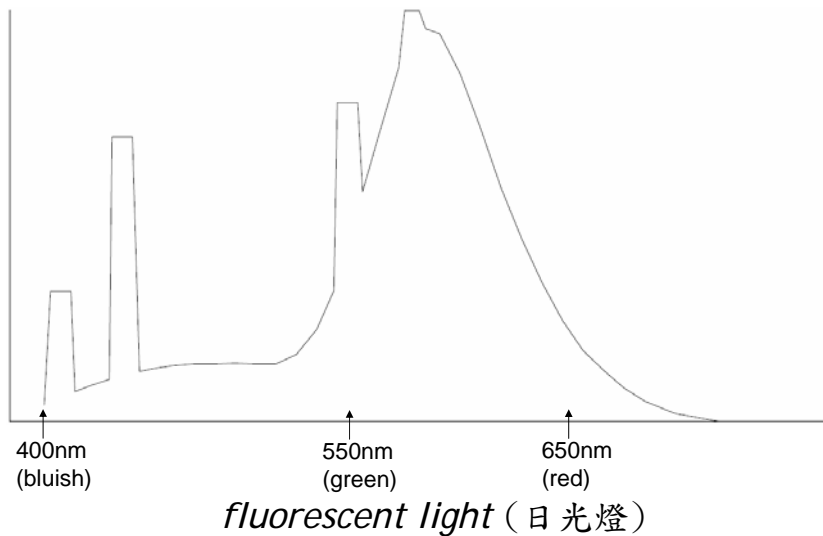


Radiometry

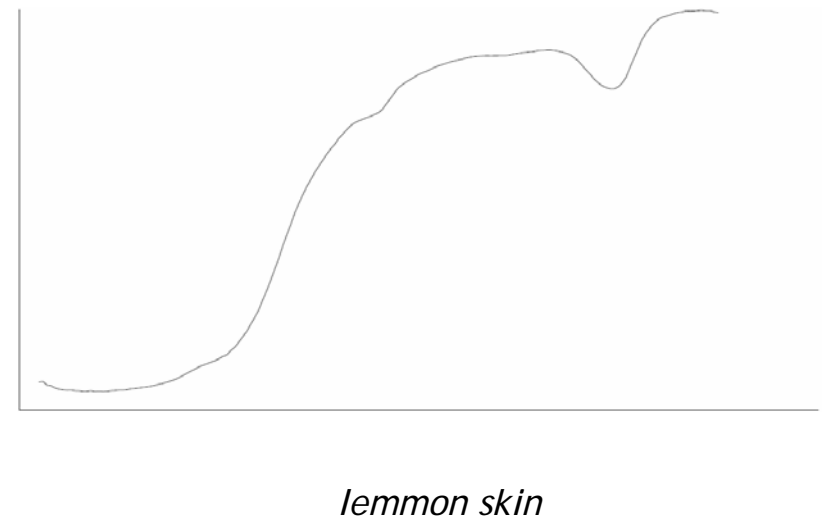
- Radiometry: study of the propagation of electromagnetic radiation in an environment
- Four key quantities: flux, intensity, irradiance and radiance
- These radiometric quantities are described by their spectral power distribution (SPD)
- Human visible light ranges from 370nm to 730nm



Spectral power distribution



Spectral power distribution



Color



- Need a compact, efficient and accurate way to represent functions like these
- Find proper basis functions to map the infinite-dimensional space of all possible SPD functions to a low-dimensional space of coefficients
- For example, $B(\lambda)=1$, a bad approximation

Spectrum



- In `core/color.*`
- Not a plug-in, to use inline for performance
- `Spectrum` stores a fixed number of samples at a fixed set of wavelengths. Better for smooth functions. **Why is this possible? Human vision system**

```
#define COLOR_SAMPLE 3 We actually sample RGB
class COREDLL Spectrum {
public:
    <arithmetic operations>           component-wise
private:                               + - * / comparison...
    float c[COLOR_SAMPLES];
    ...
}
```

Human visual system



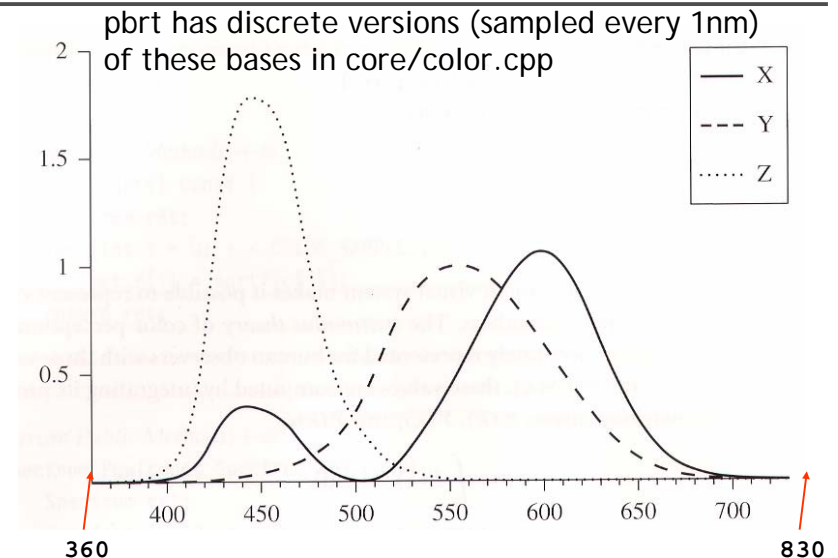
- Tristimulus theory: all visible SPDs can be accurately represented for human observers with three values, x_λ , y_λ and z_λ .
- The basis are the *spectral matching curves*, $X(\lambda)$, $Y(\lambda)$ and $Z(\lambda)$.

$$x_\lambda = \int_\lambda S(\lambda)X(\lambda)d\lambda$$

$$y_\lambda = \int_\lambda S(\lambda)Y(\lambda)d\lambda$$

$$z_\lambda = \int_\lambda S(\lambda)Z(\lambda)d\lambda$$

XYZ basis



XYZ color



- It's, however, not good for spectral computation. A product of two SPD's XYZ values is likely different from the XYZ values of the SPD which is the product of the two original SPDs.
- Hence, we often have to convert our samples (RGB) into XYZ

```
void XYZ(float xyz[3]) const {
    xyz[0] = xyz[1] = xyz[2] = 0.;
    for (int i = 0; i < COLOR_SAMPLES; ++i) {
        xyz[0] += XWeight[i] * c[i];
        xyz[1] += YWeight[i] * c[i];
        xyz[2] += ZWeight[i] * c[i];
    }
}
```

Conversion between XYZ and RGB



```
float Spectrum::XWeight[COLOR_SAMPLES] = {
    0.412453f, 0.357580f, 0.180423f
};
float Spectrum::YWeight[COLOR_SAMPLES] = {
    0.212671f, 0.715160f, 0.072169f
};
float Spectrum::ZWeight[COLOR_SAMPLES] = {
    0.019334f, 0.119193f, 0.950227f
};
Spectrum FromXYZ(float x, float y, float z) {
    float c[3];
    c[0] = 3.240479f * x + -1.537150f * y + -
0.498535f * z;
    c[1] = -0.969256f * x + 1.875991f * y +
0.041556f * z;
    c[2] = 0.055648f * x + -0.204043f * y +
1.057311f * z;
    return Spectrum(c);
}
```

Basic radiometry



- pbrt is based on radiative transfer: study of the transfer of radiant energy based on radiometric principles and operates at the geometric optics level (light interacts with objects much larger than the light's wavelength)
- It is based on the particle model. Hence, **diffraction** and **interference** can't be easily accounted for.

Basic assumptions about light behavior



- **Linearity**: the combined effect of two inputs is equal to the sum of effects
- **Energy conservation**: scattering event can't produce more energy than they started with
- **No polarization**: that is, we only care the frequency of light but not other properties
- **No fluorescence or phosphorescence**: behavior of light at a wavelength doesn't affect the behavior of light at other wavelengths
- **Steady state**: light is assumed to have reached equilibrium, so its radiance distribution isn't changing over time.

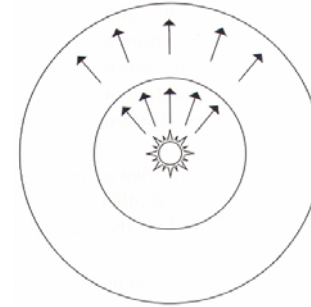
Fluorescent materials



Flux (Φ)



- Radiant flux, power
- Total amount of energy passing through a surface per unit of time (J/s,W)



Irradiance (E)



- Area density of flux (W/m^2) $E = \frac{d\Phi}{dA}$

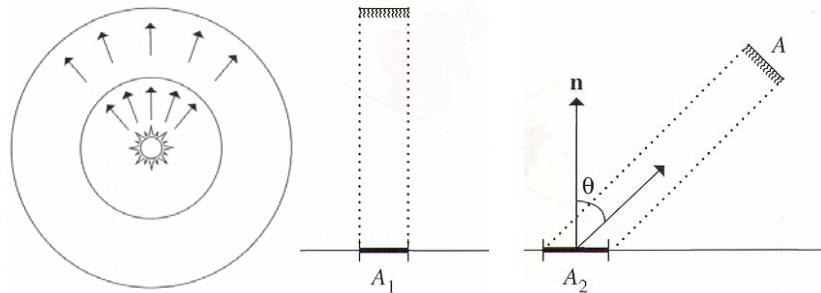
Inverse square law

$$E = \frac{\Phi}{4\pi r^2}$$

Lambert's law

$$E = \frac{\Phi}{A}$$

$$E = \frac{\Phi \cos \theta}{A}$$



Angles and Solid Angles



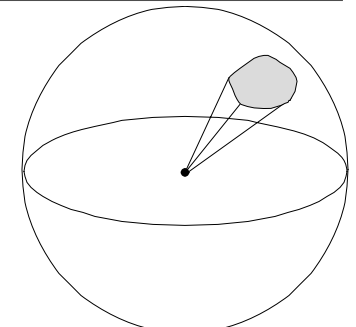
- Angle $\theta = \frac{l}{r}$

⇒ circle has 2π radians

- Solid angle $\Omega = \frac{A}{R^2}$

The solid angle subtended by a surface is defined as the surface area of a unit sphere covered by the surface's projection onto the sphere.

⇒ sphere has 4π steradians

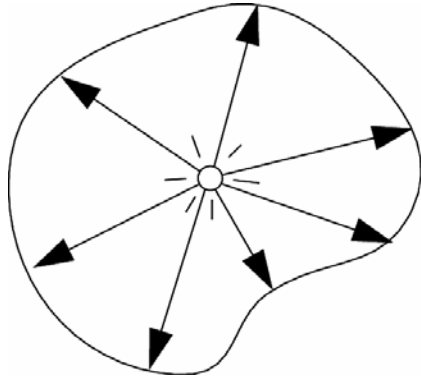


Intensity (I)



- Flux density per solid angle $I = \frac{d\Phi}{d\omega}$
- Intensity describes the directional distribution of light

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$



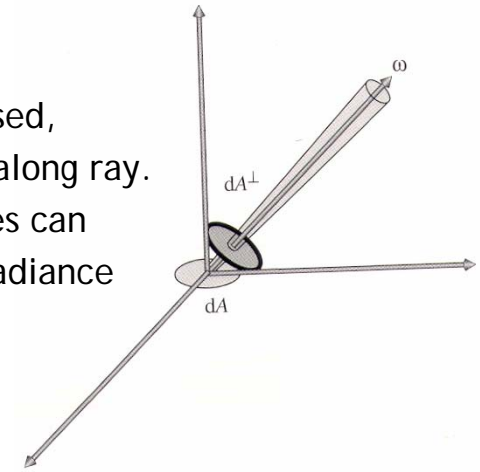
Radiance (L)



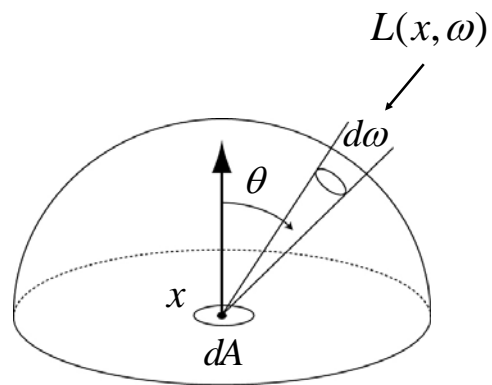
- Flux density per unit area per solid angle

$$L = \frac{d\Phi}{d\omega dA^\perp}$$

- Most frequently used, remains constant along ray.
- All other quantities can be derived from radiance



Calculate irradiance from radiance

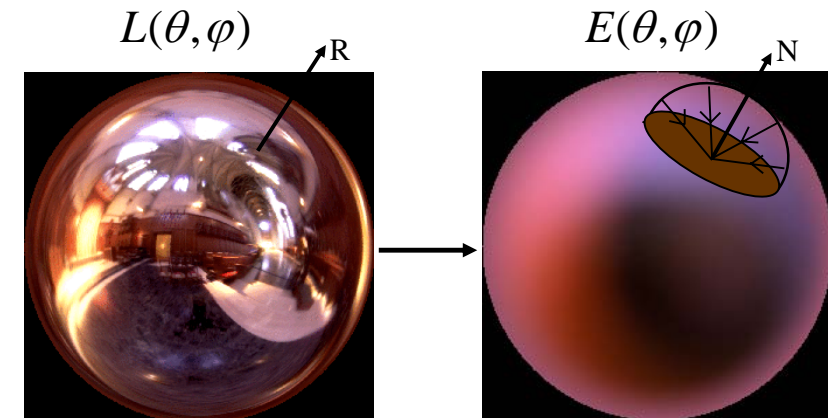


$$E(x) = \frac{d\Phi}{dA} = \int_{\Omega} L(x, \omega) \cos \theta d\omega$$



Light meter

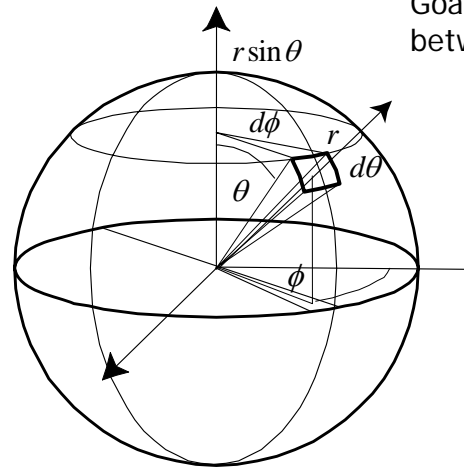
Irradiance Environment Maps



Radiance Environment Map

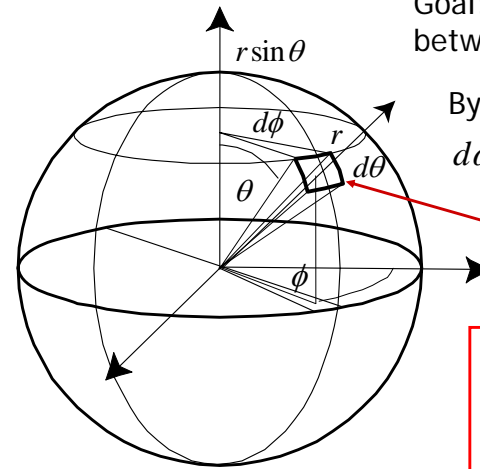
Irradiance Environment Map

Differential solid angles



Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

Differential solid angles



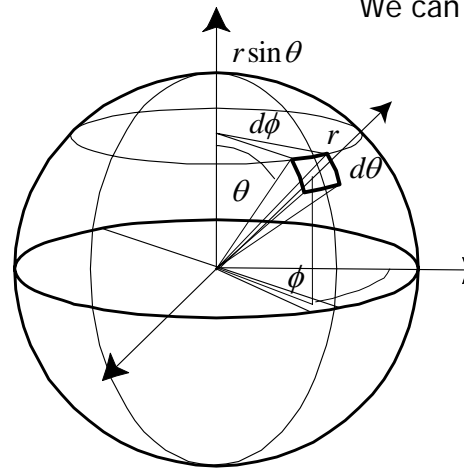
Goal: find out the relationship between $d\omega$ and $d\theta$, $d\phi$

By definition, we know that $d\omega = dA$ when $r = 1$

$$dA = (r d\theta)(r \sin \theta d\phi) \\ = r^2 \sin \theta d\theta d\phi$$

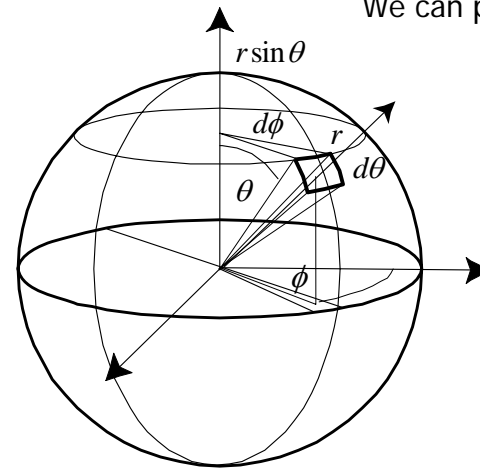
$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi \\ = -d \cos \theta d\phi$$

Differential solid angles



We can prove that $\Omega = \int_{s^2} d\omega = 4\pi$

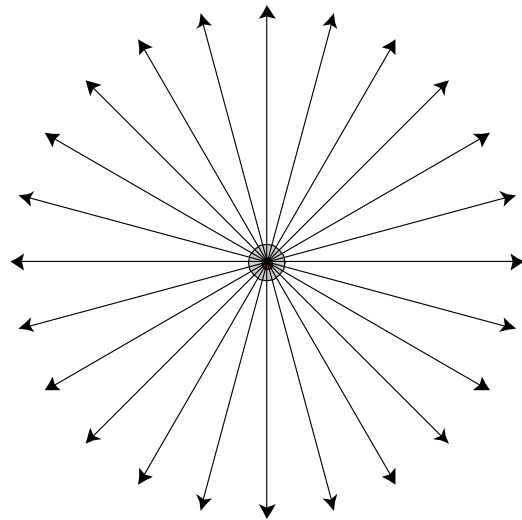
Differential solid angles



We can prove that $\Omega = \int_{s^2} d\omega = 4\pi$

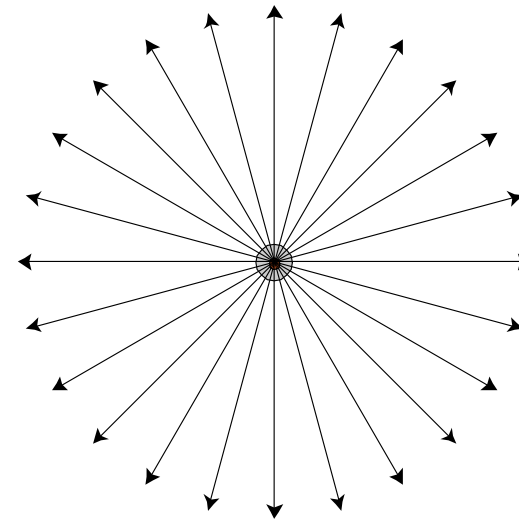
$$\Omega = \int_{s^2} d\omega \\ = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi \\ = \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \\ = 2\pi \int_1^{-1} -d \cos \theta \\ = 4\pi$$

Isotropic point source



If the total flux of the light source is Φ , what is the intensity?

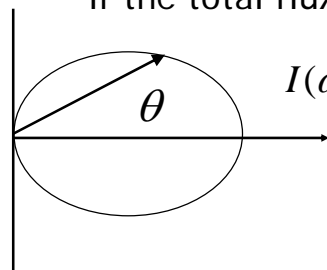
Isotropic point source



If the total flux of the light source is Φ , what is the intensity?

$$\begin{aligned}\Phi &= \int_{S^2} I d\omega \\ &= 4\pi I \\ I &= \frac{\Phi}{4\pi}\end{aligned}$$

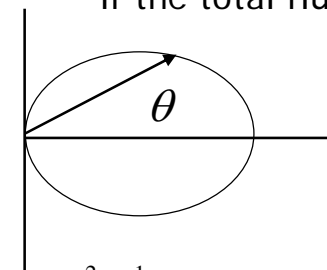
Warn's spotlight



If the total flux is Φ , what is the intensity?

$$I(\omega) \propto \cos^S \theta$$

Warn's spotlight

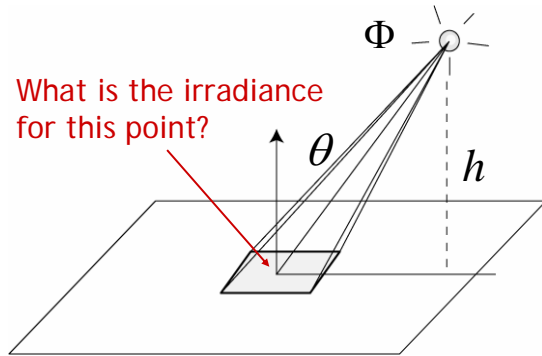


If the total flux is Φ , what is the intensity?

$$I(\omega) = \begin{cases} c \cos^S \theta & \theta \geq \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

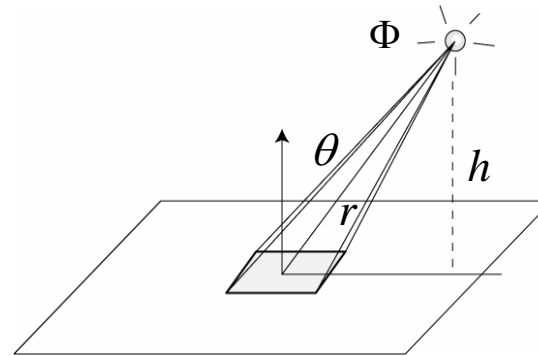
$$\begin{aligned}\Phi &= c \int_0^{2\pi} \int_0^1 \cos^S \theta d \cos \theta d\phi = 2\pi c \int_0^1 \cos^S \theta d \cos \theta \\ &= 2\pi c \frac{y^{S+1}}{S+1} \Big|_{y=0}^{y=1} = \frac{2\pi c}{S+1} \longrightarrow c = \frac{S+1}{2\pi} \Phi\end{aligned}$$

Irradiance: isotropic point source



What is the irradiance for this point?

Irradiance: isotropic point source



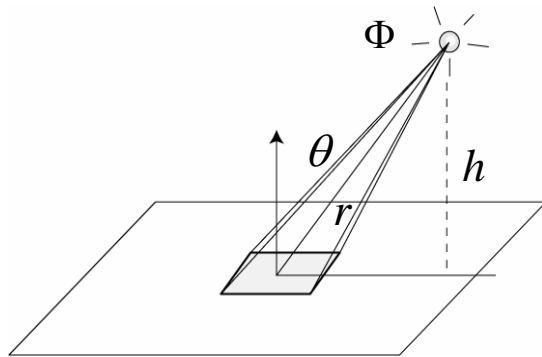
$$r = \frac{h}{\cos \theta}$$

$$I = \frac{d\Phi}{d\omega} = \frac{\Phi}{4\pi}$$

$$E = \frac{d\Phi}{dA} = \frac{Id\omega}{dA} = \frac{\Phi}{4\pi} \frac{d\omega}{dA} = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2}$$

Lambert law
Inverse square law

Irradiance: isotropic point source

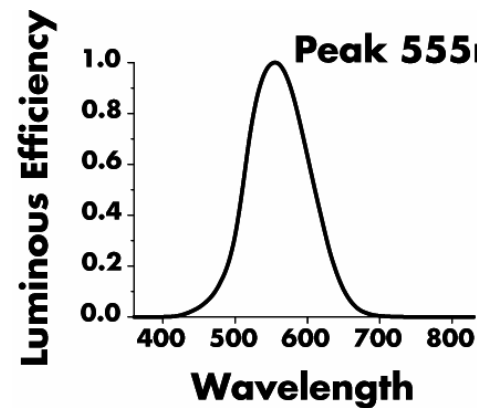


$$r = \frac{h}{\cos \theta}$$

$$I = \frac{d\Phi}{d\omega} = \frac{\Phi}{4\pi}$$

$$E = \frac{d\Phi}{dA} = \frac{Id\omega}{dA} = \frac{\Phi}{4\pi} \frac{d\omega}{dA} = \frac{\Phi}{4\pi} \frac{\cos^3 \theta}{h^2}$$

Photometry



Luminance

$$Y = \int V(\lambda)L(\lambda)d\lambda$$