

# Realistic Camera Model

Shan-Yung Yang

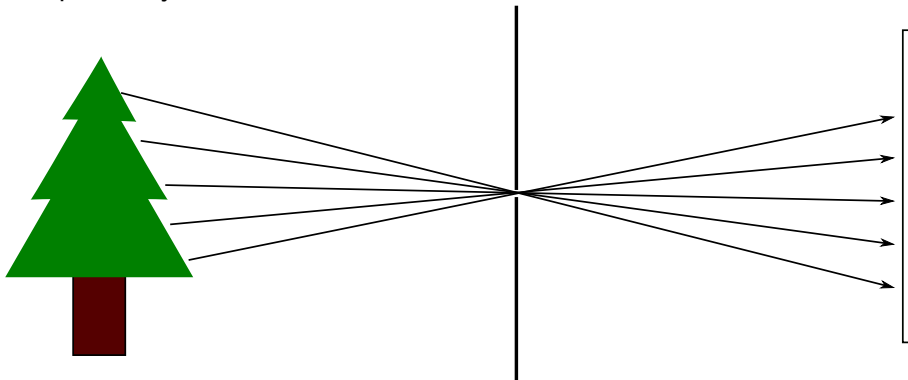
November 2, 2006

# Outline

- Introduction
- Lens system
- Thick lens approximation
- Radiometry
- Sampling
- Assignment #2

# Introduction

Until now we have only discussed the pinhole camera model, which is not physically correct.



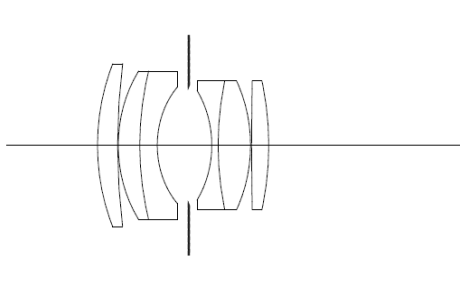
# Why We Need Realistic Model

- Physical correctness is our goal.
- Combining real images with synthetic ones is very common in digital visual effects.
- Machine vision and scientific applications need to simulate camera correctly.
- Users of 3D graphics system are familiar with cameras.

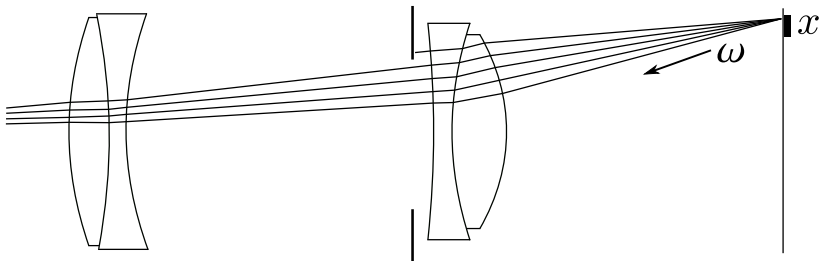
# Lens Systems

Lens systems are typically constructed from a series of individual spherical lenses.

| radius  | thick  | $n_d$ | V-no | ap   |
|---------|--------|-------|------|------|
| 58.950  | 7.520  | 1.670 | 47.1 | 50.4 |
| 169.660 | 0.240  |       |      | 50.4 |
| 38.550  | 8.050  | 1.670 | 47.1 | 46.0 |
| 81.540  | 6.550  | 1.699 | 30.1 | 46.0 |
| 25.500  | 11.410 |       |      | 36.0 |
|         | 9.000  |       |      | 34.2 |
| -28.990 | 2.360  | 1.603 | 38.0 | 34.0 |
| 81.540  | 12.130 | 1.658 | 57.3 | 40.0 |
| -40.770 | 0.380  |       |      | 40.0 |
| 874.130 | 6.440  | 1.717 | 48.0 | 40.0 |
| -79.460 | 72.228 |       |      | 40.0 |



# Measurement Equation



$$R = \int \int \int \int L(T(x, \omega, \lambda); \lambda) S(x, t) P(x, \lambda) \cos \theta \, dx \, d\omega \, dt \, d\lambda$$

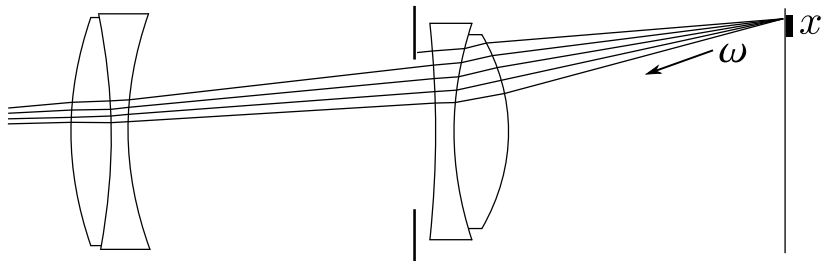
$L$ : radiance

$T$ : image to object space transformation

$S$ : shutter function

$P$ : sensor response characteristics

# Measurement Equation



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Solution: Monte Carlo method:

$$\int_{\Omega} f(x) dx \approx \left[ \frac{1}{N} \sum_{i=1}^N f(x_i) \right] \cdot \int_{\Omega} dx$$

where  $x_1, x_2, \dots, x_N$  are uniform distributed random samples in  $\Omega$ .

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- 3 Shoot the ray according to the result of  $T(x_i, \omega_i)$  into the scene, and calculate the radiance.

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- 4 Set the pixel value to the average of radiance.

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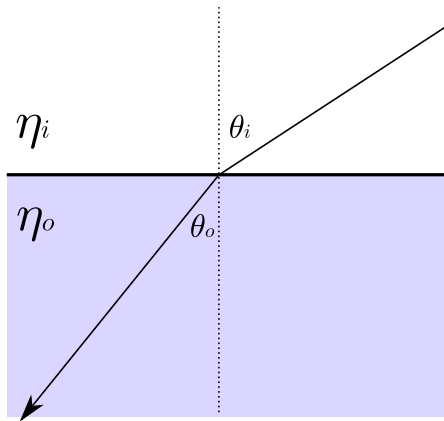
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  - 2 Compute the new direction by Snell's law if the medium is different.

# Snell's Law



$$\eta_i \sin \theta_i = \eta_o \sin \theta_o$$

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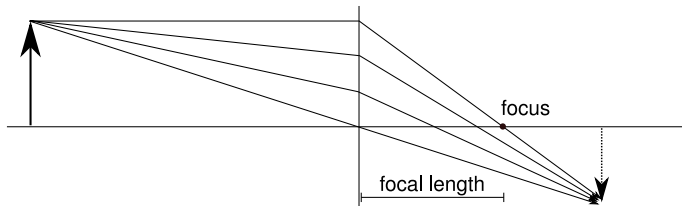
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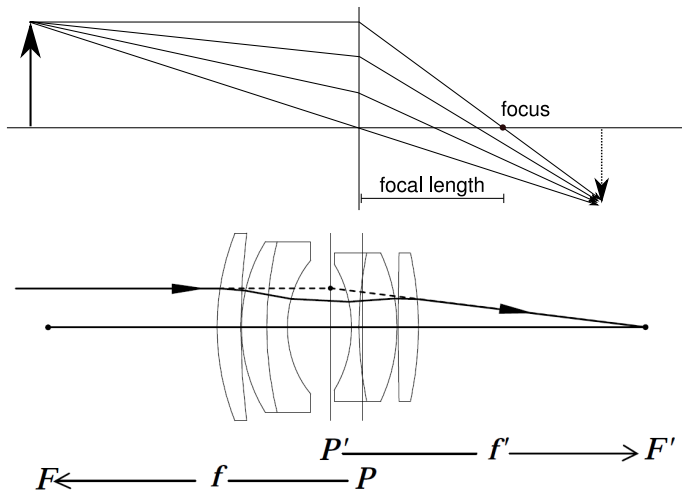
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  - ▶ Ideal lens: each point in object space is imaged onto a single point in the image space.
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- Thick lens approximation has additional parameter of thickness.

# Thin Lens and Thick Lens

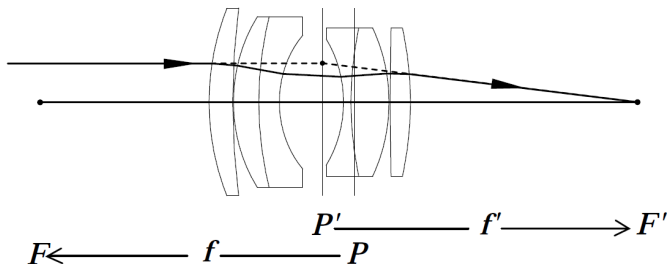




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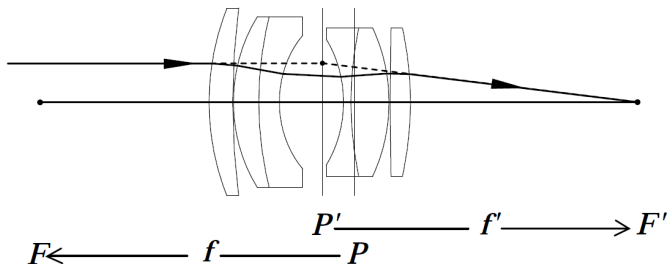


# Finding Thick Lens Approximation



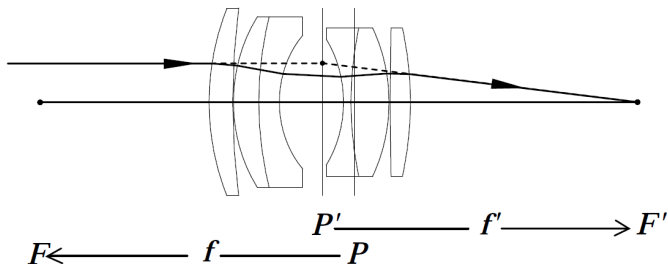
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- 1 Shoot a ray parallel to the axis to find the focus.
- 2 Find the principal plane by intersecting the refracted ray and parallel one.
- 3 Find the secondary principal plane by tracing from another side.

# Application of Thick Lens Approximation

- A faster way to calculate  $T(x_i, \omega_i)$ .

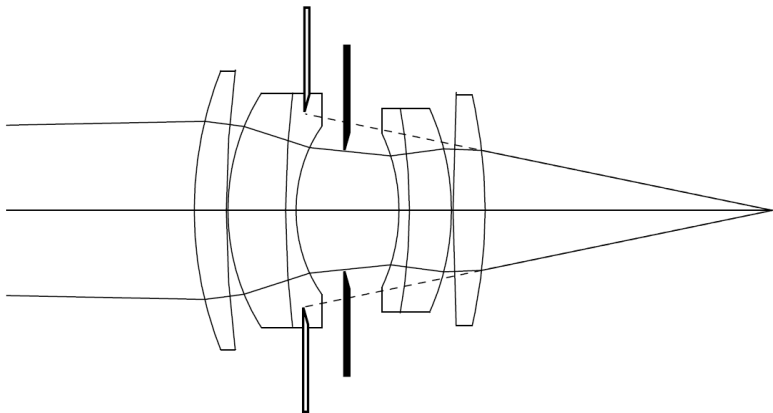
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- Calculate the **exit pupil**.

## Exit Pupil (1/2)



The exit pupil is the effective aperture stop in the image space which allows ray incidence.



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- Finding the exit pupil:
  - 1 For each aperture stop, calculate its image by thick lens approximation.
  - 2 Find the aperture stop whose image subtends the smallest solid angle.
- You may also use the aperture of the nearest lens as the exit pupil.

# Exposure

- Assume that the irradiance is constant over the exposure period:

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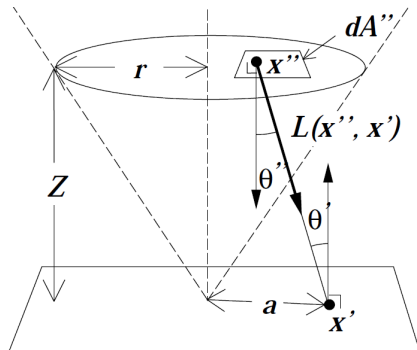
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- In practice, we only need to integrate over the **exit pupil** instead of the whole semisphere.
- Let

$$E(x') = \int L(T(x', \omega)) \cos \theta' d\omega$$

$$R = \Delta t \cdot \int E(x') dx'$$

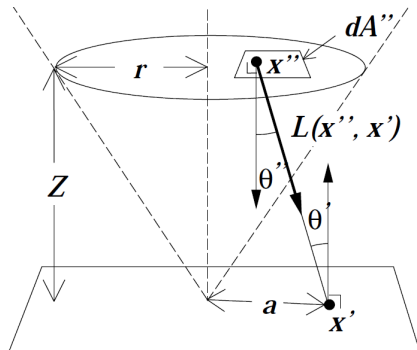
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$$E(x') = \int_{x'' \in D} L(x'', x') \cos \theta' d\omega$$

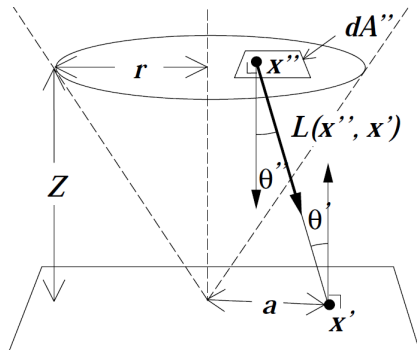


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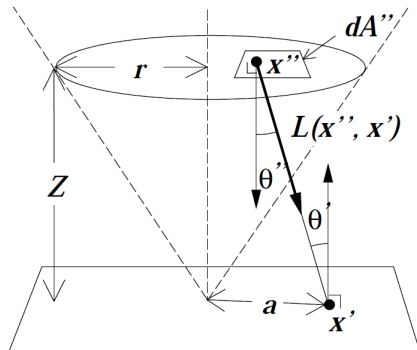
$$E(x') = \int_{x'' \in D} L(x'', x') \cos \theta' d\omega$$
$$d\omega = \frac{\cos \theta''}{r^2} dA''$$

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$$E(x') = \int_{x'' \in D} L(x'', x') \frac{\cos \theta' \cos \theta''}{\|x'' - x'\|^2} dA''$$

# Integral over the Exit Pupil



$$E(x') = \frac{1}{Z^2} \int_{x'' \in D} L(x'', x') \cos^4 \theta' dA''$$

# Sampling a Disk Uniformly

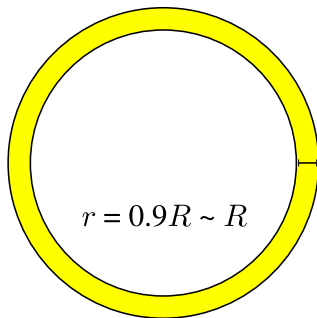
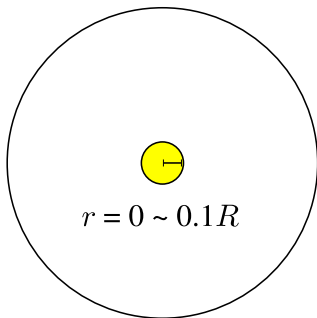
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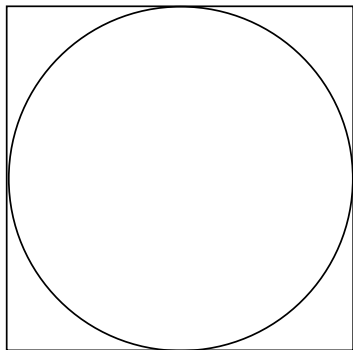
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  - ▶ The result is not uniform due to coordinate transformation.



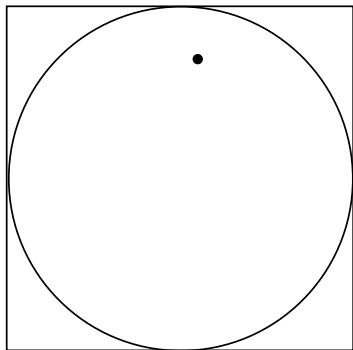
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- 1 Uniformly sample a point in the bounding square of the disk.
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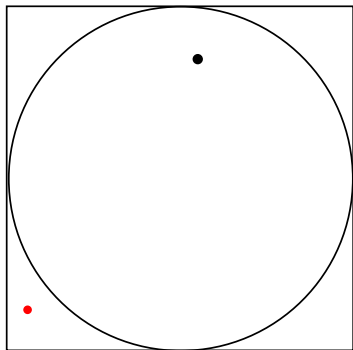
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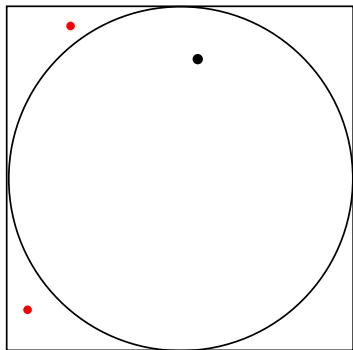
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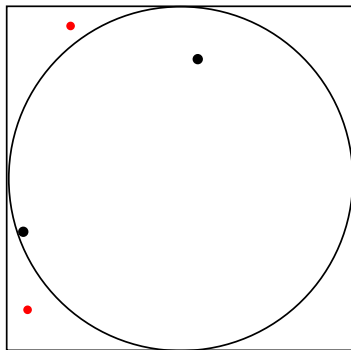
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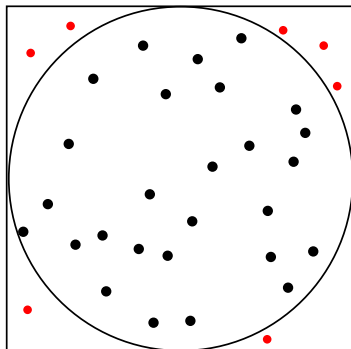
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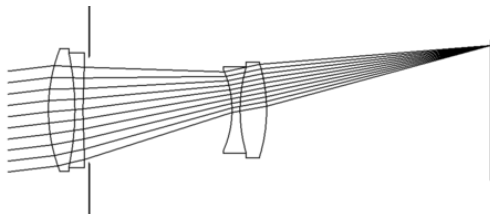
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- This produce uniform samples on a disk after coordinate transformation. We will prove it later in chapter 14 “Monte Carlo integration”.

# Results



200mm Telescope

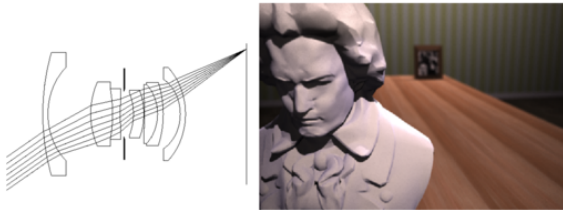


# Results



50mm General

# Results



35mm wide-angle

# Results



16mm Fisheye

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  - ▶ PBRT generate rays by calling `GenerateRay()`, which is a virtual function of `Camera`.

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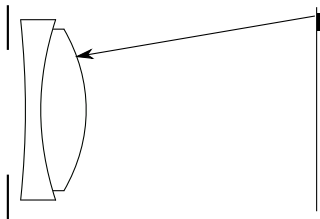
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  - ▶ PBRT will give you pixel location in `sample`.
  - ▶ You need to fill the content of `ray` and return a value for its weight.

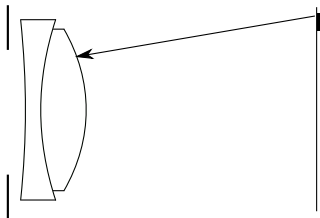


# Inside GenerateRay()



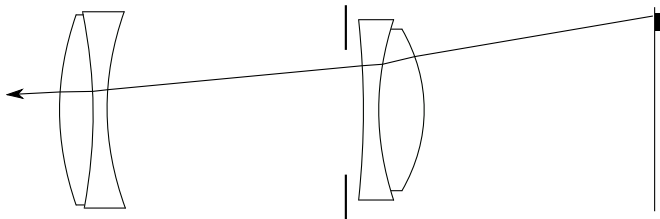
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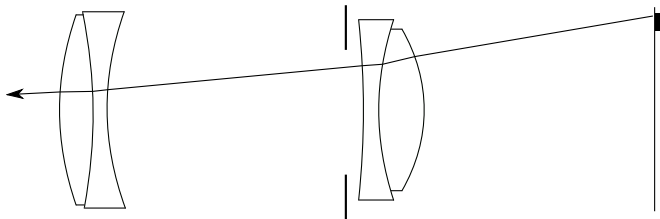
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- 3 Fill `ray` with the result and return  $\frac{\cos^4 \theta'}{Z^2}$  as its weight.

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- The rendered result.
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- My email address:
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  - [littleshan@gmail.com](mailto:littleshan@gmail.com)