

## Homework 1

October 4, 2012

Due date: October 17, 2012

- (12%) What is the binary representation of the following hexadecimal numbers? What are the decimal numbers they represent when interpreted as unsigned and signed integers.
  - 5C
  - B2
- (8%) Prove that (a) {NOT, OR} and (b) {NOR} are universal.
- (10%) Prove (a)  $A + \overline{A}B = A + B$  (b)  $(A+B)(A+C)=A+BC$  both algebraically and using the truth table.
- (10%) A leading-1 circuit has three inputs  $A_2, A_1, A_0$  and two outputs  $Z_1, Z_0$ . The circuit returns the binary encoding of the position of the leading 1-bit. For example, if the input is 010, the output is 01 since the first 1 appears at  $A_1$ . For another example, for input 101, the output is 10 (because  $A_2$  contains the leading 1-bit). If there is no 1 in the inputs (000), it returns 11. Implement the circuit with logic gates and components in the class.
- (20%) Design a binary multiplier that multiplies two 3-bit unsigned integers,  $X = X_2X_1X_0$  and  $Y = Y_2Y_1Y_0$ , and a 6-bit output  $Z = Z_5Z_4 \dots Z_0$  and  $Z = X \times Y$ , where  $X_0, Y_0$  and  $Z_0$  are LSBs. You may use the notation  $X[n..m]$  to identify a portion of wires. For example,  $X[2..1]$  means the set of wires,  $X_2X_1$ .
- (20%) Design an 8-bit increase-by-k circuit. The circuit is similar to the counter we mentioned in the class. However, it is different in two ways. First, it only counts up and the increment is a 4-bit constant  $k$  specified by the user rather than 1 as the counter in the class. Second, it checks for saturation. That is, if the number after increment is larger than 255, it keeps the number before increment. It has the following interface. For inputs, it has an 8-bit input  $D_{in}$ , a 4-bit  $k_{in}$  for the new increment and a 2-bit op for the opcode. The output is the 8-bit content of the counter. Its operations are listed in the following table. (Hint: A full-adder has an output for carry out.)

op[1]	op[0]	operation	semantics
0	0	<b>reset</b>	$C \leftarrow 0$
0	1	<b>load</b>	$C \leftarrow D_{in}$
1	0	<b>set k</b>	$k \leftarrow k_{in}$
1	1	<b>inc</b>	if $(C + k < 256)$ then $C \leftarrow C + k$

- (20%) Refer to the following figure for TOY architecture. Please specify the control signals,  $MUX_A, MUX_B, MUX_C, MUX_D, MUX_E, REG_W, MEM_W$  and  $ALU_{OP}$  during the execution stage for the instructions "shift left", "store", "load indirect" and "jump register". For example, the answer for "jump and link" would be  $MUX_A = 0, MUX_B = *, MUX_C = *, MUX_D = 1, MUX_E = 01, REG_W = 1, MEM_W = 0, ALU_{OP} = *$ . Please note that the numbering of architecture could be different from the one in the class. Please refer to the following figure rather than slides in the class.

