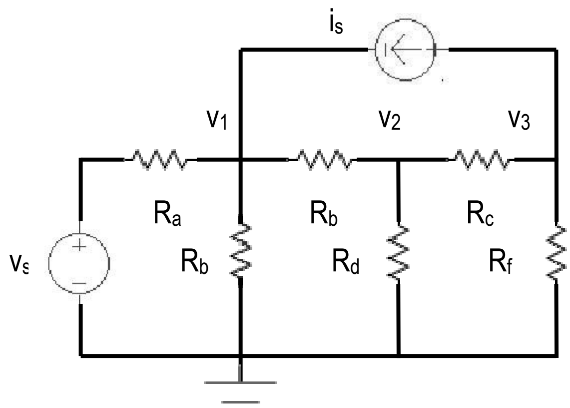
Engineering Mathematics – The Inverse of a Matrix with Complex Entries

- Motivations

When we are tackling a DC circuit, we recall the famous Ohm's law and Kirchhoff's law we learned in high school, and the latter forms the systematic analysis methods to deal with a resistive circuit.

For example,



According to the Kirchhoff's Current Law (KCL) of v_1, we will have

$$G_a(v_1 - v_s) + G_b v_1 + G_c(v_1 - v_2) - i_s = 0$$

By the same token, here come

$$G_c(v_2 - v_1) + G_d v_2 + G_e(v_2 - v_3) = 0$$

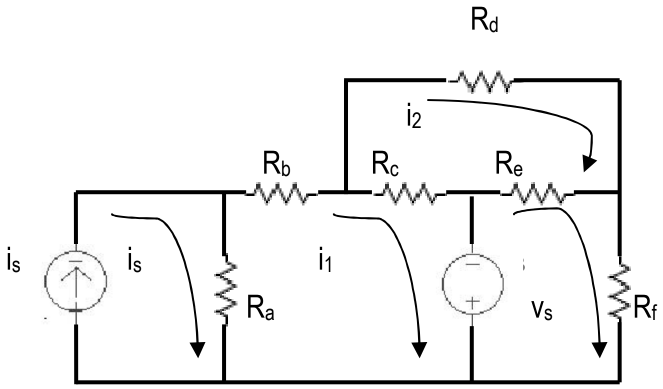
$$G_e(v_3 - v_2) + G_f v_3 + i_s = 0$$

To rearrange the system of equations with matrix, we get

$$\begin{bmatrix} G_a + G_b + G_c & -G_c & 0 \\ -G_c & G_c + G_d + G_e & -G_e \\ 0 & -G_e & G_e + G_f \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} G_a v_s + i_s \\ 0 \\ -i_s \end{bmatrix}$$

In fact, a method exists to finish the matrices without using KCL, but there is no much point to mention the tedious process.

Here is another case.



This time we use Kirchhoff's Voltage Law (KVL) on the mesh rounded by i_1, and we find

$$R_a(i_1 - i_s) + R_b i_1 + R_c(i_1 - i_2) - v_s = 0$$

In a similar manner, we come to the result that

$$R_c(i_2 - i_1) + R_d i_2 + R_e(i_2 - i_3) = 0$$

$$R_e(i_3 - i_2) + R_f i_3 + v_3 = 0$$

Once again, we rewrite these equations into matrices.

$$\begin{bmatrix} R_a + R_b + R_c & -R_c & 0 \\ -R_c & R_c + R_d + R_e & -R_e \\ 0 & -R_e & R_e + R_f \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} R_a i_s + v_s \\ 0 \\ -v_s \end{bmatrix}$$

As is expected, there is a rapid way to fill the matrices out, but we will skip it here.

Compare these two matrix equation with each other, we may find that the coefficient matrices are composed of the resistors or their inverses, the variable matrices consist of branch voltages or currents, and the constant matrices are made up of driving forces. To find the unknowns out, we can use Gaussian reduction or multiply the inverse of the coefficient matrices on both sides of the equation on condition that the matrices themselves are nonsingular. In the following session we will focus mainly on the coefficient matrices.

Things are no longer easy as we apply an AC source to the circuit with either capacitors or inductors. We can still solve the circuit by ODEs based on the property that

$$i_c = C\frac{dv_c}{dt}$$

$$v_L = L\frac{di_L}{dt}$$

However, a smarter way is to use matrices again with the assumption that all branch variables have the same frequency as that of the AC source. We will just take advantage of the conclusion that the equivalent impedance of resistors, capacitors, and inductors are respectively

$$Z_R = R$$

$$Z_c = \frac{1}{j\omega C}$$

$$Z_L = j\omega L$$

With this substitution, we can complete our **complex** coefficient matrices with ease, whereas it is more tiring to solve a complex matrix because its echelon form cannot be calculated so simply as a matrix with real entries. We will consequently find out its inverse matrix to evaluate the unknowns.

If we are facing a nonsingular real matrix, to find its inverse is a piece of cake if we use a calculator, but actually most calculators do not support a complex matrix and that is the reason why this GBA program is developed.

- Features

This program features a GBA interface. In addition, the value of the multiple of each element in the same circuit does not exceed 100 in practical cases. This program supports inputs with real part and imaginary parts ranging from 99.9 to -99.9. Given the value of each entries of the matrix, its simplified inverse complex matrix will be output. Since ARM does not support floating point arithmetic, we multiply each input by 100 to keep the first digit after the decimal point accurate after conducting chop-off division. Before printing the result, we have each number divided by 100 to fit the format demanded.

- Operation

A & B: To increase and to decrease the current digit by one

←&→: To switch digits within the same part of an element

L & R: To switch adjacent parts of the elements

Select: To negate the current part

Start: To start calculating

- Drawbacks

Due to the awkward input style of GBA keyboard, the user will spend much more time pressing button A and B to key in the value of each entry and button L and R to access each element of the matrix. Moreover, the biggest challenge is the setting of output in GBA programming. It will definitely cost tremendous efforts to output an inverse of a 3*3 complex matrix. Therefore the 3*3 version is still developed in pure C.

In terms of Algorithm, on account of the strenuous process to reach the echelon form of a complex matrix, we will instead make use of cofactors in 3*3 or higher-order matrices. To evaluate the determinant via recurrence time plenty of time.

Furthermore, sometimes a larger denominator will make the whole part zero, and that poses a great threat to the accuracy.

- This is a 2-membered group including only B95902024 金迺安 and B95902041 洪晧瑜. One who asserts his belonging to this group is lying.