Training Support Vector Machines: Status and Challenges

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Outline

- SVM is popular
  But its training isn't easy
- We check existing techniques
- Large data sets
  We show several approaches, and discuss various considerations
- Will try to partially answer why there are controversial comparisons

Outline

- Introduction to SVM
- Solving SVM Quadratic Programming Problem
- Training large-scale data
- Linear SVM
- Discussion and Conclusions

Support Vector Classification

- Training data $(\mathbf{x}_i, y_i), i = 1, \ldots, l, \mathbf{x}_i \in R^n, y_i = \pm 1$
- Maximizing the margin
  [Boser et al., 1992, Cortes and Vapnik, 1995]

$$\begin{aligned} \min_{\mathbf{w}, b, \boldsymbol{\xi}} \quad & \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l} \xi_i \\ \text{subject to} \quad & y_i(\mathbf{w}^T\phi(\mathbf{x}_i) + b) \geq 1 - \xi_i, \\ & \xi_i \geq 0, \; i = 1, \ldots, l. \end{aligned}$$

- High dimensional (maybe infinite) feature space

$$\phi(\mathbf{x}) = (\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \ldots).$$

Support Vector Classification (Cont'd)

- **w**: maybe infinite variables
- The dual problem (finite # variables)

$$\begin{aligned} \min_{\boldsymbol{\alpha}} \quad & \frac{1}{2}\boldsymbol{\alpha}^T Q \boldsymbol{\alpha} - \mathbf{e}^T \boldsymbol{\alpha} \\ \text{subject to} \quad & 0 \leq \alpha_i \leq C, i = 1, \ldots, l \\ & \mathbf{y}^T \boldsymbol{\alpha} = 0, \end{aligned}$$

  where $Q_{ij} = y_i y_j \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ and $\mathbf{e} = [1, \ldots, 1]^T$
- At optimum

$$\boxed{\mathbf{w} = \sum_{i=1}^{l} \alpha_i y_i \phi(\mathbf{x}_i)}$$

- Kernel: $\boxed{K(\mathbf{x}_i, \mathbf{x}_j) \equiv \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)}$

Outline

Large Dense Quadratic Programming

$$\begin{aligned} \min_{\boldsymbol{\alpha}} \quad & \frac{1}{2}\boldsymbol{\alpha}^T Q \boldsymbol{\alpha} - \mathbf{e}^T \boldsymbol{\alpha} \\ \text{subject to} \quad & 0 \le \alpha_i \le C, i = 1, \ldots, l \\ & \mathbf{y}^T \boldsymbol{\alpha} = 0 \end{aligned}$$

- $Q_{ij} \neq 0$, Q : an l by l fully dense matrix
- 50,000 training points: 50,000 variables:
  $(50,000^2 \times 8/2)$ bytes $=$ 10GB RAM to store Q
- Traditional methods:
  Newton, Quasi Newton cannot be directly applied

Decomposition Methods

- Working on some variables each time (e.g., [Osuna et al., 1997, Joachims, 1998, Platt, 1998])
- Similar to coordinate-wise minimization
- Working set B, $N = \{1, \ldots, l\} \backslash B$ fixed
- Sub-problem at the kth iteration:

$$\begin{aligned}\min_{\boldsymbol{\alpha}_B} \quad & \frac{1}{2} \begin{bmatrix} \boldsymbol{\alpha}_B^T & (\boldsymbol{\alpha}_N^k)^T \end{bmatrix} \begin{bmatrix} Q_{BB} & Q_{BN} \\ Q_{NB} & Q_{NN} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_B \\ \boldsymbol{\alpha}_N^k \end{bmatrix} - \\ & \begin{bmatrix} \mathbf{e}_B^T & (\mathbf{e}_N^k)^T \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_B \\ \boldsymbol{\alpha}_N^k \end{bmatrix} \\ \text{subject to} \quad & 0 \leq \alpha_t \leq C, t \in B, \ \mathbf{y}_B^T \boldsymbol{\alpha}_B = -\mathbf{y}_N^T \boldsymbol{\alpha}_N^k \end{aligned}$$

Avoid Memory Problems

- The new objective function

$$\frac{1}{2}\boldsymbol{\alpha}_B^T Q_{BB} \boldsymbol{\alpha}_B + (-\mathbf{e}_B + Q_{BN}\boldsymbol{\alpha}_N^k)^T \boldsymbol{\alpha}_B + \text{ constant}$$

- Only B columns of Q needed ($|B| \geq 2$)
- Calculated when used

  Trade time for space
- Popular software such as SVM^{light}, LIBSVM, SVMTorch are of this type

How Decomposition Methods Perform?

- Convergence not very fast
- But, no need to have very accurate $\boldsymbol{\alpha}$

  Prediction not affected much
- In some situations, $\#$ support vectors $\ll$ $\#$ training points

  Initial $\boldsymbol{\alpha}^1 = 0$, some instances never used

- An example of training 50,000 instances using LIBSVM

  ```
  $svm-train -c 16 -g 4 -m 400 22features
  Total nSV = 3370
  Time 79.524s
  ```

- On a Xeon 2.0G machine
- Calculating Q may have taken more time
- $\#$SVs $= 3{,}370 \ll 50{,}000$

  A good case where some remain at zero all the time

Issues of Decomposition Methods

Techniques for faster decomposition methods

- store recently used kernel elements
- working set size/selection
- theoretical issues: convergence
- and others (details not discussed here)

But training large data still difficult

- Kernel square to the number of data

  Training millions of data time consuming
- Will discuss some possible approaches

Outline

Parallel: Multi-core/Shared Memory

- Most computation of decomposition methods: kernel evaluations
- Easily parallelized via openMP
- One line change of LIBSVM

  Each core/CPU calculates part of a kernel column

Multicore		Shared-memory	
1	80	1	100
2	48	2	57
4	32	4	36
8	27	8	28

  Same 50,000 data (kernel evaluations: 80% time)
- Using GPU [Catanzaro et al., 2008]

Parallel: Distributed Environments

What if data data cannot fit into memory?

Use distributed environments

- PSVM: [Chang et al., 2007]
  `http://code.google.com/p/psvm/`
- π-SVM: `http://pisvm.sourceforge.net`,
- Parallel GPDT [Zanni et al., 2006]
- All use MPI
- They report good speed-up
- But on certain environments, communication cost a concern

Approximations

Subsampling

- Simple and often effective

Many more advanced techniques

- Incremental training: (e.g., [Syed et al., 1999])
  Data $\Rightarrow$ 10 parts
  train 1st part $\Rightarrow$ SVs, train SVs + 2nd part, ...
- Select and train good points: KNN or heuristics
  e.g., [Bakır et al., 2005]

Approximations (Cont'd)

- Approximate the kernel; e.g., [Fine and Scheinberg, 2001, Williams and Seeger, 2001]
- Use part of the kernel; e.g., [Lee and Mangasarian, 2001, Keerthi et al., 2006]
- And many others

  Some simple but some sophisticated

Parallelization or Approximation

- Difficult to say
- Parallel: general
- Approximation: simpler in some cases
- We can do both
- For certain problems, approximation doesn't easily work

Parallelization or Approximation (Cont'd)

- covtype: 500k training and 80k testing
  rcv1: 550k training and 14k testing

covtype		rcv1	
Training size	Accuracy	Training size	Accuracy
50k	92.5%	50k	97.2%
100k	95.3%	100k	97.4%
500k	98.2%	550k	97.8%

- For large sets, selecting a right approach is essential
- We illustrate this point using linear SVM for document classification

Outline

Linear Support Vector Machines

- Data not mapped to another space
- In theory, RBF kernel with certain parameters
  $\Rightarrow$ as good geleralization performance as linear [Keerthi and Lin, 2003]
- But sometimes can easily solve much larger linear SVMs
- Training of linear/nonlinear SVMs should be separately considered

Linear Support Vector Machines (Cont'd)

- Linear SVM useful if accuracy similar to nonlinear
- Will discuss an example of linear SVM for document classification

Linear SVM for Large Document Sets

Document classification

- Bag of words model (TF-IDF or others)

  A large # of features
- Can solve larger problems than kernelized cases

Recently an active research topic

- *SVM*^{perf} [Joachims, 2006]
- Pegasos [Shalev-Shwartz et al., 2007]
- LIBLINEAR [Lin et al., 2007, Hsieh et al., 2008]
- and others

Linear SVM

- Primal without the bias term b

$$\min_{\mathbf{w}} \quad \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l} \max\left(0, 1 - y_i\mathbf{w}^T\mathbf{x}_i\right)$$

- Dual

$$\begin{array}{ll} \min\limits_{\boldsymbol{\alpha}} & f(\boldsymbol{\alpha}) = \frac{1}{2}\boldsymbol{\alpha}^T Q\boldsymbol{\alpha} - \mathbf{e}^T\boldsymbol{\alpha} \\ \text{subject to} & 0 \le \alpha_i \le C, \forall i \end{array}$$

- No linear constraint $\mathbf{y}^T\boldsymbol{\alpha} = 0$
- $Q_{ij} = y_i y_j \mathbf{x}_i^T \mathbf{x}_j$

A Comparison: LIBSVM and LIBLINEAR

- rcv1: # data: $>$ 600k, # features: $>$ 40k
  TF-IDF
- Using LIBSVM (linear kernel)
  $>$ 10 hours
- Using LIBLINEAR
  Computation: $<$ 5 seconds; I/O: 60 seconds
- Same stopping condition
- Accuracy similar to nonlinear

Revisit Decomposition Methods

- The extreme: update one variable at a time
- Reduced to

$$\alpha_i \leftarrow \min\left(\max\left(\alpha_i - \frac{\nabla_i f(\boldsymbol{\alpha})}{Q_{ii}}, 0\right), C\right)$$

where

$$\nabla_i f(\boldsymbol{\alpha}) = (Q\boldsymbol{\alpha})_i - 1 = \sum_{j=1}^{l} Q_{ij}\alpha_j - 1$$

- $O(nl)$ to calculate ith row of Q
  n: # features, l: # data

- For linear SVM, define

$$\mathbf{w} = \sum_{j=1}^{l} y_j \alpha_j \mathbf{x}_j,$$

- Much easier: $O(n)$

$$\nabla_i f(\boldsymbol{\alpha}) = y_i \mathbf{w}^T \mathbf{x}_i - 1$$

- All we need is to maintain **w**. If

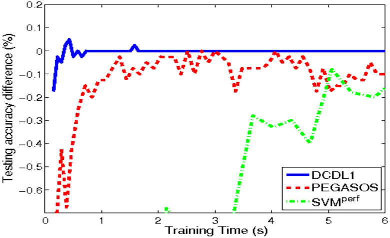
$$\bar{\alpha}_i \leftarrow \alpha_i$$

then

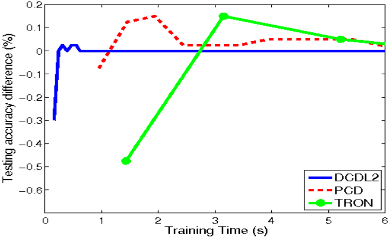
$$\mathbf{w} \leftarrow \mathbf{w} + (\alpha_i - \bar{\alpha}_i) y_i \mathbf{x}_i$$

Still $O(n)$

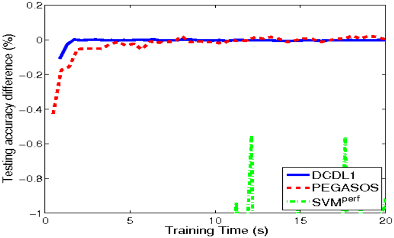
Testing Accuracy (Time in Seconds)



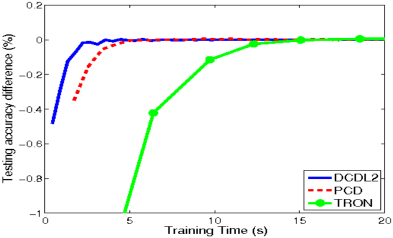
L1-SVM: news20



L2-SVM: news20



L1-SVM: rcv1



L2-SVM: rcv1

Analysis

- Other implementation details in [Hsieh et al., 2008]
- Decomposition method for linear/nonlinear kernels: $O(nl)$ per iteration
- New way for linear: $O(n)$ per iteration

  Faster if # iterations not l times more
- A few seconds for million data; Any limitation?
- Less effective if

  # features small: should solve primal

  Large penalty parameter C

Analysis (Cont'd)

- One must be careful on comparisons
- Now we have two decomposition methods (nonlinear and linear)
- Similar theoretical convergence rates
- Very different practical behaviors for certain problems
- This partially explains controversial comparisons in some recent work

Analysis (Cont'd)

- A lesson: different SVMs

  To handle large data $\Rightarrow$ may need different training strategies
- Even just for linear SVM

  # data $\gg$ # features

  # data $\ll$ # features

  # data, # features both large
- Should use different methods
- For example, # data $\gg$ # features

  primal based method; (but why not nonlinear?)

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Discussion and Conclusions

- Linear versus nonlinear

  In this competition, most use linear (wild track)

  Even accuracy may be worse
- Recall I mention "parallelization" & "approximation"

  Linear is essentially an approximation of nonlinear
- For large data, selecting a right approach seems to be essential

  But finding a suitable one is difficult

Discussion and Conclusions (Cont'd)

- This (i.e., "too many approaches") is indeed bad from the viewpoint of designing machine learning software
- The success of LIBSVM and *SVM*^{light}

  Simple and general
- Developments in both directions (general and specific) will help to advance SVM training