- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

1. (20%) Assume $A \in R^{n \times n}$ is a sparse matrix stored in the column format and $x \in R^{n \times 1}$ is stored in a regular array. Write a code to do $c = A^T x$.

2. (30%) Assume $A, B \in R^{n \times n}$ are sparse matrices stored in the column format. Write a code to do $C = A \times B$. We store C in the column format and assume C's `c, crow_ind, ccol_ptr` have been allocated (i.e., we do not worry about memory management).

   You are allowed to use sparse matrix and dense vector product as an intermediate step. Your total time complexity cannot exceed $O(n \cdot \text{nnz})$

3. (50%) Consider a linear system $Ax = b$:

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

   - Assume $x_0 = [0, 0]^T$. Do two CG iterations and show what x_1 and x_2 are?
   - Check if p_1 and p_2 are A-conjugate
   - Let r_1 and p_1 be vectors from the above procedure. Solve the following optimization problem with the variable p:

$$\begin{aligned} \min_{p} \quad & \|p - r_1\|^2 \\ \text{subject to} \quad & p \in \text{span}\{Ap_1\}^{\perp} \end{aligned} \tag{1}$$

     How is the solution of this problem connected to p_2 obtained in the CG procedure

   - If we denote the solution of (**??**) as $\bar{p}_2$ and calculate $\bar{\alpha}_2$, x_2 by

$$\bar{\alpha}_2 = \frac{\bar{p}_2^T r_1}{\bar{p}_2^T A \bar{p}_2}, \qquad x_2 = x_1 + \bar{\alpha}_2 \bar{p}_2,$$

     what are $\bar{\alpha}_2$ and x_2?

   It is true that one may make mistakes in doing the calculation. However, if you understand the concept of CG, you should be able to easily validate your results.