

0.11

Find the error in the following proof that all horses are the same color.

CLAIM: In any set of h horses, all horses are the same color.

PROOF: By induction on h .

Basis: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

Induction step: For $k \geq 1$ assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in the set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2 . By the same argument, all the horses in H_2 are the same color. Therefore all the horses in H must be the same color, and the proof is complete.

1.5

Each of the following language is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts $\Sigma = \{a, b\}$.

c. $\{w \mid w \text{ contains neither the substring } ab \text{ nor } ba\}$.

h. $\{w \mid w \text{ is any string except } a \text{ and } b\}$.

1.6

Give state diagram of DFAs recognizing the following languages. In all parts $\Sigma = \{0, 1\}$.

j. $\{w \mid w \text{ contains at least two 0s and at most one 1}\}$