- **Please give details of your calculation. A direct answer without explanation is not counted.**
- **Your answers must be in English.**
- **Please carefully read problem statements.**
- **During the exam you are not allowed to borrow others' class notes.**
- **Try to work on easier questions first.**

1. (a) (12%) In using the pumping lemma, we use a property that

$$\text{not } (A \text{ and } B \text{ and } C) \tag{1}$$

is equivalent to

$$B \text{ and } C \rightarrow \text{ not } A, \tag{2}$$

where A, B, and C are any statements.

Use a (detailed) truth table to prove the above property (i.e., prove (1) is equivalent to (2). Please consider all 8 combinations of A, B, C; don't put "B and C" as a variable).

(b) (13%) In almost all Pumping Lemma examples we discussed, we consider

- A: $xy^iz \in$ the language, $\forall i \geq 0$
- B: $|y| > 0$
- C: $|xy| \leq p$

and prove (2). Then from the above property that (1) and (2) are equivalent, we obtain the desired result (1). Assume that instead of (2), we are now able to prove that

$$B \rightarrow \text{ not } A. \tag{3}$$

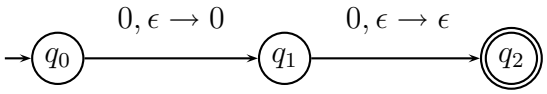
Is (3) then sufficient to imply (1)? If your answer is yes, show your argument and give an example (i.e., give a language and in the use of pumping lemma you use (3)). If your answer is no, give your detailed argument.

2. (20%) Using pumping lemma to prove that the following language is not regular:

$$\{0^r1^s \mid r \geq 2s, r \geq 0, s \geq 0, r \text{ and } s \text{ are integers}\}.$$

We require you to directly use the standard pumping lemma. Don't use something else (e.g., do complement of this language or union it with something else; and then prove blah blah).

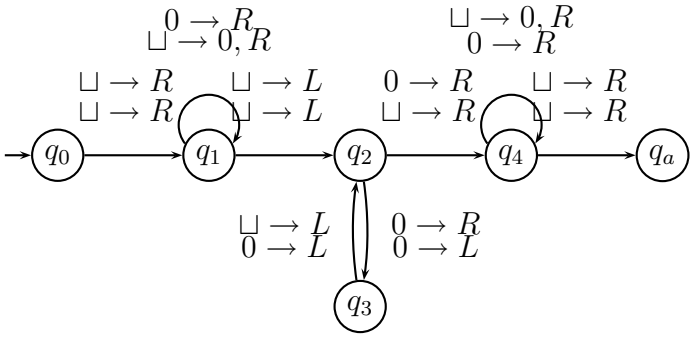
3. (25%) Consider the following PDA with $\Sigma = \{0\}$:



Find CFG of this PDA's language. You are required to follow the **same procedure** in Lemma 2.27 to generate rules (Lemma 2.27 proves that if a PDA recognizes some language, then it is context free. Your notes should have provided enough materials regardless of whether you have the textbook or not.)

4. (20%) Assume $\Sigma = \{0, 1\}$. We would like to design a (one-tape deterministic) Turing machine to shift a string. That is, if w is the input, after running the machine, we have $\sqcup w$ in the tape. Give the transition diagram as well as the formal definition of this TM.

5. (10%) Consider the following two-tape TM



Assume the initial configuration is $\sqcup 0000$ in the 1st tape and $\sqcup\sqcup\ldots$ in the 2nd tape. Run the TM and give the **sequence of configurations**.