Link Scheduling with QoS Guarantee for Wireless Relay Networks

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***Abstract*—The emerging *wireless relay networks* (WRNs) are expected to provide significant improvement on throughput and extension of coverage area for next-generation wireless systems. We study an optimization problem for multi-hop link scheduling with bandwidth and delay guarantee over WRNs. Our optimization problem is investigated under a general interference model with a generic objective. The objective can be based on various kinds of performance indexes (*e.g.*, throughput, fairness and capacity) determined by service providers. Through the theoretical analysis, the intractability and in-approximability of the optimization problem are shown. Due to the intractable computational complexity, we present efficient algorithms to practically provide a small approximation factor against any optimal solution even for a worst-case input. Furthermore, the experimental results indicate that our presented algorithms yield near-optimal performance in the average case.**

I. Introduction

With the advance of broadband wireless access technologies, the emerging *wireless relay networks* (WRNs) are expected to provide significant improvement on throughput and extension of coverage area for next-generation wireless systems. As real-time applications (*e.g.*, voice over IP and video streaming) rapidly grow, provisioning *quality-of-service* (QoS) to these applications is one of the most important issues for WRNs. In WRNs, contention-based *medium access control* (MAC) protocols, such as *IEEE 802.11*, are hard to quantitatively guarantee QoS due to its unpredictable and uncontrollable packet collisions. Compared to the contention-based MAC protocols, polling-based channel access (*e.g.*, *IEEE 802.16*) can provide fine-granularity resource control and management. Moreover, the centralized control provided by *base stations* (BSs) facilitates the feasibility of QoS provisioning to the real-time applications.

Recently, *IEEE 802.16j* task group has been devoted to develop a multi-hop relay mode for *IEEE 802.16* point-to-multi-point networks. The increasing system throughput and coverage area through low-cost IEEE 802.16j *relay stations* (RSs) accelerate the deployment of the real-time applications over broadband wireless networks. Although the communication protocols of resource allocation for IEEE 802.16 networks have been specified in [1], the design of resource allocation policies is still an open issue. Without an appropriate allocation policy, some critical service flows might suffer from the lack of available bandwidth, and the service quality of these flows cannot be guaranteed due to the increase of delay, jitter and packet loss. The issue is further complicated by *IEEE 802.16j*-based multihop relay transmission because of spatial reuse, and has been referred to as *link scheduling* problem in wireless multihop networks. The interference-aware link scheduling algorithms have been extensively developed in *mobile ad hoc networks* (MANETs) and *wireless mesh networks* (WMNs). In [2], the link scheduling problem that deals with the maximization of system throughput has been proven to be $\mathcal{NP}$-hard for a multi-hop wireless network. Based on [2], several heuristic algorithms were presented to handle multi-hop link scheduling for throughput improvement [3], [4], [5], [6]. Furthermore, some approximation algorithms with worst-case performance bounds were developed for multi-hop link scheduling [7], [8], [9].

In addition to the consideration of the best-effort traffic for multi-hop link scheduling, the QoS issue was also studied in MANETs and WMNs for resource allocation. Specifically, Harish and Vinod studied the QoS routing and link scheduling with a fixed route scenario for IEEE 802.16-based WMNs [10]. In [11], a link scheduler was presented to minimize the maximal delay of requests for a TDMA-based multi-hop network by adopting the spatial reuse. Unfortunately, both of the studies can not provide explicit QoS guarantee to real-time applications. To solve the problem, Tang *et al.* [12] proposed a mechanism of interference-aware topology control and QoS routing for multi-channel WMNs. This mechanism ensures that all data deliveries are satisfied with their allocated bandwidth. In [13], an approximation algorithm with a worst-case bound was proposed to provide a hard guarantee for bandwidth allocation for MANET users. In addition to the ensuring of bandwidth allocation, Lee *et al.* [14] proposed a tree-based routing algorithm with a delay guarantee for multi-hop wireless backhaul networks. However, the design of link scheduler was not addressed in this work.

In this paper, we study an optimization problem for interference-aware link scheduling with QoS guarantee over WRNs. To the best of our knowledge, for any multihop wireless networks, our work presents the first attempt at studying the link scheduling problem based on explicit delay requirements. The contributions of this work are described as follows.

- **Explicit QoS guarantee:** We present efficient link scheduling algorithms to provide delay guarantee for WRNs over arbitrary routing.
- **Flexible performance metric:** The performance metrics (*e.g.*, throughput, capacity) can be specified by service providers.
- **Appropriate system model:** We assume that a link scheduler has the information of service flows (such as traffic demand, delay constraint, routing path) instead of the information of links. Although the link information could be measured, it requires substantial efforts and extra overheads, and may be highly inaccurate. The flow information is relatively easy to obtain in 802.16-based WRNs because SSs shall request resources for their requesting flows to the link scheduler in the BS [1].
- **Provable problem hardness:** We show the computational complexity of the optimization problem. Specifically, the analysis on intractability and in-approximability is presented.
- **Worst-case Performance bound:** The worst-case performance of the presented polynomial-time algorithms is proven

to be within a practically small factor of the optimal algorithm. The approximation factor is independent of the number of requesting flows and of the number of subscribers.

The remainder of the paper is organized as follows. Section II formally presents our system model and problem formulation. The proof of problem intractability and in-approximability is given in Section II. In Section III, we present our link scheduling algorithms and the worst-case performance analysis of the algorithms. Section IV conducts the simulation study. Finally, the paper is concluded in Section V.

II. System Model and Problem Formulation

This section elaborates on our system model based on IEEE 802.16j WRNs. By adopting the model, the optimization problem for link scheduling with explicit QoS guarantee is formally defined, and the interference model used by the optimization problem is described. Also, the proof of problem intractability and in-approximability is presented in this section.

A. System Model

The model we study is based on a *static* WRN, which is centrally controlled by a BS equipped with wired interfaces to connect the IP-based backbone networks. The BS supervises one or more RSs through wireless links, and each SS communicates with the BS directly or through RSs. In our model, we assume that each station is equipped with one radio interface and there is only a single channel in the network. Our work can be easily extended to accommodate the case of multiple subchannels as in [9]. To comply the IEEE 802.16 specification, data transmissions among stations are assumed to be synchronized on frame basis. A frame consists of several timeslots, and a timeslot is the basic time unit for transmissions.

In this study, we focus on the designing of a QoS scheduler to support *real-time services* on a frame-by-frame transmission manner. In IEEE 802.16-based networks [1], [15], there are three service types for real-time traffic, namely *unsolicited grant service* (UGS), *real-time polling service* (rtPS) and *enhanced real-time polling service* (ertPS). The mandatory QoS service flow parameters of those service types include the minimum reserved traffic rate (*i.e.*, the required bandwidth) and the maximum latency (*i.e.*, the deadline). Without loss of generality, we consider *uplink* service flows (from SSs to their BS) in this paper. Our model can be easily extended to accommodate the case for downlink service flows, since the uplink data and downlink data are scheduled separately in IEEE 802.16 networks [1], [16]. For uplink real-time service flows, the BS periodically allocates the timeslots to SSs for resource requests. When any real-time service flow is initiated, the corresponding SS uses the allocated slot to request the resource with their required QoS such as bandwidth and deadline. The link scheduler in the BS accepts the request of a service flow only when the *bandwidth requirement* and the *delay requirement* of the flow can be guaranteed in the following frames. Thus a admitted real-time service flow can be exclusively transmitted in its allocated bursts of the following frames until the flow is terminated. To assure the availability of resource allocation during transmission, our scheduler is assumed to be aware of interference [12], [8], [11], and the interference model adopted for our problem will be described in the following subsection.

B. Problem Formulation

Based on the system model, our optimization problem for link scheduling (also called the `OPT-LS` problem) with explicit QoS guarantee is formulated as follows, and the notations used in the problem formulation are summarized in Table I. A WRN is modeled as a *simple directed* graph $G = (V, E)$, where V represents all stations (including a BS, r RSs and $(|V| - 1 - r)$ SSs) and E is a set of wireless links in the WRN. A wireless link (α,β) is an element of E if and only if station β is within the maximum transmission range of station α. Let the frame period ranges between $[0, c)$. In each frame, the set of requesting real-time flows and previously admitted flows is denoted as $\mathbb{F} = \{F_1, \ldots, F_n\}$. Suppose that the transmission paths of the flows have been determined by a routing algorithm/protocol. Then each flow i can be associated with a quintuple $(b_i, d_i, w_i, a_i, \mathbb{T}_i)$, where b_i, d_i, and w_i are, respectively, the required bandwidth (Mbits), the maximum latency (ms) (*i.e.*, deadline), and a *positive integer weight* specified by its service provider. a_i indicates the status of a flow, *i.e.*,

$$a_i = \begin{cases} 1, & \text{flow } i \text{ is an admitted flow} \\ 0, & \text{flow } i \text{ is a requesting flow} \end{cases} \tag{1}$$

$\mathbb{T}_i$ is the routing path of flow i, and is denoted as an *ordered* set $\{T_{i,1}, \ldots, T_{i,\kappa_i}\}$ of transmissions, where κ_i denotes the number of wireless links in the path, *i.e.*, the *hop count* of flow i. Then the maximum hop count is denoted by $\varphi = \max_{i\in\mathbb{F}}(\kappa_i)$. The jth transmission of the routing path for flow i (*i.e.*, $T_{i,j}$) can be characterized by a quadruple $(TX_{i,j}, RX_{i,j}, \tau_{i,j}, \mathbb{I}_{i,j})$. $TX_{i,j}$ and $RX_{i,j}$ are, respectively, the transmitting station and the receiving station of $T_{i,j}$, where $TX_{i,j} \in V$, $RX_{i,j} \in V$, and $(TX_{i,j}, RX_{i,j}) \in E$. $\tau_{i,j}$ is the transmission latency of $T_{i,j}$. The value of $\tau_{i,j}$ can be calculated by taking b_i divided by the transmission rate of link $(TX_{i,j}, RX_{i,j})$[1].

For brevity, we introduce the following abbreviations to be used in the remainder of this paper. "for $i \in \mathbb{F}$" represents "for each flow $F_i \in \mathbb{F}$". "for $(i, j) \in \mathbb{T}$" signifies "for each flow $F_i \in \mathbb{F}$ and its transmission $T_{i,j} \in \mathbb{T}_i$". "for $(i, j \neq j') \in \mathbb{T}$" indicates "for each flow $F_i \in \mathbb{F}$ and for two different transmissions $T_{i,j} \in \mathbb{T}_i$ and $T_{i,j'} \in \mathbb{T}_i$". "for $(i \neq i', j \neq j') \in \mathbb{T}$" expresses "for two different flows $F_i \in \mathbb{F}$ and $F_{i'} \in \mathbb{F}$ and for two different transmissions $T_{i,j} \in \mathbb{T}_i$ and $T_{i',j'} \in \mathbb{T}_{i'}$".

TABLE I
Summary of Main Notations

Symbol	Semantics
n and r	The number of service flows and of relay stations
p	The number of requesting service flows
$\mathbb{F}$	The set of all real-time service flows
F_i	The ith service flow of $\mathbb{F}$
b_i, d_i, and w_i	The required bandwidth, deadline, and weight of flow i
W	The total weight of all requesting flows
κ_i	The number of transmissions of flow i
φ	The maximum hop count
$\mathbb{T}_i$	The set of transmissions of flow i
$T_{i,j}$	The jth transmissions of flow i
$TX_{i,j}$	The transmitting station of the jth transmission of flow i
$RX_{i,j}$	The receiving station of the jth transmission of flow i
$\tau_{i,j}$	The transmission latency of the jth transmission of flow i
$I_{i,j}$	The interference list of the jth transmission of flow i
$\hat{\mathbb{F}}$	The set of scheduled flows
$\hat{t}_{i,j}$	The schedule instance of the jth transmission of scheduled flow i
$\Gamma(\hat{\mathbb{F}})$	The profit of a scheduled flow set $\hat{\mathbb{F}}$ for $\mathbb{F}$
$\Gamma(\tilde{\mathbb{F}})$	The profit of an optimal scheduled flow set $\tilde{\mathbb{F}}$ for $\mathbb{F}$

For $(i, j) \in \mathbb{T}$, the interference list $I_{i,j}$ is a set of the wireless links that suffer from the interference of link $(TX_{i,j}, RX_{i,j})$. In MANETs and WMNs, a considerable number of interference models, *e.g.*, fixed-radius interference model [12], protocol interference model, fixed power protocol interference model, and RTS/CTS model [8], are considered. However, these models are not suitable for WRNs due to the following reason. Stations in MANETs and WMNs are generally assumed to be identical and equipped with an omnidirectional antenna. On the other hand, there are different physical capabilities for different types of stations in WRNs. For example, a BS could support *multi-user adaptive antenna system* (AAS) technique to provide multi-beam adaptive beamforming

[1]We assume that the BS is aware of the transmission rate of each link. The link rate can be determined by the transmission power, modulation and coding scheme [17].

and null-steering [18]. Also, an RS could use directional antenna or sectorized antenna to strengthen signal strength [19]. In the above cases, the interference zone can be significantly reduced. Moreover, an omni-directional antenna is possibly applied for low-cost RSs [19] and SSs. Due to the variety of antenna technologies adopted by WRN stations, our interference model $I_{i,j} = U_{i,j} \cup S_{i,j}$ consists of two types of interference, *primary interference* $U_{i,j}$ and *secondary interference* $S_{i,j}$. The *primary interference* occurs when a transceiver transmits and receives packets at the same time [20], *i.e.*,

$$\begin{aligned} U_{i,j} &= \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \alpha = TX_{i,j} \text{ or } \alpha = RX_{i,j} \\ &\quad \text{or } \beta = TX_{i,j} \text{ or } \beta = RX_{i,j}\} \,. \end{aligned} \tag{2}$$

The *secondary interference* occurs when a receiving station is interfered by other transmissions which are not intended for this station. We assume that $S_{i,j}$ can be an arbitrary set of wireless links depending on its own communication characteristic. Notice that with a general assumption for secondary interference, our interference model can be applicable to any other models.

A schedule is defined as $\hat{\mathbb{F}}$, where $\hat{\mathbb{F}} \subseteq \mathbb{F}$, and the service flows in $\hat{\mathbb{F}}$ can be scheduled and be satisfied with their required QoS. For $i \in \hat{\mathbb{F}}$, the set of transmissions of the flow i is expressed as $\hat{\mathbb{T}}_i$. Let $\hat{\ell}_{i,j} \geq 0$ be the schedule instance for $(i,j) \in \hat{\mathbb{T}}$. Then the jth transmission of flow i is scheduled at the time interval $[\hat{\ell}_{i,j}, \hat{\ell}_{i,j} + \tau_{i,j})$. A *feasible* schedule of the service flow set $\mathbb{F}$ is referred to as a schedule $\hat{\mathbb{F}}$ that includes all admitted flows in $\mathbb{F}$ and meets both the *transmission constraints* and *interference-free constraints*. The *transmission constraints* indicate that the transmissions of any service flow in the schedule 1) follow a strict order and 2) meet their deadline and the frame boundary. Thus $\hat{\mathbb{F}}$ satisfies the *transmission constraints* if and only if each schedule instance $\hat{\ell}_{i,j}$ is bounded by

$$\hat{\ell}_{i,j} \leq \begin{cases} \hat{\ell}_{i,j+1} - \tau_{i,j}, & 1 \leq j \leq \kappa_i - 1 \\ \min\{d_i, c\} - \tau_{i,j}, & j = \kappa_i \end{cases} \tag{3}$$

For the purpose of spatial reuse as well as collision avoidance, the *interference-free constraints* are defined as follows. Let $\hat{T}_{i,j} \Leftrightarrow \hat{T}_{i',j'}$ and $\hat{T}_{i,j} \not\Leftrightarrow \hat{T}_{i',j'}$ respectively represent $\hat{T}_{i,j}$ and $\hat{T}_{i',j'}$ are and are not overlapped with each other. The transmissions $\hat{T}_{i,j}$ and $\hat{T}_{i',j'}$ are not overlapped if and only if the time intervals $[\hat{\ell}_{i,j}, \hat{\ell}_{i,j} + \tau_{i,j})$ and $[\hat{\ell}_{i',j'}, \hat{\ell}_{i',j'} + \tau_{i',j'})$ are disjoint. Let $\hat{T}_{i,j} \vDash \hat{T}_{i',j'}$ denote the transmission link of $\hat{T}_{i,j}$ interfere with that of $\hat{T}_{i',j'}$, *i.e.*, $(TX_{i',j'}, RX_{i',j'}) \in I_{i,j}$. Then $\hat{\mathbb{F}}$ satisfies the *interference-free constraints* if and only if $\hat{T}_{i,j} \not\Leftrightarrow \hat{T}_{i',j'}$ for $(i \neq i', j \neq j') \in \hat{\mathbb{T}}$, $\hat{T}_{i,j} \vDash \hat{T}_{i',j'}$.

In this paper, we are interested in the derivation of a feasible schedule for OPT-LS such that the schedule profit in a frame is maximized. The profit $\Gamma(\hat{\mathbb{F}})$ of a feasible schedule $\hat{\mathbb{F}}$ is defined by the summation of the weights of scheduled flows, *i.e.*, $\Gamma(\hat{\mathbb{F}}) = \sum_{i \in \hat{\mathbb{F}}} w_i$. An *optimal schedule* $\tilde{S}(\mathbb{F}) = \tilde{\mathbb{F}}$ is defined as a *feasible schedule* such that $\Gamma(\tilde{\mathbb{F}})$ is maximized for a given flow set $\mathbb{F}$. Note that the weight in our problem can be a general performance measure depending on the practical requirements from WRN service providers. For example, if the weight is set as the bandwidth requirement, the problem is to maximize the system throughput.

Now, we formally formulate our OPT-LS problem as follows:

$$\begin{aligned} \textit{Maximize} \quad & \Gamma(\hat{\mathbb{F}}) = \textstyle\sum_{i \in \hat{\mathbb{F}}} w_i \\ \textit{Subject to:} \quad & F_i \in \hat{\mathbb{F}}, \text{ for } F_i \in \mathbb{F} \text{ and } a_i = 1 \\ & \hat{\ell}_{i,j} \leq \begin{cases} \hat{\ell}_{i,j+1} - \tau_{i,j}, & 1 \leq j \leq \kappa_i - 1 \\ d_i - \tau_{i,j}, & j = \kappa_i \end{cases}, \text{ for } (i,j) \in \hat{T}_{i,j} \\ & \hat{T}_{i,j} \not\Leftrightarrow \hat{T}_{i',j'}, \text{ for } (i \neq i', j \neq j') \in \hat{\mathbb{T}} \text{ and } \hat{T}_{i,j} \vDash \hat{T}_{i',j'} \end{aligned}$$

Figure 1 depicts an example to illustrate our problem OPT-LS. Specifically, the network topology including one BS (Station 1), two RSs (Stations 2 and 3) and three SSs (Stations 4, 5, and 6) is shown in Figure 1(a). A interference graph is shown in Figure 1(b), where the vertices represent the wireless links in E and the edges indicate the interference between the wireless links. Figure 1(c) illustrates the data rates for relay (BS-to-RS or RS-to-RS) and access links (BS-to-SS or RS-to-SS) in the example, and the service flows F_1, F_2 and F_3 are shown in Figure 1(d).

Figures 1 (e)-(g) illustrate three infeasible schedules, where the charcoal areas represent the transmissions and the gray areas signifies the interfered area. Figure 1(e) shows the case that the transmission $T_{2,2}$ precedes $T_{2,1}$, which violates the transmission constraints. Moreover, since the admitted flow F_1 have not been scheduled, the schedule in Figure 1 cannot be feasible. Figure 1(f) shows that the schedule does not satisfy the transmission constraints because the completion time of F_3, 85 ms, exceeds $d_3 = 80$ ms. Although Figure 1(g) meets the transmission constraints, the *interference-free constraints* cannot be fulfilled due to the interference between $T_{2,2}$ and $T_{3,2}$.

Clearly, the exhausted method for OPT-LS is not realistic since there are an exponential number of possible subsets of $\mathbb{F}$. Now, we present a theoretical analysis of intractability and inapproximability for our OPT-LS. Consider a case of OPT-LS that all service flows are requesting flows and have the same weight, deadline, and total transmission time. Even in such a special case, OPT-LS is proven to be $\mathcal{NP}$-hard, and no polynomial-time algorithm can approximate OPT-LS within a factor of $n^{1-\epsilon}$ for any $\epsilon > 0$.

Theorem 1: OPT-LS is $\mathcal{NP}$-hard, and no polynomial-time algorithm can approximate OPT-LS within a factor of $n^{1-\epsilon}$ for any $\epsilon > 0$ unless $\mathcal{NP} \nsubseteq \mathcal{ZPP}$.

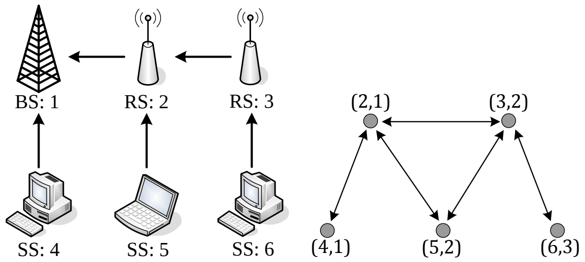
Proof: We will reduce MAX CLIQUE [21] to our OPT-LS as follows. In MAX CLIQUE, we are given a simple graph $G_c = (V_c, E_c)$ with $|V_c|$ vertices $v_1, v_2, \ldots, v_{|V_c|}$. Based on G_c, we construct a complement graph $\overline{G}_c = (\overline{V}_c, \overline{E}_c)$ such that $\overline{V}_c = V_c$ and $\overline{E}_c = \{(v_i, v_j) \mid v_i \in V_c, v_j \in V_c, (v_i, v_j) \notin E_c\}$. To conduct the reduction, a set of $|V_c|$ service flows $\mathbb{F} = \{F_1, F_2, \ldots, F_{|V_c|}\}$ is constructed. For $i \in \mathbb{F}$, the number of transmissions κ_i is assigned by $|\overline{E}_c| + 1$. For jth transmission $(1 \leq j \leq \kappa_i - 1)$, the transmission latency $\tau_{i,j} = \hbar$ is an arbitrary positive constant. τ_{i,κ_i} is assigned by an arbitrary value $\Im$ where $0 < \Im < \frac{\hbar}{|V_c|}$. Then $\sum_{(i,j) \in \mathbb{T}} \tau_{i,j} = (\kappa_i - 1)\hbar + \Im$. $d_i = (\kappa_i - 1)\hbar + |V_c|\Im$, and $w_i = 1$. The interference list $I_{i,j}$ is equivalent to $S_{i,j}$ shown in (2). Furthermore, the routing path of each transmission $T_{i,j}$ is assigned based on Algorithm 1, and an example of route assignment is shown in Figure 2. Figure 2(a) and Figure 2(b) illustrate an example of G_c and $\overline{G}_c$, respectively. Based on G_c and $\overline{G}_c$, Figure 2(c) shows the route assignment for $\mathbb{F}$ (as refers to lines 5-12 in Algorithm 1), and Figure 2(d) shows the results after the RS merging process (as refers to lines 13-16 in Algorithm 1).

Then we will show that there is a maximal clique of u vertices in G_c if and only if there is an *optimal schedule* $\tilde{\mathbb{F}}$ such that $\Gamma(\tilde{\mathbb{F}}) = \sum_{i \in \tilde{\mathbb{F}}} w_i = |\tilde{\mathbb{F}}| = u$ is maximized. In "only if" part, suppose that such a maximal clique V_c^* exists and $|V_c^*| = u$. By our reduction, V_c^* is mapped to a subset $\mathbb{F}^* \in \mathbb{F}$ of u service flows. It is obvious that a feasible schedule with u service flows can be obtained by assigning each schedule instance as follows.

1) For the case of first $\kappa_i - 1$ transmissions $T^*_{i,j}$ $(1 \leq i \leq u, 1 \leq j \leq \kappa_i - 1)$, we have $\hat{\ell}^*_{i,j} = (j-1) \times \hbar$.
2) For the case of the last transmission T^*_{i,κ_i} $(1 \leq i \leq u)$, we have $\hat{\ell}^*_{i,\kappa_i} = (\kappa_i - 1) \times \hbar + (i-1) \times \Im$.

Now suppose by contradiction, a feasible schedule $\hat{\mathbb{F}}$ with $u' > u$ flows exists. $\hat{\mathbb{F}}$ satisfies the *transmission constraints* since $\tau_{i,\kappa_i} = \Im < \frac{\hbar}{|V_c|} < \hbar = \tau_{i,j}$ for any $\hat{F}_i \in \hat{\mathbb{F}}$, $1 \leq j \leq \kappa_i - 1$. Hence, for any two different service flows $\hat{F}_i \in \hat{\mathbb{F}}$ and $\hat{F}_{i'} \in \hat{\mathbb{F}}$, we have $\hat{F}_{i,j} \Leftrightarrow \hat{T}_{i',j}$ for any $1 \leq j \leq \kappa_i - 1$. Then $r_{i,j} \neq r_{i',j}$ implies that

$$\xi(i+1,m)=\begin{cases}\min\{\xi(i,m-w_{i+1})+P_{i+1},\xi(i,m)\} & \text{if } \xi(i,m-w_{i+1})+P_{i+1}\le\min\{d_{i+1},c\}\\ \xi(i,m) & \text{otherwise}\end{cases}\tag{4}$$



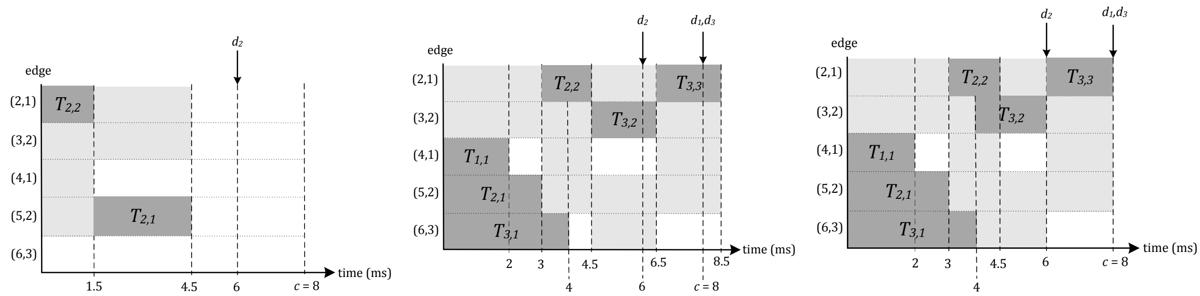
(a) Network topology (b) Interference graph

Link	Data Rate
Relay Link (2,1), (3,2)	12Mbps
Access Link (4,1), (5,2), (6,3)	6Mbps

(c) Link rates

F_i	a_i	d_i (ms)	b_i (Mbits)	κ_i	$T_{i,j}$	$(TX_{i,j}, RX_{i,j})$	$p_{i,j}$ (ms)
F_1	1	8	2.4	1	$T_{1,1}$	(4, 1)	2
F_2	0	6	1.8	2	$T_{2,1}$	(5, 2)	3
					$T_{2,2}$	(2, 1)	1.5
F_3	0	8	2.4	3	$T_{3,1}$	(6, 3)	4
					$T_{3,2}$	(3, 2)	2
					$T_{3,3}$	(2, 1)	2

(d) Input instance



(e) Illegal schedule 1 (f) Illegal schedule 2 (g) Illegal schedule 3

Fig. 1. An example to illustrate the problem OPT-LS.

Algorithm 1 ROUTE ASSIGNMENT

construct $|V_c||\overline{E}_c|$ relay stations $r_{i,j}$ $(1 \le i \le |V_c|, 1 \le j \le \kappa_i - 1 = |\overline{E}_c|)$
construct $|V_c|$ subscriber stations s_i $(1 \le i \le |V_c|)$
construct a base station
$E^\sharp \leftarrow \overline{E}_c$
for all service flow $F_i \in \mathbb{F}$ **do**
  $TX_{i,1} \leftarrow s_i$
  $RX_{i,\kappa_i} \leftarrow$ the base station
  for $j = 1$ to $\kappa_i - 1$ **do**
    $RX_{i,j} \leftarrow r_{i,j}$
    $TX_{i,j+1} \leftarrow r_{i,j}$
  end for
end for
for $z = 1$ to $|\overline{E}_c|$ **do**
  remove an arbitrary edge (α,β) from $E^\sharp$
  $r_{\alpha,z} = r_{\beta,z}$
end for

$(TX_{i,j}, RX_{i,j}) \notin I_{i',j}$ and $(TX_{i',j}, RX_{i',j}) \notin I_{i,j}$. Thus, an independent set of $\overline{G}_c$ with u' vertices exists, and we have a clique of size $u' > u$. The "if" part can be derived in the same way, and the details are omitted.

To show the in-approximability of our OPT-LS, a kind of reduction, *L-reduction*, is used. *L-reduction* preserves a relative error of approximation within a constant factor [22]. Suppose that both Π and Π' are the maximization problems. Π *can be L-reduced to* Π' if there exist two constants $\vartheta_1, \vartheta_2 > 0$ and a polynomial-time transformation f satisfies the following conditions:

1) For each instance x of Π, $f(x)$ is an instance of Π'.
2) The optimum solutions of x and $f(x)$, $OPT(x)$ and $OPT(f(x))$, satisfy $OPT(f(x)) \le \vartheta_1 \times OPT(x)$.
3) For any solution of $f(x)$, $c(f(x))$, we can find in polynomial time a solution of x, $c(x)$, satisfying $c(x) \ge OPT(x) + \vartheta_2[c(f(x)) - OPT(f(x))]$.

In our reduction, the optimum value in the two instances is the same. Let $\vartheta_1 = \vartheta_2 = 1$, we have an L-reduction from MAX CLIQUE to the special case of OPT-LS. Then by the known result of in-approximability of MAX CLIQUE [23], OPT-LS cannot be approximated within $n^{1-\epsilon}$ in polynomial time for any $\epsilon > 0$. ■

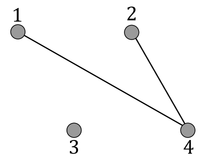
Notice that in WRNs, frame resources are not dedicated to real-time service flows, and shall be shared with other types of data transmission such as best-effort traffic and signaling control message. In this case, a frame would have some unavailable intervals that can not be used for scheduling the real-time service flows. To consider this situation for our OPT-LS, a modification of deadlines of real-time service flows is needed. The deadlines are shifted backward such that the unavailable intervals are virtually removed for real-time service flows. Figure 3 illustrates an example of deadline shift for real-time service flows originally with deadlines d_1 and d_2. In Figure 3(a), there are two unavailable intervals, i.e., U1 and U2. U1 and U2 are virtually removed when d_1 and d_2 are respectively changed to $\underline{d}_1$ and $\underline{d}_2$ (see Figure 3(b)).

III. Scheduling Algorithms

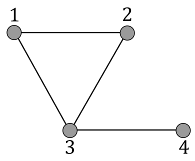
Since OPT-LS problem is $\mathcal{NP}$-hard, we can only need to find viable heuristic to solve it. This section presents two heuristic scheduling algorithms to support QoS guarantee over WRNs. First, we develop a Dynamic-Programming-based Scheduling (DPS) algorithm for OPT-LS. Then, we present a DPS-based algorithm with Spatial Reuse (DPS-SR). The worst-case performance analysis of Algorithms DPS and DPS-SR is also given in this section.

A. Algorithm DPS

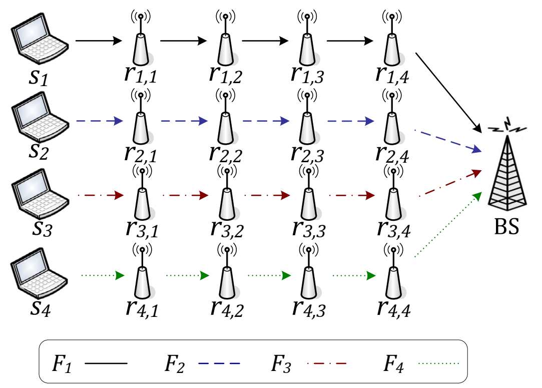
The algorithm DPS consists of the following three phases: 1) *Flow Sequencing*, 2) *Flow Selection*, and 3) *Assignment of Sched-*



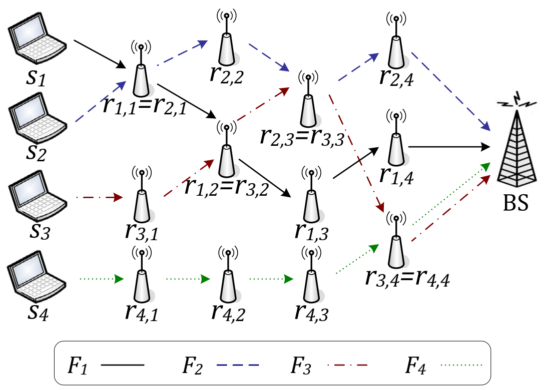
(a) G_c



(b) $\bar{G}_c$

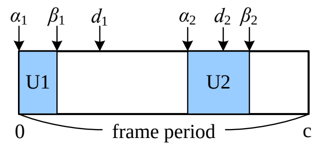


(c) The route assignment, where each service flow F_i is assigned a transmission path $s_i \rightsquigarrow r_{i,1} \rightsquigarrow r_{i,2} \rightsquigarrow \ldots \rightsquigarrow r_{i,|\bar{E}_c|} \rightsquigarrow$ BS

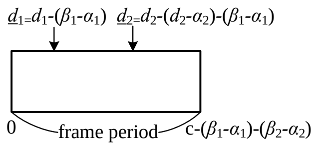


(d) The result after performing the RS merging process

Fig. 2. An example of route assignment.



(a) Frame with unavailable intervals



(b) Frame after removing unavailable intervals

Fig. 3. An illustration of the removing of unavailable intervals.

ule Instance. In the first phase, all flows will be re-sequenced. Based on the sequence of the flows, a subset of the flows will be selected in the second phase, and the schedule instances of the selected flows will be assigned in the last phase.

1) **Flow Sequencing:** In the first phase, the flows in $\mathbb{F}$ are renumbered in non-decreasing order of their deadlines. The Earliest-Due-Date (EDD) rule is used because it has been proven to be optimal for the minimization of the maximum tardiness (*i.e.*, the time difference between job completion time and its deadline) for single-machine job scheduling [24]. This feature facilitates the selection of the flows in the second phase.
2) **Flow Selection:** Since the completion time of each service flow is close to its deadline by using EDD, a feasible schedule could be obtained by rejecting only a small number of requesting service flows with small weights. To ensure that the previously admitted service flows are selected, the flow weights are modified to $\underline{w}_i$, and $\underline{w}_i$ will be

$$\underline{w}_i = \begin{cases} w_i + WA, & \text{if } a_i = 1 \\ w_i, & \text{otherwise} \end{cases} \qquad (6)$$

where $WA = (\sum_{i \in \mathbb{F}, a_i = 0} w_i) + 1$.

In this phase, the dynamic programming technique is adopted to achieve a better schedule profit. For each i, m ($i = 0, 1, \ldots, n$ and $m = 0, 1, \ldots, \sum_{i \in \mathbb{F}} \underline{w}_i$), let the state variable $\xi(i, m)$ be the minimal completion time among the m-profit subsets (*i.e.*, the total value of the subsets equals to m) of the set of the first i flows $\{F_1, F_2, \ldots, F_i\}$ in $\mathbb{F}$. For $i \in F$, let P_i be the total transmission time of F_i, *i.e.*, $P_i = \sum_{T_{i,j} \in \mathbb{T}_i} \tau_{i,j}$. Initially, $\xi(i, 0)$ is set to 0, and $\xi(i, m) = \infty$ for $m > 0$. Then each $\xi(i+1, m)$ can be computed in a constant time by (4). Let m^* be the maximum value of m such that $\xi(i, m) < \infty$. The corresponding selected flows with the profit m^* is determined by the backtracking of the above computation process. The concept of the design of the selection process comes from the single-machine job scheduling, and the correctness of our recurrence (4) can be easily shown [25].
3) **Schedule Instance Assignment:** Suppose that from the second phase, we select a set of ϱ flows, say $\bar{\mathbb{F}}$, such that $\sum_{i \in \bar{\mathbb{F}}} w_i = m^*$ and $d_i \le d_{i'}$ if $i < i'$. Let $\hat{\mathbb{F}} = \bar{\mathbb{F}}$. Then the schedule instances $\hat{\mathbb{L}}$ of $\hat{\mathbb{F}}$ can be obtained by Algorithm 2. Specifically, the time is initialized to 0, and the schedule instances of the service flows in $\hat{\mathbb{F}}$ are set based on the schedule sequence and ξ determined in the second phase.

Algorithm 2 `Assignment of Schedule Instances`

Require: a schedule sequence for requesting flows $\hat{\mathbb{F}} = \{F_1, F_2, \ldots, F_\varrho\}$
Ensure: a set of schedule instances $\hat{\mathbb{L}}$

$t \leftarrow 0$
for $i = 1$ to ϱ **do**
  for $j = 1$ to κ_q **do**
    $\hat{\ell}_{i,j} \leftarrow t$
    $t \leftarrow t + \tau_{i,j}$
  end for
end for
return $\hat{\mathbb{L}}$

Theorem 2: The schedule produced by Algorithm `DPS` is a feasible solution for `OPT-LS`.

Proof: The resulting schedule satisfies the transmission constraints because Algorithm 2 assigns the instances of scheduled flows by following its strict transmission order. Also, the deadlines of the flows are definitely met based on (4). In addition to the transmission constraints, the resulting schedule satisfies the interference-free constraints because only one of the flows is assigned to transmit at any time instance. Furthermore, we will prove by contradiction that all previously admitted flows will be chosen by Algorithm `DPS`. Notice that the admitted flows are feasibly scheduled by Algorthm `DPS` in last frame. Suppose that there exists an admitted service flow $F_i \notin \hat{\mathbb{F}}$. Since Algorithm `DPS` selects a subset of $\mathbb{F}$ with a maximum total weight, there must exist a set of requesting flows whose sum of weights is no less than $\underline{w}_i$. We reach the contradiction because $\underline{w}_i > \sum_{i \in \mathbb{F}, a_i = 0} w_i$ by (6). ■

Theorem 3: With the constant weight w_i, the time complexity of `DPS`, dominated by the second-phase operation, is

$$\xi(i+1,m) = \begin{cases} \min\{\xi(i,m-w_{i+1}) + Q_{i+1,m}, \xi(i,m)\} & \text{if } \xi(i,m-w_{i+1}) + Q_{i+1,m} \le \min\{d_{i+1}, c\} \\ \xi(i,m) & \text{otherwise} \end{cases} \tag{5}$$

$O(n \sum_{i \in \mathbb{F}} \underline{w}_i) = O(n^2 p)$.

B. Algorithm DPS-SR

Based on our DPS algorithm, we further pursue a feasible schedule with a higher degree of spatial reuse, The heuristic, named DPS-SR, is an amendment of DPS. Recall that in DPS, the schedule sequence is determined by (4), and the exclusive transmission of the selected flows is provided in Algorithm 2. However, allowing multiple transmissions to proceed simultaneously would enhance the schedule profit. To achieve a higher degree of spatial reuse, each transmission could be scheduled "as early as possible" while maintaining the *transmission constraints* and the *interference-free constraints*.

For any i, m ($1 \le i \le n, 1 \le m \le \sum_{i \in \mathbb{F}} w_i$), let $\xi(i,m)$ be associated with a corresponding schedule $\hat{\mathbb{F}}(i,m)$. Instead of adopting (4), our DPS-SR computes each $\xi(i,m)$ by using the recurrence shown in (5). Specifically, $Q_{i,m}$ is defined as the increase of the completion time for inserting the κ_i transmissions of F_i into $\hat{\mathbb{F}}(i-1, m-w_{i+1})$ while the transmission constraints and the interference-free constraints are satisfied. Then the resulting schedule can be obtained by $\hat{\mathbb{F}}(i,m^*)$, where m^* is the maximum value of all possible m such that $\xi(i,m^*) < \infty$.

Theorem 4: The schedule produced by Algorithm DPS-SR is a feasible solution for OPT-LS.

Proof: The proof is similar to that of Theorem 2 and omitted here. ■

Theorem 5: The schedule profit of DPS-SR is not less than DPS.

Proof: Since $Q_{i,m} \le P_i$ for each possible i, m, any $\xi(i,m)$ of Algorithm DPS-SR is less than or equal to $\xi(i,m)$ of Algorithm DPS. It implies that any schedule sequence produced by Algorithm DPS is also a candidate sequence of Algorithm DPS-SR. ■

Theorem 6: The time complexity of DPS-SR is $O(n^3 p \times \varphi^2)$.

C. Worst Case Analysis of Algorithms

This section studies the worst-case performance of Algorithm DPS and DPS-SR. Due to the analysis of in-approximability of OPT-LS shown in Section 1, it is hard to design an algorithm with a "theoretically" small approximation factor. Here, we are interested in finding out whether a "practically" small approximation factor is supported for Algorithms DPS/DPS-SR. We focus on the deriving of a "practically" small approximation factor for DPS. The factor can be also applied to DPS-SR since for any input instance, the schedule profit of DPS-SR is at least equal to that of DPS.

Recall that r represents the number of relay stations within a WRN, while φ are the maximum hop count of requesting service flows, respectively. We will show that the approximation factors of DPS are respectively $(1+r)$ and $(r\lceil\frac{\varphi}{2}\rceil)$ for the case of $\varphi \le 2$ (via one RS) and $\varphi > 2$ (through more than one RS). In WRNs, the values of r and φ are mostly dominated by network topology/configuration, not by system load. Thus the worst-case performance of our scheduling algorithms can be determined in the network planning stage and is stable in runtime. Furthermore, in the common implementation of QoS-supported WRN, the values of r and φ are pretty small due to the following reasons.

1) With large r or φ, service flows would pass through many RSs, and the considerably large transmission delay of the flows is unacceptable for QoS guarantee.
2) The operational complexity of a WRN will be significantly increased when the hop count is larger than 2 [26]. Consequently, the deployment of $\varphi \le 2$ is mandatory for IEEE 802.16j, while the multi-hop relay is optionally implemented [27].
3) The effective system capacity is about inversely proportional to the number of RSs associated with a BS [28]. If a BS is required to support a large number of RSs, the demands of SSs could not be fully satisfied. The existing usage scenarios of RS deployment in IEEE 802.16j are based on small values of r and φ ($r \le 16$ and $\varphi \le 3$) [29], [30], [31].

The following theorem provides inspiration for our worst-case analysis.

Theorem 7: Algorithm DPS produces an optimal profit for a special case of input where the concurrent transmissions for different service flows are not allowed, *i.e.*, $I_{i,j}$ is equal to E for $(i,j) \in \mathbb{T}$.

Proof: If $I_{i,j}$ is equal to E for $(i,j) \in \mathbb{T}$, the service flows are referred to as single-machine jobs. Then the problem OPT-LS can be reduced to minimize the weighted number of tardy jobs on a single machine, and the proof has been presented by [25]. ■

For any input instance $\mathbb{F}$, we can construct a corresponding set of flows, say $\mathbb{F}^{(3)}$, such that $I^{(3)}_{i,j}$ is equal to E for $(i,j) \in \mathbb{T}^{(3)}$. By Theorem 7, we have $\Gamma(\tilde{\mathbb{F}}^{(3)}) = \Gamma(\hat{\mathbb{F}})$. If different service flows are allowed to be simultaneously transmitted, we would like to extend the above idea to obtain a practically small h such that

$$\frac{\Gamma(\tilde{\mathbb{F}})}{h} \le \Gamma(\tilde{\mathbb{F}}^{(3)}) \tag{7}$$

Before the analysis is presented in detail, the intuition behind our following proofs is summarized as follows. In WRNs, the receiver of any uplink transmission can be the BS or RS, not the SS. For any time, an optimal algorithm can schedule at most $(1+r)$ transmissions concurrently. In order to find a proper h, we try to divide the service flows into $(1+r)$ regions such that any two links within different regions will not interfere with each other. However, due to our general interference model, each link could potentially interfere with any other links. Moreover, a service flow could consist of more than one transmission, and the transmissions of a service flow might belong to different regions. Also, the transmissions of a flow should follow a strict order under the delay constraints. The characteristics of delay constraints and general interference model lead to the difficulty of analyzing the approximation bound for Algorithm DPS. To conquer this, we start the analysis with the subproblem of OPT-LS: each flow has at most two transmissions, *i.e.*, $\varphi \le 2$. A flow set $\mathbb{F}^{(1)}$ will be constructed by merging multiple transmissions of each flow in $\mathbb{F}$ into one. Then *we intend to show that*

$$\frac{\Gamma(\tilde{\mathbb{F}})}{1+r} \le \frac{\Gamma(\tilde{\mathbb{F}}^{(1)})}{1+r} \le \Gamma(\tilde{\mathbb{F}}^{(3)}) \tag{8}$$

In our analysis, there is one base station and r relay stations. Let station 1 be the base station, and stations α ($2 \le \alpha \le r+1$) be the relay stations. For any input instance $\mathbb{F}$, we construct $\mathbb{F}^{(1)} = \{F^{(1)}_1, F^{(1)}_2, \ldots, F^{(1)}_n\}$ such that for $i \in \mathbb{F}^{(1)}$, $b^{(1)}_i = b_i$; $d^{(1)}_i = d_i$; $\kappa^{(1)}_i = \lceil \kappa_i/2 \rceil$; $w^{(1)}_i = w_i$. Also, for $(i,j) \in \mathbb{T}^{(1)}$, we have $TX^{(1)}_{i,j} = TX_{i,2j-1}$ and $RX^{(1)}_{i,j} = RX_{i,2j-1}$. The interference set $I^{(1)}_{i,j}$ is constructed as $I^{(1)}_{i,j} = \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \beta = RX^{(1)}_{i,j}\} \subseteq I_{i,2j-1}$. $\tau^{(1)}_{i,j}$ is set as $\tau_{i,2j-1} + \tau_{i,2j}$ if $2j \le \kappa_i$. Otherwise, $\tau^{(1)}_{i,j} = \tau_{i,2j-1}$.

Theorem 8: Let $\tilde{\mathbb{F}}$ and $\tilde{\mathbb{F}}^{(1)}$ be the optimal schedules for an input instance $\mathbb{F}$ and the constructed $\mathbb{F}^{(1)}$, respectively. Then $\Gamma(\tilde{\mathbb{F}}) \le \Gamma(\tilde{\mathbb{F}}^{(1)})$.

Proof:

For any input instance $\mathbb{F}$, we try to construct a transformation $f : \tilde{\mathbb{F}} \to \hat{\mathbb{F}}^{(1)}$ where $\hat{\mathbb{F}}^{(1)}$ is a feasible schedule for $\mathbb{F}^{(1)}$ such that $\Gamma(\tilde{\mathbb{F}}) = \Gamma(\hat{\mathbb{F}}^{(1)})$. If the transformation can be done, then the theorem holds since $\Gamma(\hat{\mathbb{F}}^{(1)}) \le \Gamma(\tilde{\mathbb{F}}^{(1)})$. For $(i, j) \in \mathbb{T}^{(1)}_{i,j}$, let $\hat{\ell}^{(1)}_{i,j} = \hat{\ell}_{i,2j-1}$. Now we prove $\hat{\mathbb{F}}^{(1)}$ is a feasible schedule. $\hat{\mathbb{F}}^{(1)}$ satisfies the *transmission constraints* because for all i, j $(1 \le i \le |\hat{\mathbb{F}}^{(1)}|, 1 \le j \le \kappa^{(1)}_i - 1)$

$$\begin{aligned} \hat{\ell}^{(1)}_{i,j} &= \hat{\ell}_{i,2j-1} \\ &\ge \hat{\ell}_{i,2j-2} + \tau_{i,2j-2} \\ &\ge \hat{\ell}_{i,2j-3} + \tau_{i,2j-2} + \tau_{i,2j-3} \\ &\ge \hat{\ell}_{i,2(j-1)-1} + \tau_{i,2(j-1)} + \tau_{i,2(j-1)-1} \\ &= \hat{\ell}^{(1)}_{i,(j-1)} + \tau^{(1)}_{i,j-1} \end{aligned}$$

and for all i $(1 \le i \le |\hat{\mathbb{F}}^{(1)}|)$,

$$\begin{aligned} \hat{\ell}^{(1)}_{i,\kappa^{(1)}_i} &= \hat{\ell}_{i,2\kappa^{(1)}_i-1} \\ &= \hat{\ell}_{i,2\lceil\kappa_i/2\rceil-1} \\ &= \begin{cases} \ell_{i,\kappa_i-1} & \text{if } \kappa_i \text{ is even} \\ \ell_{i,\kappa_i} & \text{otherwise} \end{cases} \\ &\le \begin{cases} d_i - \tau_{i,\kappa_i-1} - \tau_{i,\kappa_i} & \text{if } \kappa_i \text{ is even} \\ d_i - \tau_{i,\kappa_i} & \text{otherwise} \end{cases} \\ &= d^{(1)}_i - \tau^{(1)}_{i,\kappa^{(1)}_i} \end{aligned}$$

In addition to the *transmission constraints*, we show that $\hat{\mathbb{F}}^{(1)}$ satisfies the *interference-free constraints*. To prove this, each transmission $\hat{T}^{(1)}_{i,j}$ of $\hat{F}^{(1)}_i$ is separated into $\hat{T}^{(1)}_{i,j,1}$ and $\hat{T}^{(1)}_{i,j,2}$ such that $\tau^{(1)}_{i,j,1} = \tau_{i,2j-1}$, $\tau^{(1)}_{i,j,2} = \tau_{i,2j}$, $\hat{\ell}^{(1)}_{i,j,1} = \hat{\ell}_{i,2j-1}$, and $\hat{\ell}^{(1)}_{i,j,2} = \hat{\ell}_{i,2j-1}$[2]. $\hat{\mathbb{F}}^{(1)}$ satisfies the *interference-free constraints* if the following statement holds. If any two transmissions $\hat{T}^{(1)}_{i,j,\gamma}$ and $\hat{T}^{(1)}_{i',j',\gamma'}$ are overlapped, then we have $(TX^{(1)}_{i,j}, RX^{(1)}_{i,j}) \notin I^{(1)}_{i',j'}$ and $(TX^{(1)}_{i',j'}, RX^{(1)}_{i',j'}) \notin I^{(1)}_{i,j}$. The proof can be classified into the following three cases based on γ and γ'.

1) Case 1: $\gamma = \gamma' = 1$
   Suppose that there exists a transmission $\hat{T}^{(1)}_{i,j,1}$, and the transmission $\hat{T}^{(1)}_{i,j,1}$ is interfered by another transmission $\hat{T}^{(1)}_{i',j',1}$ or vice vera. That is, $(TX^{(1)}_{i,j,1}, RX^{(1)}_{i,j,1}) \in I^{(1)}_{i',j',1}$ or $(TX^{(1)}_{i',j',1}, RX^{(1)}_{i',j',1}) \in I^{(1)}_{i,j,1}$. Then we can find the corresponding transmissions in $\tilde{\mathbb{F}}$ such that $(TX_{i,2j-1}, RX_{i,2j-1}) \in I_{i',2j'-1}$ or $(TX_{i',2j'-1}, RX_{i',2j'-1}) \in I_{i,2j-1}$. If $\hat{T}^{(1)}_{i,j,1}$ and $\hat{T}^{(1)}_{i',j',1}$ are overlapped, $\tilde{\mathbb{F}}$ violates the *interference-free constraints* because $\hat{T}_{i,2j-1}$ and $\hat{T}_{i',2j'-1}$ are overlapped, which makes the contradiction.
2) Case 2: $\gamma = 1$, $\gamma' = 2$
   Assume that $\hat{T}^{(1)}_{i,j,1}$ and $\hat{T}^{(1)}_{i',j',2}$ are overlapped. By our construction, $\hat{T}_{i,2j-1}$ and $\hat{T}_{i',2j'}$ are also overlapped. If $(TX^{(1)}_{i,j,1}, RX^{(1)}_{i,j,1}) \in I^{(1)}_{i',j',2}$, then we obtain

$$\begin{aligned} I_{i',2j'} &\supseteq \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \\ &\qquad \alpha = TX_{i',2j'} \text{ or } \alpha = RX_{i',2j'} \\ &\qquad \text{or } \beta = TX_{i',2j'} \text{ or } \beta = RX_{i',2j'}\} \\ &\supseteq \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \beta = TX_{i',2j'}\} \\ &= \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \beta = RX_{i',2j'-1}\} \\ &= \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \beta = RX^{(1)}_{i',j'}\} \\ &= I^{(1)}_{i',j'} \\ &= I^{(1)}_{i',j',2} \\ &\ni (TX^{(1)}_{i,j,1}, RX^{(1)}_{i,j,1}) \\ &= (TX_{i,2j-1}, RX_{i,2j-1}) \end{aligned}$$

   which makes the contradiction.
   Similarly, if $(TX^{(1)}_{i',j',2}, RX^{(1)}_{i',j',2}) \in I^{(1)}_{i,j,1}$, then we have

$$\begin{aligned} I_{i,2j-1} &\supseteq \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \\ &\qquad \beta = TX_{i,2j-1} \text{ or } \beta = RX_{i,2j-1} \\ &\qquad \text{or } \alpha = TX_{i,2j-1} \text{ or } \alpha = RX_{i,2j-1}\} \\ &\supseteq \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \beta = RX_{i',2j'-1}\} \\ &= \{(\alpha,\beta) \mid (\alpha,\beta) \in E; \beta = RX^{(1)}_{i',j'}\} \\ &= I^{(1)}_{i',j'} \\ &= I^{(1)}_{i',j',1} \\ &\ni (TX^{(1)}_{i',j',1}, RX^{(1)}_{i',j',1}) \\ &= (TX^{(1)}_{i',j',2}, RX^{(1)}_{i',j',2}) \\ &= (TX_{i',2j'}, RX_{i,2j'}) \end{aligned}$$

   which makes the contradiction.
3) Case 3: $\gamma = \gamma' = 2$
   The proof in this case is omitted since it is similar to that in case 2.

■

[2] $\hat{T}^{(1)}_{i,j,2}$ exists if and only if $\hat{T}_{i,2j}$ exists.

For any input instance $\mathbb{F}^{(1)}$, we construct a transformation $g : \mathbb{F}^{(1)} \to \mathbb{F}^{(3)}$ such that each flow $F^{(3)}_i$ is equal to $F^{(1)}_i$ except for $I^{(3)}_{i,j} = E$ for $(i, j) \in \mathbb{T}^{(3)}$. Then we derive the following theorem.

Theorem 9: For $\varphi \le 2$, given the *optimal schedules* $\tilde{\mathbb{F}}^{(1)}$ and $\tilde{\mathbb{F}}^{(3)}$ for the input instance $\mathbb{F}^{(1)}$ and its transformed instance $\mathbb{F}^{(3)}$, we have $\Gamma(\tilde{\mathbb{F}}^{(1)}) \le (1 + r)\Gamma(\tilde{\mathbb{F}}^{(3)})$.

Proof: Given $\varphi \le 2$, we have $\varphi^{(1)} = \kappa^{(1)}_i = 1$ for $i \in \mathbb{F}^{(1)}$. $\mathbb{F}^{(1)}$ is divided into $1 + r$ disjoint subsets $\mathbb{F}^{(1,1)}, \mathbb{F}^{(1,2)}, \ldots, \mathbb{F}^{(1,1+r)}$ such that $RX^{(1,u)}_{i,j}$ is station u for any $1 \le u \le (1 + r)$, $(i, j) \in \mathbb{T}^{(1,u)}$. Then

$$\Gamma(\tilde{\mathbb{F}}^{(1)}) \le \sum_{1 \le u \le (1+r)} \Gamma(\tilde{\mathbb{F}}^{(1,u)}). \tag{9}$$

The above inequality holds due to the following reason. We divide $\hat{\mathbb{F}}^{(1)}$ into $1+r$ disjoint subsets $\hat{\mathbb{F}}^{[1,1]}, \ldots, \hat{\mathbb{F}}^{[1,1+r]}$ such that $RX^{[1,u]}_{i,1}$ is station u for $i \in \hat{\mathbb{F}}^{[1,u]}$. Assigning $\hat{\ell}^{[1,u]}_{i,1}$ to $\hat{\ell}^{(1,u)}_{i,1}$ produces a feasible schedule for each $\mathbb{F}^{(1,u)}$. Thus every feasible schedule for $\mathbb{F}^{(1)}$ can be applicable to $\mathbb{F}^{(1,u)}$ for any $1 \le u \le 1 + r$.

Moreover, every transmission in $\mathbb{T}^{(1,u)}$ has the same receiver, and the overlapping is not allowed for any feasible schedule $\hat{\mathbb{F}}^{(1,u)}$. Thus, $I^{(1,u)}_{i,1}$ can be set to E for each i, u $(i \in \mathbb{F}^{(1,u)}, 1 \le u \le 1 + r)$, and the resulting $\Gamma(\tilde{\mathbb{F}}^{(1,u)})$ is still maintained. However, $\mathbb{F}^{(1,u)}$ is currently a subset of (or included in) $\mathbb{F}^{(3)}$. It is sufficient to show

$$\Gamma(\tilde{\mathbb{F}}^{(1,u)}) \le \Gamma(\tilde{\mathbb{F}}^{(3)}). \tag{10}$$

for any $1 \le u \le 1 + r$. Substituting (10) into (9),

$$\Gamma(\tilde{\mathbb{F}}^{(1)}) \le \sum_{1 \le u \le (1+r)} \Gamma(\tilde{\mathbb{F}}^{(3)}) = (1 + r) \times \Gamma(\tilde{\mathbb{F}}^{(3)}) \tag{11}$$

■

By Theorem 8 and Theorem 9, we conclude the following result.

Theorem 10: For $\varphi \le 2$, Algorithm DPS is an $(1 + r)$-approximation algorithm for $\mathbb{F}$.

Considering $\varphi > 2$, Theorem 9 can not hold because $\kappa^{(1)}_i$ can be greater than 1 for some $i \in \mathbb{F}^{(1)}$. Then the separation of $\mathbb{F}^{(1)}$ described in the proof of Theorem 9 is not allowed since there probably exist j and j' such that $RX_{i,j} \ne RX_{i,j'}$. Then for any $\mathbb{F}^{(1)}$, a flow set $\mathbb{F}^{(2)}$ will be constructed, and *we intend to show that*

$$\frac{\Gamma(\tilde{\mathbb{F}}^{(1)})}{(1 + r)\zeta} \le \frac{\Gamma(\tilde{\mathbb{F}}^{(2)})}{(1 + r)} \le \Gamma(\tilde{\mathbb{F}}^{(3)}) \tag{12}$$

for some constant ζ. The main difference between $\mathbb{F}^{(1)}$ and $\mathbb{F}^{(2)}$ is that the transceivers of transmissions of each flow in $\mathbb{F}^{(1)}$ will be modified such that the transceivers of any transmissions belong to the same flow in $\mathbb{F}^{(2)}$ are the same. Specifically, for any $\mathbb{F}^{(1)}$, we try to construct $\mathbb{F}^{(2)}$ such that for $i \in \mathbb{F}^{(2)}$, $F_i^{(2)}$ is equal to $F_i^{(1)}$ except that the transceivers in $\mathbb{F}^{(2)}$ are constrained to the following properties:

Property 1) $RX_{i,j}^{(2)} = RX_{i,j'}^{(2)}$ for $(i, j \neq j') \in \mathbb{T}^{(2)}$

Property 2) $TX_{i,j}^{(2)} = TX_{i,j'}^{(2)}$ for $(i, j \neq j') \in \mathbb{T}^{(2)}$

Property 3) $(TX_{i,j}^{(2)}, RX_{i,j}^{(2)}) \in E$ for $(i, j) \in \mathbb{T}^{(2)}$

With these properties, each transmission of any flow in $\mathbb{F}^{(2)}$ has the same transmitter and receiver, and then we have $I_{i,j}^{(2)} = I_{i,j'}^{(2)}$ for $(i, j \neq j') \in \mathbb{T}^{(2)}$ due to the primary interference.

Theorem 11: Given the optimal schedules $\tilde{\mathbb{F}}^{(2)}$ and $\tilde{\mathbb{F}}^{(3)}$ for input instance $\mathbb{F}^{(2)}$ and its transformed instance $\mathbb{F}^{(3)}$, we have $\Gamma(\tilde{\mathbb{F}}^{(2)}) \leq (1 + r)\Gamma(\tilde{\mathbb{F}}^{(3)})$.

Proof: The proof is similar to that of Theorem 9 and omitted here. ■

Moreover, for any $\tilde{\mathbb{F}}^{(1)}$, if there exists a transformation $f' : \tilde{\mathbb{F}}^{(1)} \to \hat{\mathbb{F}}^{(2)}$ such that $\hat{\mathbb{F}}^{(2)}$ is a feasible schedule for $\mathbb{F}^{(2)}$ and $\Gamma(\tilde{\mathbb{F}}^{(1)}) \leq \zeta \times \Gamma(\hat{\mathbb{F}}^{(2)})$ for some constant ζ, we obtain $\Gamma(\tilde{\mathbb{F}}^{(1)}) \leq \zeta \times \Gamma(\tilde{\mathbb{F}}^{(2)})$. In order to construct a legitimate $\mathbb{F}^{(2)}$ such that ζ is small,

Algorithm 3 $\mathbb{F}^{(2)}$ CONSTRUCTION

```
Require: F^(1), F~^(1), G, Ğ^(1)
Ensure: F^(2)
F^(2) ← F^(1)
for i ∈ F^(1) do
    if there exist i' ∈ F~^(1) and β such that F_i^(1) = F~_i'^(1) and i ∈ M̌_β then
        find any α such that (α,β) ∈ E
    else
        arbitrarily select an edge (α,β) ∈ E
    end if
    for (i, j) ∈ T^(1) do
        TX_{i,j}^(2) ← the station β
        RX_{i,j}^(2) ← the station α
    end for
end for
return F^(2)
```

Algorithm 4 $\hat{\mathbb{F}}^{(2)}$ CONSTRUCTION

```
Require: F^(2), F^(1), F~^(1), Ğ^(1)
Ensure: F^^(2)
1: construct F^^(2) = {F^_1^(2), ..., F^_{|V̌|}^(1)}
2: θ ← 1
3: for i ∈ F^(1) do
4:     if there exist i' ∈ F~^(1) and β such that F_i^(1) = F~_i'^(1), and i ∈ M̌_β then
5:         F^_θ^(2) ← F_i^(2)
6:         ℓ^_{θ,j}^(2) ← ℓ~_{i,j}^(1) for all possible j
7:         θ ← θ + 1
8:     end if
9: end for
10: return F^^(2); L^(2)
```

we model $\tilde{\mathbb{F}}^{(1)}$ as a vertex-weighted graph $G^{(1)} = (M, J)$. Each vertex i in $G^{(1)}$ represents $\tilde{F}_i^{(1)} \in \tilde{\mathbb{F}}^{(1)}$, and $|\tilde{\mathbb{F}}^{(1)}| = |M|$. Each vertex i is assigned a weight $z_i = w_i^{(1)}$, while the total weight is denoted as $W = \sum_{i \in M} z_i$. Let $J = \left\{(i, j) \mid i \in M, j \in M, \exists i', j', \tilde{T}_{i,i'}^{(1)} \Leftrightarrow \tilde{T}_{j,j'}^{(1)}\right\}$. The degree of each vertex i is defined by δ_i, while the weighted average degree of $G^{(1)}$ is $\bar{\delta} = \frac{\sum_{i \in M}(z_i \times \delta_i)}{W}$. Suppose that there exists a subset $\check{M} \subseteq M$ such that the induced subgraph $\check{G}^{(1)} = (\check{M}, \check{J})$ is $(1+r)$-colorable and $\sum_{i \in M} z_i \leq \zeta \times \sum_{i \in \check{M}} z_i$. Then we have $(1+r)$ disjoint independent vertex sets $\check{M}_1, \ldots, \check{M}_{1+r}$ and $\bigcup_{1 \leq u \leq 1+r} \check{M}_u = \check{M}$.

For any $\tilde{\mathbb{F}}^{(1)}$, as shown in Algorithm 3, we modify the transceivers of $\tilde{\mathbb{F}}^{(1)}$ to make $\mathbb{F}^{(2)}$ conform to the three properties listed above. Based on the $\tilde{\mathbb{F}}^{(1)}$ and the derived $\mathbb{F}^{(2)}$, Algorithm 4 computes $\hat{\mathbb{F}}^{(2)}$ such that any flow is in $\hat{\mathbb{F}}^{(2)}$ if and only if there exists a corresponding vertex in $\check{G}^{(1)}$, *i.e.*, $\sum_{i \in M} z_i = \Gamma(\hat{\mathbb{F}}^{(2)})$. By this construction, if $\hat{\mathbb{F}}^{(2)}$ is feasible for $\mathbb{F}^{(2)}$, the following equation can be derived.

$$\Gamma(\tilde{\mathbb{F}}^{(1)}) = \sum_{i \in M} z_i \leq \zeta \times \sum_{i \in \check{M}} z_i = \zeta \times \Gamma(\hat{\mathbb{F}}^{(2)}) \leq \zeta \times \Gamma(\tilde{\mathbb{F}}^{(2)}) \qquad (13)$$

Theorem 12: $\hat{\mathbb{F}}^{(2)}$ produced by Algorithm 4 is a feasible schedule for $\mathbb{F}^{(2)}$.

Proof: For $i \in \hat{\mathbb{F}}^{(2)}$, there exists a service flow $\tilde{F}_{\pi_i}^{(1)}$ such that $\hat{\ell}_{i,j}^{(2)} = \tilde{\ell}_{\pi_i,j}^{(1)}$ (see line 5 of Algorithm 4). $\hat{\mathbb{F}}^{(2)}$ satisfies the *transmission constraints* since $\tilde{\mathbb{F}}^{(1)}$ is a feasible schedule. Also, $I_{i,j}^{(2)} = I_{\pi_i,j}^{(1)} = \{(\beta, \alpha) \mid (\beta, \alpha) \in E; \alpha \text{ is station } i\}$. Then, for $(i \neq i', j \neq j') \in \hat{\mathbb{T}}^{(2)}$,

1) When $RX_{i,j}^{(2)} \neq RX_{i',j'}^{(2)}$, we have $(TX_{i,j}^{(2)}, RX_{i,j}^{(2)}) \notin I_{i',j'}^{(2)}$ and $(TX_{i',j'}^{(2)}, RX_{i',j'}^{(2)}) \notin I_{i,j}^{(2)}$.
2) When $RX_{i,j}^{(2)} = RX_{i',j'}^{(2)}$, then $\tilde{T}_{\pi_{i,j}}^{(1)} \not\Leftrightarrow \tilde{T}_{\pi_{i',j'}}^{(1)}$ since $M_{RX_{i,j}^{(2)}}$ is an independent set. Thus we have $\hat{T}_{i,j}^{(2)} \not\Leftrightarrow \hat{T}_{i',j'}^{(2)}$.

■

Since $z_i = w_i^{(1)}$, by (13), Theorem 11, and Theorem 12, we conclude the following corollary.

Corollary 1: $\Gamma(\tilde{\mathbb{F}}^{(1)}) \leq \zeta \times (1 + r) \times \Gamma(\tilde{\mathbb{F}}^3)$ if for any $G^{(1)}$ the induced subgraph $\check{G}^{(1)}$ is $(1+r)$-colorable and $\sum_{i \in M} z_i \leq \zeta \times \sum_{i \in \check{M}} z_i$.

Now we will determine the lower bound ζ of the maximum weighted cardinality of the family of $(1 + r)$ disjoint independent sets from $G^{(1)}$. Before determining ζ, the following theorem gives an important property for $G^{(1)}$.

Theorem 13: For any $\tilde{\mathbb{F}}^{(1)}$, $\bar{\delta} \leq (1 + r)\left\lceil \frac{\varphi}{2} \right\rceil$.

Proof: For any $\tilde{\mathbb{F}}^{(1)}$, let $\tilde{\mathbb{T}}^{(1)} = \bigcup_{(i,j) \in \tilde{\mathbb{T}}^{(1)}} \tilde{T}_{i,j}^{(1)}$. We separate $\tilde{\mathbb{T}}^{(1)}$ into $(1 + r)$ disjoint subsets $\tilde{\mathbb{T}}^{(1,1)}, \ldots, \tilde{\mathbb{T}}^{(1,1+r)}$ such that $RX_{i,j}^{(1)}$ is station α for $(i, j) \in \tilde{\mathbb{T}}^{(1,\alpha)}$, *i.e.*, $\tilde{\mathbb{T}}^{(1,\alpha)} = \{\tilde{T}_{i,j}^{(1)} \mid \tilde{T}_{i,j}^{(1)} \in \tilde{\mathbb{T}}^{(1)}; RX_{i,j}^{(1)} = \text{ is station } \alpha\}$. Since $I_{i,j}^{(1)} = \{(\alpha, \beta) \mid (\alpha, \beta) \in E; \beta = \text{the station } i\}$, we have $\tilde{T}_{i,j}^{(1,\alpha)} \not\Leftrightarrow \tilde{T}_{i',j'}^{(1,\alpha)}$ for $(i \neq i', j \neq j') \in \tilde{\mathbb{T}}^{(1,\alpha)}$. We denote $\sigma(\tilde{\mathbb{T}}^{(1,\alpha)})$ as the completion time of the last finished transmission of $\tilde{\mathbb{T}}^{(1,\alpha)}$, *i.e.*, $\sigma(\tilde{\mathbb{T}}^{(1,\alpha)}) = \max_{(i,j) \in \tilde{\mathbb{T}}^{(1,\alpha)}}\{\tilde{\ell}_{i,j}^{(1)} + \tau_{i,j}^{(1)}\}$. Similarly, the starting time of the first transmission of $\tilde{\mathbb{T}}^{(1,\alpha)}$ is signified by $\varsigma(\tilde{\mathbb{T}}^{(1,\alpha)}) = \min_{(i,j) \in \tilde{\mathbb{T}}^{(1,\alpha)}}\{\tilde{\ell}_{i,j}^{(1)}\}$. We denote $x(\tilde{\mathbb{T}}^{(1,\alpha)}, \sigma(\tilde{\mathbb{T}}^{(1,\alpha)}), \varsigma(\tilde{\mathbb{T}}^{(1,\alpha)}))$ be the weight of flows that have at least a transmission in $\tilde{\mathbb{T}}^{(1,\alpha)}$ satisfying that $\tilde{\ell}_{i,j}^{(1)} + \tau_{i,j}^{(1)} \geq t$ or $\tilde{\ell}_{i,j}^{(1)} < t'$. For any α $(1 \leq \alpha \leq 1 + r)$, let $\Omega(\tilde{\mathbb{T}}^{(1,\alpha)}) = \left\{(i, i') | \tilde{T}_{i,j}^{(1)} \Leftrightarrow \tilde{T}_{i',j'}^{(1)}, \tilde{T}_{i,j}^{(1)} \in \tilde{\mathbb{T}}^{(1,\alpha)}, \tilde{T}_{i',j'}^{(1)} \in \tilde{\mathbb{T}}^{(1)} \setminus \tilde{\mathbb{T}}^{(1,\alpha)}\right\}$ and let

$$X(\tilde{\mathbb{T}}^{(1,\alpha)}) = \sum_{(i,i') \in \Omega(\tilde{\mathbb{T}}^{(1,\alpha)})} w_{i'} \qquad (14)$$

By induction on $|\tilde{\mathbb{T}}^{(1,\alpha)}|$, we prove that $X(\tilde{\mathbb{T}}^{(1,\alpha)})$ is less than or equal to $\sum_{(i,j) \in \tilde{\mathbb{T}}^{(1,\alpha)}} w_i + \sum_{1 \leq u \leq (1+r),\ u \neq \alpha} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u)}), \sigma(\tilde{\mathbb{T}}^{(1,u)})\right\}$. As $|\tilde{\mathbb{T}}^{(1,\alpha)}| = 1$, it trivially holds because

$$X(\tilde{\mathbb{T}}^{(1,\alpha)}) \quad \leq \sum_{1 \leq u \leq (1+r),\ u \neq \alpha} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u)}), \sigma(\tilde{\mathbb{T}}^{(1,u)})\right\}$$

Suppose that the hypothesis of induction holds as $|\tilde{\mathbb{T}}^{(1,\alpha)}| = l$. As $|\tilde{\mathbb{T}}^{(1,\alpha)}| = l + 1$, there exists a timepoint m such that the earliest transmission $\tilde{T}^{(1,\alpha)}_{i,j} \in \tilde{\mathbb{T}}^{(1,\alpha)}$ ends at m. We divide $\tilde{\mathbb{T}}^{(1,\alpha)}$ into $\tilde{\mathbb{T}}^{(1,\alpha,1)}$ and $\tilde{\mathbb{T}}^{(1,\alpha,2)}$ such that $\tilde{\mathbb{T}}^{(1,\alpha,1)} = \{\tilde{T}^{(1,\alpha)}_{i,j}\}$ and $\tilde{\mathbb{T}}^{(1,\alpha,2)} = \tilde{\mathbb{T}}^{(1,\alpha)} \setminus \tilde{\mathbb{T}}^{(1,\alpha,1)}$. Since $|\tilde{\mathbb{T}}^{(1,\alpha,1)}| = 1$ and $|\tilde{\mathbb{T}}^{(1,\alpha,2)}| = |\tilde{\mathbb{T}}^{(1,\alpha)}| - 1$, we have

$$\begin{aligned} X(\tilde{\mathbb{T}}^{(1,\alpha)}) &= X(\tilde{\mathbb{T}}^{(1,\alpha,1)}) + X(\tilde{\mathbb{T}}^{(1,\alpha,2)}) \\ &\le \sum_{1\le u\le(1+r),\ u\ne\alpha} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u,1)}), \sigma(\tilde{\mathbb{T}}^{(1,u,1)})\right\} \\ &\quad + \sum_{(i,j)\in\tilde{\mathbb{T}}^{(1,u,2)}} w_i + \sum_{1\le u\le(1+r),\ u\ne\alpha} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u,2)}), \sigma(\tilde{\mathbb{T}}^{(1,u,2)})\right\} \\ &\le \sum_{(i,j)\in\tilde{\mathbb{T}}^{(1,\alpha)}} w_i + \sum_{1\le u\le(1+r),\ u\ne\alpha} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u)}), \sigma(\tilde{\mathbb{T}}^{(1,u)})\right\}. \end{aligned}$$

Then we have

$$\begin{aligned} \bar{\delta} &= \frac{1}{W}\sum_{i\in M}(z_i\delta_i) \\ &= \frac{1}{W}\sum_{(i,j)\in J}(z_i + z_j) \\ &\le \frac{1}{W}\sum_{\alpha=1}^{r+1} X(\tilde{\mathbb{T}}^{(1,\alpha)}) \\ &\le \frac{1}{W}\sum_{\alpha=1}^{r+1}\left[\sum_{(i,j)\in\tilde{\mathbb{T}}^{(1,\alpha)}} w_i + \sum_{1\le u\le(1+r),\ u\ne\alpha} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u)}), \sigma(\tilde{\mathbb{T}}^{(1,u)})\right\}\right] \\ &\le \frac{1}{W}\sum_{\alpha=1}^{r+1}\left[\sum_{(i,j)\in\tilde{\mathbb{T}}^{(1,\alpha)}} w_i\right] + \frac{r}{W}\sum_{u=1}^{1+r} x\left\{u, \varsigma(\tilde{\mathbb{T}}^{(1,u)}), \sigma(\tilde{\mathbb{T}}^{(1,u)})\right\} \\ &= \frac{1}{W}\sum_{i\in\tilde{\mathbb{F}}^{(1)}}(\kappa_i^{(1)} \times w_i) + \frac{r}{W}\sum_{i\in\tilde{\mathbb{F}}^{(1)}}(\kappa_i^{(1)} \times w_i) \\ &\le \frac{1+r}{W}\sum_{i\in\tilde{\mathbb{F}}^{(1)}}(\varphi^{(1)} \times w_i) \\ &= (1+r)\varphi^{(1)} \\ &= (1+r)\left\lceil\frac{\varphi}{2}\right\rceil \end{aligned}$$

■

Theorem 14: There exists $\check{M} \subseteq M$ such that the induced subgraph $\check{G}^{(1)} = (\check{M}, \check{J})$ is $(1+r)$-colorable and $\sum_{i\in M} z_i \le \frac{2r\times\lceil\frac{\varphi}{2}\rceil}{1+r} \times \sum_{i\in\check{M}} z_i$.

Proof:

In [32], it had been shown that for any vertex-weighted graph $G^{(1)} = (M, Z)$, there exists an independent set whose weight is equal or greater than $W/(1+\bar{\delta})$. Then the theorem can be proven by iteratively performing `MAX WEIGHTED INDEPENDENT SET` $(r+1)$ times as follows. In the first iteration, the input is the graph $G^{(1)} = (M, J)$ with the weighted average degree $\bar{\delta}$ while the output is a maximum independent set $M_D \subseteq M$. By removing the derived independent set from $G^{(1)}$, we have the remaining vertex set $M_R = M \setminus M_D$ and the remaining edge set $J_R = J \setminus \{(\alpha,\beta) \mid \alpha \in M_D \text{ or } \beta \in M_D\}$. The remaining graph $G_R = (M_R, J_R)$ with the weighted average degree $\bar{\delta}_R$ will be formed as the input of the next iteration. Note that $\bar{\delta}_R \le \bar{\delta} - 1$ because any vertex in M will be deprived of at least one edge after an iteration and $z_i \ge 1$. If any vertex $i \in M_R$ has the same weighted degree as that in M, the vertex will be chosen as a vertex in M_D. Let Ψ_χ be the weight of the family of χ disjoint independent sets after the χth iteration. Then we have

$$\Psi_\chi \ \ge\ \Psi_{\chi-1} + \frac{W - \Psi_{\chi-1}}{\bar{\delta} - \chi}, \text{ for } \chi \ge 1$$

Then we prove $\Psi_\chi \ge (\chi W)/(1+\bar{\delta})$ by the induction on χ. Let $\Psi_0 = 0$. By the inductive hypothesis for $\Psi_{\chi-1}$,

$$\begin{aligned} \Psi_\chi &\ge \Psi_{\chi-1} + \frac{W - \Psi_{\chi-1}}{\bar{\delta} - \chi} \\ &\ge \frac{(\chi-1)W}{1+\bar{\delta}} + \frac{W - \frac{(\chi-1)W}{1+\bar{\delta}}}{\bar{\delta} - \chi} \\ &\ge \chi W/(1+\bar{\delta}) \end{aligned}$$

Thus, letting $\sum_{i\in\check{M}} z_i = \Psi_{r+1}$, we have

$$\begin{aligned} \sum_{i\in\check{M}} z_i &= \Psi_{1+r} \\ &\ge \frac{(1+r)W}{1+\bar{\delta}} \\ &\ge \sum_{i\in M} z_i \times \frac{1}{\left\lceil\frac{\varphi}{2}\right\rceil} \quad \text{(by Theorem 13)} \end{aligned}$$

The theorem is proven with $\zeta = \left\lceil\frac{\varphi}{2}\right\rceil$.

■

By Theorem 13, Theorem 14 and Colloary 1, we have $\Gamma(\tilde{\mathbb{F}}^{(1)}) \le (2r\lceil\frac{\varphi}{2}\rceil) \times \Gamma(\tilde{\mathbb{F}}^{(3)})$. Then we conclude the following result.

Theorem 15: For $\varphi > 2$, `DPS` is an $(2r\lceil\frac{\varphi}{2}\rceil)$-approximation algorithm, where φ is the maximum hop count of requesting flows.

D. Tradeoff of Time Complexity and Approximation Factor

This section discusses the tradeoff of time complexity and approximation factor for our algorithms `DPS` and `DPS-SR` with the consideration of weight setting of service flows. If the weight is not a constant, `DPS` will be a pseudo polynomial-time algorithm where the running time is $O(n^2pY)$, where $Y = \max(w_1, w_2, \ldots, w_n)$. With a large constant Y, the worst-case running time of `DPS` would be accordingly large, and a link scheduler could not afford to execute such a heavy-computation task. To reduce the time complexity, we can limit the number of precision bits of the weight by truncating the last b bits of w_i for all flow i in $\mathbb{F}$. Then the truncated weight will be

$$w_i^\eta = 2^b\left\lceil\frac{w_i}{2^b}\right\rceil \tag{15}$$

Suppose that $\hat{\mathbb{F}}$ and $\hat{\mathbb{F}}^\eta$ are the feasible schedules obtained by Algorithm `DPS` for any $\mathbb{F}$ and the corresponding $\mathbb{F}^\eta$, respectively. Since any feasible schedule of $\mathbb{F}$ is also a feasible schedule of $\mathbb{F}^\eta$, the schedule profit of Algorithm `DPS` for the requesting flow set $\mathbb{F}^\eta$, $\Gamma(\hat{\mathbb{F}}^\eta) = \sum_{i\in\hat{\mathbb{F}}^\eta} w_i^\eta$, is bounded by

$$\Gamma(\hat{\mathbb{F}}^\eta) \ \ge\ \sum_{i\in\hat{\mathbb{F}}}\left(w_i - 2^b\right) \tag{16}$$

Let $b = \left\lfloor\log_2[Y(1-\frac{1}{\epsilon})]\right\rfloor$. Then (16) can be rewritten as

$$\begin{aligned} \Gamma(\hat{\mathbb{F}}^\eta) &\ge \sum_{i\in\hat{\mathbb{F}}}\left[w_i - Y(1-\frac{1}{\epsilon})\right] \\ &\ge \sum_{i\in\hat{\mathbb{F}}}\left[w_i - w_i(1-\frac{1}{\epsilon})\right] \\ &= \sum_{i\in\hat{\mathbb{F}}}\frac{w_i}{\epsilon} = \frac{\Gamma(\hat{\mathbb{F}})}{\epsilon} \end{aligned} \tag{17}$$

Based on (17), the approximation factor of `DPS` is $\epsilon(1+r)$ for $\varphi \le 2$ and $\epsilon(1 + r \times 2 \times \lceil\frac{\varphi}{2}\rceil)$ for $\varphi \ge 3$, while the running time of `DPS` is reduced to $O(\frac{n^2pY}{2^b})$. The same technique can be applied to Algorithm `DPS-SR` so that the running time of `DPS-SR` with non-constant or large weight will be improved.

IV. Performance Evaluation

This section studies the performance of DPS and DPS-SR through our developed Monte-Carlo simulation. Since the existing link schedulers can not guarantee the schedulability of previously admitted service flows, the schedule profits obtained by DPS and DPS-SR are only compared with that of the optimal solution (OPT).

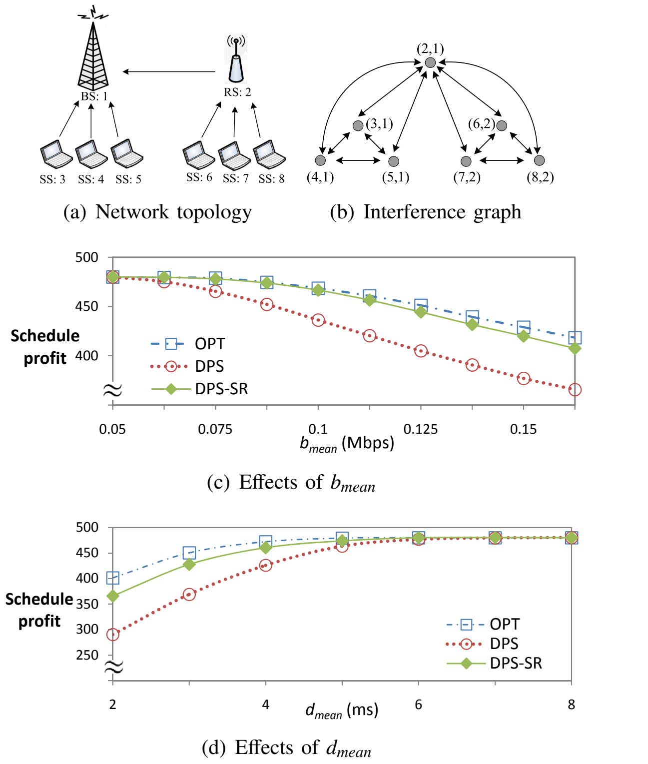
In our experiments, the IEEE 802.16 OFDM is assumed as the underlying modulation technology. The coding rate is 3/4. The data rates of relay links and of access links are respectively set to 18.36 Mbps and 6 Mbps. 18.36 Mbps and 6 Mbps are the raw data rates by adopting the modulation schemes of 64-quadrature amplitude modulation (64-QAM) and quadrature phase-shift keying (QPSK), respectively [17]. The frame duration is set to 10 ms [1]. We assume that each SS has four requesting flows and four admitted flows. In order to assure that the admitted flows can be scheduled in the frame, a schedulability test shall be done in advance for the admitted flows. For each flow, the Gamma distribution is adopted to generate its required bandwidth, deadline and weight. The Gamma distribution is used since it can approximate many other distributions as well as experimental data [33]. The parameters of the distributions of service flows are set as Table II.

In Figure 4(a), a network topology of the one-hop relay scenario is shown. Based on the topology, Figure 4(b) demonstrates the interference graph, where the vertices represent the wireless links and the edges indicate the interference between the wireless links. Figure 4(c) and Figure 4(d) show the experimental results under an one-hop relay scenario (*i.e.*, $\varphi = 2$) for our DPS, DPS-SR, and optimal solution (OPT). The optimal schedule is obtained by using the brute-force search. In Figure 4(c), we study the effect of b_{mean} on the schedule profit for our DPS, DPS-SR, and OPT. In this figure, b_{mean} ranges from 0.05 Mbps (close to the average bandwidth requirement of a high-quality voice call with silence compression) to 0.175 Mbps (close to mean data rate for compressed flash video). As b_{mean} increases, the schedule profit gradually degrades because network resources are exhausted. Figure 4(d) indicates the effect of d_{mean} on the schedule profit. The decrease of d_{mean} results in the reduction of the schedule profit because the service flows with urgent deadline are hard to be satisfied. From Figure 4(c) and Figure 4(d), we observe that the schedule profits of DPS and DPS-SR are quite close to that of OPT.

TABLE II
The Setting of the Distribution

Attribute	Parameter	Value
Bandwidth	Mean	b_{mean} = 50 Kbps
	Shape	b_{shape} = 14
Deadline	Mean	d_{mean} = 7 ms
	Shape	d_{shape} = 14
Weight	Mean	w_{mean} = 10
	Shape	w_{shape} = 14

Now the experiments are extended from one-hop relay to two-hop relay (*i.e.*, $\varphi = 3$). Figures 5(a) and 5(b) respectively show the network topology and the interference graph of the two-hop relay scenario. Figure 5(c) and Figure 5(d) respectively show the schedule profits for different b_{mean} and d_{mean} for our DPS, DPS-SR, and OPT. Compared with the case of one-hop relay, the schedule profits in the two-hop relay scenario are degraded for all algorithms under investigation. When the hop count is increased, the time for data relaying is increased, and hence the delay requirements of the flows are hard to fulfill. However, the performance of our DPS-SR is quite close to that of OPT in this case since link resources are effectively utilized by adopting spatial reuse. Although a larger difference of schedule profits between DPS and OPT, the lower time complexity of DPS makes DPS preferred for the WRNs where the BSs have low computational capability.

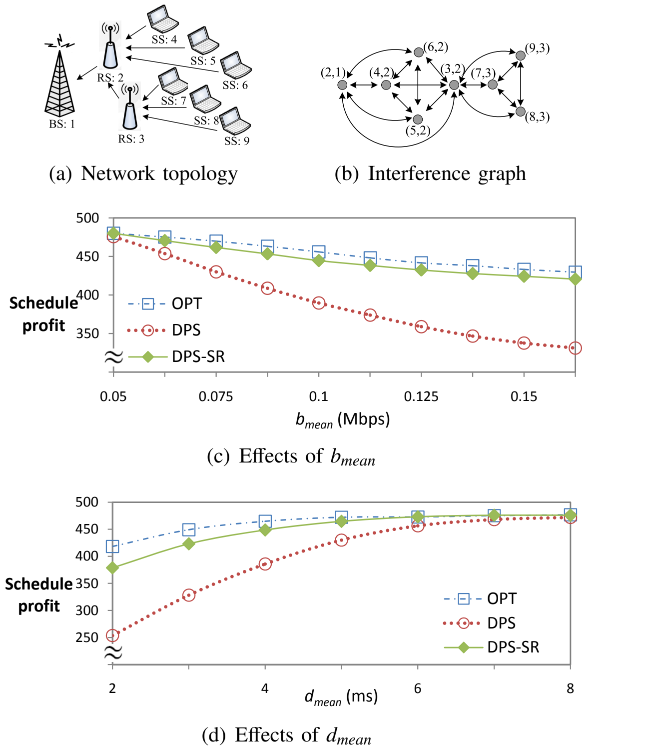


(a) Network topology (b) Interference graph

(c) Effects of b_{mean}

(d) Effects of d_{mean}

Fig. 4. Performance comparison of our DPS, DPS-SR, and OPT in an one-hop relay scenario.



(a) Network topology (b) Interference graph

(c) Effects of b_{mean}

(d) Effects of d_{mean}

Fig. 5. Performance comparison of our DPS, DPS-SR, and OPT in a two-hop relay scenario.

V. Conclusions

This paper studied an optimization problem for multi-hop link scheduling with explicit QoS guarantee for real-time services over *wireless relay networks* (WRNs). Our optimization problem is not limited to a specific objective, and the objective can be based on various kinds of performance metrics (*e.g.*, throughput, fairness and capacity) determined by service providers. Also, a general interference model was adopted. The theoretical analysis showed that the addressed problem is $\mathcal{NP}$-hard. Due to the problem intractability and in-approximability, we presented efficient algorithms with a practically small approximation factor. The experimental results indicated that our scheduling algorithms can achieve near-optimal performance.

For the future research, we would focus on the designing of a practical cross-layer scheduling algorithm that incorporates the consideration of physical channel conditions.

References

[1] "IEEE 802.16-2004, Air interface for fixed broadband wireless access systems," Piscataway, NJ, 2004.

[2] E. Arikan, "Some complexity results about packet radio networks," *IEEE Trans. Intell. Transp. Syst.*, vol. iT-30, pp. 910–918, July 1984.

[3] H. Viswanathan and S. Mukherjee, "Throughput-range tradeoff of wireless mesh backhaul networks," *IEEE J. Sel. Areas Commun.*, vol. 24, pp. 593–602, Mar. 2006.

[4] T. Jian, X. Guoliang, C. Chandler, and Z. Weiyi, "Link scheduling with power control for throughput enhancement in multihop wireless networks," *IEEE Trans. Veh. Technol.*, vol. 55, pp. 733–742, May 2006.

[5] A. Behzad and I. Rubin, "Optimum integrated link scheduling and power control for multihop wireless networks," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 194–205, Jan. 2007.

[6] X. Lin and S. Rasool, "A distributed joint channel-assignment, scheduling and routing algorithm for multi-channel ad hoc wireless networks," in *Proc. IEEE INFOCOM'07*, May 2007, pp. 1118–1126.

[7] G. Brar, D. M. Blough, and P. Santi, "Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks," in *Proc. ACM MOBICOM*, Sept. 2006, pp. 2–13.

[8] W. Wang, Y. Wang, X.-Y. Li, W.-Z. Song, and O. Frieder, "Efficient interference-aware TDMA link scheduling for static wireless networks," in *Proc. ACM MOBICOM*, Sept. 2006, pp. 262–273.

[9] M. Alicherry, R. Bhatia, and L. E. Li, "Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks," in *Proc. ACM MOBICOM*, Aug. 2005, pp. 58–72.

[10] S. Harish and S. Vinod, "Algorithms for routing and centralized scheduling to provide QoS in IEEE 802.16 mesh networks," in *Proc. of the 1st ACM workshop on Wireless multimedia networking and performance modeling*, Oct. 2005, pp. 140–149.

[11] P. Djukic and S. Valaee, "Link scheduling for minimum delay in spatial re-use tdma," in *Proc. IEEE INFOCOM'07*, May 2007, pp. 28–36.

[12] J. Tang, G. Xue, and W. Zhang, "Interference-aware topology control and QoS routing in multi-channel wireless mesh networks," in *Proc. ACM MOBIHOC*, May 2005, pp. 68–77.

[13] S. Sriram, T. B. Reddy, B. S. Manoj, and C. S. R. Murthy, "On the end-to-end call acceptance and the possibility of deterministic QoS guarantees in ad hoc wireless networks," in *Proc. ACM MOBIHOC*, May 2005, pp. 169–180.

[14] S. Lee, G. Narlikar, M. Pal, G. Wilfong, and L. Zhang, "Admission control for multihop wireless backhaul networks with QoS support," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC'06)*, Apr. 2006, pp. 92–97.

[15] *IEEE 802.16e-2005, Air Interface for Fixed and Mobile Broadband Wireless Access Systems*, IEEE Std. 802.16e-2005, 2005.

[16] B. Han, F. P. Tso, L. Lin, and W. Jia, "Performance evaluation of scheduling in IEEE 802.16 based wireless mesh networks," in *Proc. IEEE Intl. Conf. on Mobile Adhoc and Sensor Systems*, Oct. 2006, pp. 789–794.

[17] A. Ghosh, D. R. Wolter, J. G. Andrews, and R. Chen, "Broadband wireless access with WiMax/802.16: current performance benchmarks and future potential," *IEEE Commun. Mag.*, vol. 43, pp. 129–136, Feb. 2005.

[18] D. Branlund, M. Webb, J. Norin, and R. Popoli, "AAS direct signaling methodologies to support high capacity MR-BS to RS links," IEEE C802.16j-07/080, Jan. 2007.

[19] A. Baek *et al.*, "Harmonized contribution on 802.16j (mobile multihop relay) usage models," IEEE 802.16j-06/015, Sept. 2006.

[20] S. Ramanathan and E. L. Lloyd, "Scheduling algorithms for multi-hop radio networks," *ACM SIGCOMM Computer Communication Review*, vol. 22, pp. 211–222, Oct. 1992.

[21] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of $\mathcal{NP}$-Completeness*. New York, NY: W. H. Freeman & Co., 1990.

[22] C. Papadimitriou and M. Yannakakis, "Optimization, approximation, and complexity classes," in *STOC '88: Proceedings of the twentieth annual ACM symposium on Theory of computing*. New York, NY, USA: ACM Press, 1988, pp. 229–234.

[23] J. Håstad, "Clique is hard to approximate within $n^{1-\epsilon}$," *Acta Mathematica*, vol. 182, no. 1, pp. 105–142, 1999.

[24] M. Pinedo, *Scheduling: Theory, Algorithms, and Systems*. Upper saddle river, NJ: Prentice Hall, 2002.

[25] J. H. Anderson, *Handbook of Scheduling: Algorithms, Models, and Performance Analysis*. London: Chapman & Hall, 2004.

[26] P.-Y. Kong, H. Wang, Y. Ge, C.-K. Tham, and Y.-C. Liang, "PMP tree and hybrid PMP-mesh for multi-hop relay networks," IEEE C802.16j-06/008r2, May 2006.

[27] H. Kang *et al.*, "Proposed technical requirements guideline for IEEE 802.16 Relay TG," IEEE 802.16j-06/016r1, Oct. 2006.

[28] M. Naden, D. Kitchener, W. Tong, P. Zhu, G. Senarnath, H. Zhang, D. Steer, D. Yu, and G.-Q. Wang, "Metrics for multihop systems," IEEE C802.16j-06/014r1, May 2006.

[29] I.-K. Fu, W.-H. Sheen, C.-L. Hsiao, and C.-C. Tseng, "Reverse link performance of relay-based cellular systems in manhattan-like scenario," IEEE C80216mmr-06/004r1, Jan. 2006.

[30] J. Cho, M. Lee, H. Kang, S. Lee, J. Son, P. Joo, Y. So, S. Kim, D. Hong, J. Choe, B. Shin, and H. Choi, "On the throughput enhancementof fixed relay concept in manhattan-like urban environments," IEEE C802.16mmr-06/003, Jan. 2006.

[31] I. Guvenc, M.-R. Jeong, F. Watanabe, and C.-C. Chong, "Technical comments to P802.16j baseline document: 6.3.23.3 - use of ARQ in MBS," IEEE C802.16j-07/578r1, Nov. 2007.

[32] M. M. Halldórsson, "Approximations of weighted independent set and hereditary subset problems," *Journal of Graphs Algorithms and Applications*, vol. 4, no. 1, pp. 1–16, 2000.

[33] N. L. Johnson and S. Kotz, *Continuous Univariate Distributions-1*. New York: John Wiley & Sons, 1970.