Theory of Computation

Homework 5

Due: 2015/1/06

Problem 1 Suppose that there are n jobs to be assigned to m machines. Let t_i be the running time for job $i \in \{1 \ldots n\}$, $A[i] = j$ mean that job i is assigned to machine $j \in \{1 \ldots m\}$, and $T[j] = \sum_{A[i]=j} t_i$ be the total running time for machine j. The makespan of A is the maximum time that any machine is busy, given by

$$\text{makespan}(A) = \max_j T[j].$$

The LoadBalance problem is to compute the minimal makespan of A. Note that LoadBalance problem is NP-hard. Consider the following algorithm for LoadBalance:

```
for i ← 1 to m do
    T[i] ← 0;
end for
for i ← 1 to n do
    min ← 1;
    for j ← 2 to m do
        if T[j] < T[min] then
            min ← j;
        end if
    end for
    A[i] ← min;
    T[min] ← T[min] + t_i;
end for
return A;
```

Show that this algorithm for LoadBalance is a $\frac{1}{2}$-approximation algorithm, meaning that it returns a solution that is at most $\dfrac{1}{1-\frac{1}{2}} = 2$ times the optimum.

Problem 2 Define $\mathbf{IP}^*$ as $\mathbf{IP}$ except that the prover now runs in deterministic polynomial space instead of exponential time. Show that $\mathbf{IP}^* \subseteq \mathbf{PSPACE}$. (You cannot use the known fact $\mathbf{IP} = \mathbf{PSPACE}$.)