

Theory of Computation

Homework 5

Due: 2013/12/24

Problem 1 Prove or disprove the claim that all languages which have polynomial circuit are in PSPACE.

Proof: Polynomial circuits can accept undecidable languages, but PSPACE contains only decidable languages. So the statement is false. ■

Problem 2 Let X be a random variable with mean μ_X and standard deviation σ_X . Recall that $\sigma_X^2 = \mathbf{E}(X - \mu_X)^2$. Show that for any $t \in \mathbb{R}^+$,

$$\Pr[X - \mu_X \geq t\sigma_X] \leq \frac{1}{1 + t^2}.$$

Proof: By Markov's inequality,

$$\begin{aligned} \Pr[X - \mu_X \geq t\sigma_X] &\leq \Pr[X - \mu_X \geq (t - 1)\sigma_X] \\ &= \Pr[(X - \mu_X)^2 \geq (t - 1)^2\sigma_X^2] \\ &\leq \frac{\mathbf{E}(X - \mu_X)^2}{(t - 1)^2\sigma_X^2} \\ &\leq \frac{1}{(t - 1)^2} \\ &\leq \frac{1}{1 + t^2}. \end{aligned}$$

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