# Theory of Computation

Homework 2
Due: 2012/10/23

**Problem 1.** Let

$$\phi \equiv ((a \wedge \neg b) \vee (\neg c \wedge d)) \Rightarrow (e \Rightarrow \neg f).$$

**(a)** Turn $\phi$ into a CNF.

**(b)** Draw a Boolean circuit for your CNF of $\phi$.

**Ans:**

**(a)** As $\phi_1 \Rightarrow \phi_2 \equiv \neg\phi_1 \vee \phi_2$,

$$\phi = \neg((a \wedge \neg b) \vee (\neg c \wedge d)) \vee (\neg e \vee \neg f).$$

By De Morgan's laws,

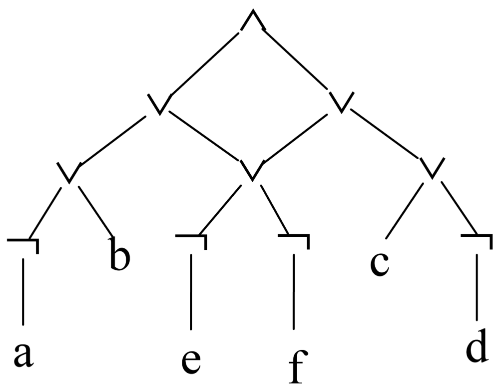
$$\begin{aligned}\phi &= (\neg(a \wedge \neg b) \wedge \neg(\neg c \wedge d)) \vee (\neg e \vee \neg f) \\ &= ((\neg a \vee b) \wedge (c \vee \neg d)) \vee (\neg e \vee \neg f) \\ &= ((\neg a \vee b) \vee (\neg e \vee \neg f)) \wedge ((c \vee \neg d) \vee (\neg e \vee \neg f)).\end{aligned}$$

Finally, a CNF of $\phi$ is[1]

$$\phi = (\neg a \vee b \vee \neg e \vee \neg f) \wedge (c \vee \neg d \vee \neg e \vee \neg f).$$

**(b)** A Boolean circuit is as follows:

[1] Your CNF may take a different but equivalent form.




**Problem 2.** We know that the halting problem

$$H = \{M; x : M(x) \neq \nearrow\}$$

is undecidable. Use this fact to prove that the following language is undecidable:

$$L = \{M : M \text{ is a TM that accepts some input}\}.$$

*Proof.* We prove that $L$ is undecidable by reducing $H$ to $L$. Suppose $L$ is decidable. For a question about the membership of $M$; $x$ in $H$, we construct a TM $M_x$ that simulates $M$ on $x$. If $M$ halts, then $M_x$ accepts; otherwise, $M_x$ rejects. Note that if $M$ halts on $x$, then $M_x$ accepts all inputs; otherwise, $M_x$ always diverges. In other words, $M; x \in H$ if and only if $M_x \in L$. So if $L$ were decidable, $H$ would be decidable, a contradiction. Hence, $L$ is undecidable. $\square$