Theory of Computation

Solutions to Homework 1

Problem 1. Given a directed graph $G = (V, E)$, a function $f : V \to \{1, 2, 3\}$ and $a, b \in V$, let the COLORED REACHABILITY problem ask whether G contains a path P from a to b such that each edge (u, v) on P satisfies $f(u) \neq f(v)$. Is COLORED REACHABILITY $\in$ P true? You may want to use the fact that REACHABILITY can be solved in polynomial time by breadth-first or depth-first search.

Proof. Yes because COLORED REACHABILITY can be solved by running a breadth-first search on the graph $G' = (V, \{(u, v) \in E \mid f(u) \neq f(v)\})$ to see whether G' contains a path from a to b. □

Problem 2. Let $\mathbb{N}$ be the set of natural numbers. Does there exist a bijection between $2^{\mathbb{N}}$ and NP?

Proof. NP is countable because there are countably many Turing machines. So no bijections exist between the countable set NP and the uncountable $2^{\mathbb{N}}$. □