Theory of Computation

Solutions to Homework 2

Problem 1. Let $L \subseteq \{0,1\}^*$ belong to $\mathrm{TIME}(2^{(n^{10})})$ and $L' \stackrel{\mathrm{def}}{=} \{x\,0^{|x|^{100}} \mid x \in L\}$ where $x\,0^{|x|^{100}}$ denotes the concatenation of x and an $|x|^{100}$ number of 0's. Show that $L' \in \mathrm{TIME}(2^n)$.

Proof. Given any $y \in \{0,1\}^*$, one can determine in $O(2^{|y|})$ time an $x \in \{0,1\}^*$ with $y = x\,0^{|x|^{100}}$, or that such an x does not exist. Then $y \in L'$ if and only if $x \in L$, whose validity can be determined in time $O(2^{(|x|^{10})}) = O(2^{|y|})$. $\square$

Problem 2. Let M be a nondeterministic polynomial-time Turing machine with alphabet set Σ. For each $x \in (\Sigma \setminus \{\sqcup\})^*$, denote by $C(M,x)$ the set of configurations that can be yielded in any number of steps from the initial configuration of M on x. Suppose that A is a deterministic polynomial-time Turing machine that, given any $x \in (\Sigma \setminus \{\sqcup\})^*$, outputs a set $S(x)$ with $C(M,x) \subseteq S(x)$. Show that $L(M) \in \mathrm{P}$.

Proof. We can decide $L(M)$ in polynomial time as follows. Given an $x \in (\Sigma \setminus \{\sqcup\})^*$, we use A to compute $S(x)$. Then we consider the directed graph G on $S(x)$ where there is an edge from $c_1 \in S(x)$ to $c_2 \in S(x)$ if c_1 and c_2 are configurations of M on x and c_1 yields c_2. Then $x \in L(M)$ if and only if there is a path in G from the initial configuration of M on x to any accepting configuration. So $L(M) \in \mathrm{P}$ follows from the fact that the depth-first search algorithm runs in polynomial time. $\square$