

# Theory of Computation

Mid-Term Examination on April 17, 2008

Spring Semester, 2008

**Problem 1** (15 points). Please answer the following questions.

1. [5 points] Is  $\text{EXP} \subseteq \text{NEXP}$  true?
2. [5 points] Does there exist a language in NP decidable in time linear in the input length?
3. [5 points] In 1975, Richard Ladner proved the famous theorem that if  $\text{NP} \neq \text{P}$ , then there is a language in NP that is neither in P nor NP-complete. Is the converse of the theorem true? That is, if some language in NP is neither in P nor NP-complete, can we conclude  $\text{NP} \neq \text{P}$ ?

**Problem 2** (20 points). Show that there exists a language  $L \subseteq \{0, 1\}^*$  that belongs neither to RE nor to coRE.

**Problem 3** (20 points). Show that there exists a language in NPSpace that is not decidable in time cubic in the input length.

**Problem 4** (20 points). Does there exist a non-recursive language in NP?

**Problem 5** (20 points). Show that it is NP-hard to determine whether a Boolean expression in conjunctive normal form has at least two satisfying assignments. (Hint: Consider adding a clause  $C$  to a Boolean expression  $F$  in conjunctive normal form where the variables in  $C$  do not appear in  $F$ .)

**Problem 6** (20 points). Prove that  $\text{NSpace}(\log^2 n) \subseteq \text{Time}(2^{(\log^5 n)})$ .