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Function Problems Cannot Be Easier than Decision Problems

- If we know how to generate a solution, we can solve the corresponding decision problem.
 - If you can find a satisfying truth assignment efficiently, then SAT is in P.
 - If you can find the best TSP tour efficiently, then TSP
 (D) is in P.
- But decision problems can be as hard as the corresponding function problems.

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Function Problems

- Decisions problem are yes/no problems (SAT, TSP (D), etc.).
- Function problems require a solution (a satisfying truth assignment, a best TSP tour, etc.).
- Optimization problems are clearly function problems.
- What is the relation between function and decision problems?
- Which one is harder?

FSAT

- FSAT is this function problem:
 - Let $\phi(x_1, x_2, \ldots, x_n)$ be a boolean expression.
 - If ϕ is satisfiable, then return a satisfying truth assignment.
 - Otherwise, return "no."
- We next show that if $SAT \in P$, then FSAT has a polynomial-time algorithm.

An Algorithm for FSAT Using SAT

1: $t := \epsilon;$

```
2: if \phi \in \text{SAT} then
 3: for i = 1, 2, ..., n do
          if \phi[x_i = \text{true}] \in \text{SAT} then
 4:
            t := t \cup \{ x_i = \texttt{true} \};
 5:
             \phi := \phi[x_i = \texttt{true}];
 6:
 7:
           else
 8:
             t := t \cup \{ x_i = \texttt{false} \};
 9:
             \phi := \phi[x_i = \texttt{false}];
           end if
10:
       end for
11:
12:
       return t:
13: else
       return "no";
14:
15: end if
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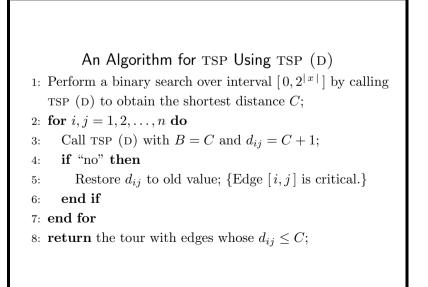
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TSP and TSP (D) Revisited

- We are given *n* cities 1, 2, ..., n and integer distances $d_{ij} = d_{ji}$ between any two cities *i* and *j*.
- The TSP asks for a tour with the shortest total distance (not just the shortest total distance, as earlier).
 - The shortest total distance must be at most $2^{|x|}$, where x is the input.
- TSP (D) asks if there is a tour with a total distance at most *B*.
- We next show that if TSP $(D) \in P$, then TSP has a polynomial-time algorithm.

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Analysis

- There are $\leq n+1$ calls to the algorithm for SAT.^a
- Shorter boolean expressions than ϕ are used in each call to the algorithm for SAT.
- So if SAT can be solved in polynomial time, so can FSAT.
- Hence SAT and FSAT are equally hard (or easy).

^aContributed by Ms. Eva Ou (R93922132) on November 24, 2004.

Analysis

- An edge that is not on *any* optimal tour will be eliminated, with its d_{ij} set to C + 1.
- An edge which is not on all remaining optimal tours will also be eliminated.
- So the algorithm ends with *n* edges which are not eliminated (why?).
- There are $O(|x| + n^2)$ calls to the algorithm for TSP (D).
- So if TSP (D) can be solved in polynomial time, so can TSP.
- Hence TSP (D) and TSP are equally hard (or easy).

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